National Qualifications 2016

2016 Mathematics Paper 2

National 5

Finalised Marking Instructions

Detailed Marking Instructions for each question

Question			Generic Scheme	Illustrative Scheme	Max Mark
1.			**Ans: 27·(25408) grams** • [1] know how to decrease by 8% • [2] know how to calculate the sugar content after 3 years • [3] evaluate	 • [1] $\times 0\cdot 92$ • [2] $35\times 0\cdot 92^3$ • [3] $27\cdot(25408)$ (grams)	**3**

Notes:

1. Correct answer without working award 3/3
2. Do not penalise incorrect rounding
3. Where an incorrect percentage is used, the working must be followed through to give the possibility of awarding 2/3
   eg For $35\times 0\cdot 08^3 = 0\cdot 01792$, with working award 2/3 ×✓✓
4. Where division is used,
   (a) along with $0\cdot 92$, •[1] is not available
   eg $35\div 0\cdot 92^3 = 44\cdot 94...$ award 2/3 ×✓✓
   (b) along with an incorrect percentage, •[1] and •[2] are not available
   eg $35\div 1\cdot 08^3 = 27\cdot 78...$ award 1/3 ××✓

Commonly Observed Responses:

Working must be shown

1. $35\times 1\cdot 08^3 = 44\cdot 0...$ award 2/3 ×✓✓
2. $35\times 0\cdot 08 = 2\cdot 8 \rightarrow 35 - 3\times 2\cdot 8 = 26\cdot 6$ award 1/3 ✓××
3. $35\times 0\cdot 92 = 32\cdot 2$ award 1/3 ✓××
4. $35\times 0\cdot 92\times 3 = 96\cdot 6$ award 1/3 ✓××
5. $35\times 0\cdot 08\times 3 = 8\cdot 4$ award 0/3

Question			Generic Scheme	Illustrative Scheme	Max Mark
2.			**Ans: 8×10^{-9} grams** $\bullet^1$ correct method $\bullet^2$ answer	 $\bullet^1$ $12\div\left(1\cdot5\times10^{9}\right)$ $\bullet^2$ 8×10^{-9}	**2**

Notes:

1. Correct answer without working award 2/2
2. $\bullet^2$ is still available if there is additional multiplication or division by 1000 (but by no other numbers).
   eg award 1/2 ×✓ for
   (a) $12\div\left(1\cdot5\times10^{9}\right)\div1000=8\times10^{-12}$
   (b) $\left(1\cdot5\times10^{9}\right)\div12\times1000=1\cdot25\times10^{11}$

Commonly Observed Responses:

No working necessary

1. $(1\cdot5\times10^{9})\div12=1\cdot25\times10^{8}$ award 1/2 ×✓
2. $\left(1\cdot5\times10^{9}\right)\div12=1\cdot2\times10^{8}$ or $1\cdot3\times10^{8}$ award 1/2 ×✓
3. $(1\cdot5\times10^{9})\times12=1\cdot8\times10^{10}$ award 1/2 ×✓

Question			Generic Scheme	Illustrative Scheme	Max Mark
3.			**Ans: v − u** •[1] correct answer	 •[1] **v** − **u** or -**u** + **v** or **v** + − **u**	**1**
Notes:					
Commonly Observed Responses:					
4.			**Ans:** $3(x+4)(x-4)$ •[1] begin to factorise •[2] factorise fully	 •[1] $3(x^2-16)$ •[2] $3(x+4)(x-4)$	**2**
Notes: 1. Correct answer without working award 2/2 2. •[1] is also available for $(3x+12)(x-4)$ or $(3x-12)(x+4)$ 3. •[1] is not available for 3 **or** (x^2-16) alone 4. All three factors must be shown **together** to obtain •[2]. 5. Special cases (a) award 1/2 for $3(x-4)^2$ or $(x+4)(x-4)$ or $3(x+8)(x-8)$ (b) award 0/2 for eg $(3x-8)(x+6)$					
Commonly Observed Responses:					
5.			**Ans: ABC = 74°** •[1] calculate the size of angle AOE or CAO •[2] calculate the size of angle CAB •[3] calculate the size of angle ABC	 •[1] 37 •[2] 53 •[3] 74	**3**
Notes: 1. Full marks may be awarded for information marked on the diagram 2. For an answer of 74° with no **relevant** working award 0/3 3. •[3] is available for correct calculation of 180 − 2 × angle CAB					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.	(a)		**Ans: mean = 13 minutes, st dev = 5·7 minutes**		**4**
			•[1] calculate mean	•[1] 13 (minutes)	
			•[2] calculate $\left(x-\bar{x}\right)^2$	•[2] 0, 9, 9, 81, 64, 1	
			•[3] substitute into formula	•[3] $\sqrt{\frac{164}{5}}$	
			•[4] calculate standard deviation	•[4] 5·7… (minutes)	
Notes: 1. For an answer of 13 and 5·7 without working award 1/4 ✓××××. 2. For use of alternative formula award •[2], •[3] and •[4] as follows: •[2] calculate $\sum x$ and $\sum x^2$ •[2] 78, 1178 •[3] substitute into formula •[3] $\sqrt{\frac{1178-\frac{78^2}{6}}{5}}$ •[4] calculate standard deviation •[4] 5·7… (minutes)					
Commonly Observed Responses:					
	(b)		**Ans: valid statements**		**2**
			•[1] compare means	•[1] On average Sophie's waiting time was longer.	
			•[2] compare standard deviations	•[2] Sophie's waiting times were more consistent.	

Question	Generic Scheme	Illustrative Scheme	Max Mark

Notes:

1. Answers must be consistent with answers to part (a).

2. Statements regarding the mean must show an understanding that mean is an average.
   (a) eg Accept
   - Sophie's average waiting time is more
   - In general her time is more
   - Sophie's waiting time is more overall

   (b) eg Do not accept
   - Sophie's mean waiting time is more
   - Sophie's waiting time is longer (this implies that all her waiting times are longer)

3. Statements regarding the standard deviation must show an understanding that standard deviation is a measure of spread.
   (a) eg Accept
   - The spread of Sophie's times is less
   - Sophie's times are more consistent
   - Her waiting is less varied

   (b) eg Do not accept
   - Sophie's standard deviation is less
   - The range of Sophie's times is less
   - On average her waiting times are less varied
   - The standard deviation is more consistent

4. Statements must refer to Sophie/Jack or she/he
   eg do not accept "on average the waiting time was longer".

5. Accept statements using 'waiting time', 'call time', 'time' or 'waiting'.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.			**Ans: 5300 cubic centimetres**		**5**
			•[1] know to find difference in two volumes	•[1] evidence of difference in two volumes	
			•[2] substitute correctly into formula for volume of large cone	•[2] $\frac{1}{3}\times\pi\times16^2\times24\ \ (=6433\cdot98\ldots)$	
			•[3] substitute correctly into formula for volume of small cone	•[3] $\frac{1}{3}\times\pi\times9^2\times13\cdot5\ \ (=1145\cdot11\ldots)$	
			•[4] carry out all calculations correctly (must involve difference or sum of two volume calculations and include a fraction)	•[4] $5288\cdot87\ldots$	
			•[5] round **final answer** to 2 significant figures **and state correct units**	•[5] $5300\,\text{cm}^3$	

Notes:

1. Correct answer without working award 0/5.
2. Accept variations in π.
   eg $\frac{1}{3}\times3\cdot14\times16^2\times24-\frac{1}{3}\times3\cdot14\times9^2\times13\cdot5=6430\cdot72-1144\cdot53=5286\cdot19=5300\,\text{cm}^3$
3. In awarding •[5]
   (a) Intermediate calculations need not be shown
   eg $\frac{1}{3}\times\pi\times16^2\times24-\frac{1}{3}\times\pi\times9^2\times13\cdot5=5300\,\text{cm}^3$ award 5/5
   (b) Where intermediate calculations are shown, they must involve at least three significant figures
   eg $6433\cdot98\ldots-1145\cdot11\ldots=6400-1100=5300\,\text{cm}^3$ award 4/5 ✓✓✓✓×
   (c) Where the volume of **only one** cone is calculated •[5] is available
   eg $\frac{1}{3}\times\pi\times16^2\times24=6400\,\text{cm}^3$ award 2/5 ×✓××✓
   (d) Accept 5300 ml or 5·3 litres.

Question	Generic Scheme	Illustrative Scheme	Max Mark
Commonly Observed Responses: **Working must be shown** 1. $\frac{1}{3}\times\pi\times16^2\times24+\frac{1}{3}\times\pi\times9^2\times13\cdot5=7600\,\text{cm}^3$ award 4/5 ×✓✓✓✓ 2. $\frac{1}{3}\times\pi\times32^2\times24-\frac{1}{3}\times\pi\times18^2\times13\cdot5=21000\,\text{cm}^3$ award 4/5 ✓×✓✓✓ 3. $\frac{1}{3}\times\pi\times16^2\times24-\frac{1}{3}\times\pi\times9^2\times10\cdot5=5500\,\text{cm}^3$ award 4/5 ✓✓×✓✓ 4. $\frac{1}{3}\times\pi\times16^2\times24+\frac{1}{3}\times\pi\times9^2\times10\cdot5=7300\,\text{cm}^3$ award 3/5 ×✓×✓✓ 5. $\frac{1}{3}\times\pi\times16^2\times10\cdot5-\frac{1}{3}\times\pi\times9^2\times13\cdot5=1700\,\text{cm}^3$ award 4/5 ✓×✓✓✓ 6. $\frac{1}{3}\times\pi\times16^2\times10\cdot5+\frac{1}{3}\times\pi\times9^2\times13\cdot5=4000\,\text{cm}^3$ award 3/5 ××✓✓✓ 7. $\pi\times16^2\times24-\pi\times9^2\times13\cdot5=16000\,\text{cm}^3$ award 3/5 ✓×✓×✓ 8. $\frac{4}{3}\times\pi\times16^3-\frac{4}{3}\times\pi\times9^3=14000\,\text{cm}^3$ award 3/5 ✓××✓✓		

Question			Generic Scheme	Illustrative Scheme	Max Mark
8.			**Ans: 78°** • [1] correct substitution into sine rule • [2] re-arrange formula • [3] find x	 • [1] $\frac{\sin x}{150}=\frac{\sin 66}{140}$ or $\frac{150}{\sin x}=\frac{140}{\sin 66}$ • [2] $\sin x=\frac{150\sin 66}{140}$ • [3] $x=78(\cdot 18\ldots)$	**3**

Notes:

1. Correct answer without working award 0/3.
2. Do not penalise incorrect rounding in the final answer
   eg $\sin x=\frac{150\sin 66}{140}=0\cdot 978\ \rightarrow x=77\cdot 9$ award 3/3
3. Premature rounding: rounded working must be to at least 2 decimal places.
4. Premature truncation: truncated working must be to at least 3 decimal places.
5. $\pm 0\cdot 028\ldots$ (uses rad) award 2 marks (working must be shown)
6. $75, 74\cdot 72\ldots$ (uses grad) award 3 marks (working must be shown)

Commonly Observed Responses:

1. Examples of premature rounding/truncation (apply notes 2 and 3)

(a) Premature rounding:

$\sin x=\frac{150\sin 66}{140}$

$=0\cdot 98\rightarrow x=78\cdot 5$

award 3/3

(b)Premature rounding:

$\sin x=\frac{150\sin 66}{140}$

$=\frac{150\times 0\cdot 9}{140}$

$=0\cdot 964\ldots\rightarrow x=74\cdot 6$

award 2/3 ✓✓×

(c) Premature truncation:

$\sin x=\frac{150\sin 66}{140}\ (=0\cdot 978\ldots)$

$=0\cdot 97\rightarrow x=75\cdot 9$

award 2/3 ✓✓×

Question			Generic Scheme	Illustrative Scheme	Max Mark
9.			**Ans:** $(x+4)^2-23$ • [1] correct bracket with square • [2] complete process	 • [1] $(x+4)^2$ • [2] $(x+4)^2-23$	**2**

Notes:
1. Correct answer without working award 2/2

Commonly Observed Responses:
No working necessary:

1. Award 2/2 for (a) $(x+4)^2+(-23)$ or $(x+4)^2+-23$
   (b) $(x+4)(x+4)-23$
2. Award 1/2 ×✓ for (a) $(x+4)-23$
   (b) $(x^2+4)-23$
   (c) $(x^2+4)^2-23$
   (d) $(x+4x)^2-23$
   (e) $(x+8)^2-71$
3. Award 0/2 for eg $(x+8)^2-23$

Question			Generic Scheme	Illustrative Scheme	Max Mark
10.			**Ans:** $\frac{1}{n^4}$ **Method 1** • [1] simplify $\left(n^2\right)^3$ • [2] simplify $n^6 \times n^{-10}$ • [3] express with a positive power **Method 2** • [1] simplify $\left(n^2\right)^3$ • [2] express with a positive power • [3] simplify $n^6 \times \frac{1}{n^{10}}$	 • [1] n^6 • [2] n^{-4} • [3] $\frac{1}{n^4}$ • [1] n^6 • [2] $\frac{1}{n^{10}}$ • [3] $\frac{1}{n^4}$	**3**
Notes: 1. Correct answer without working award 3/3					
Commonly Observed Responses:					

Question	Generic Scheme	Illustrative Scheme	Max Mark
11.	**Ans:** £4·95 **Method 1** • [1] linear scale factor • [2] know to multiply cost by the square of the linear scale factor • [3] find cost of smaller picture (calculation must involve a power of the scale factor) **Method 2** • [1] linear scale factor • [2] know to divide cost by the square of the linear scale factor • [3] find cost of smaller picture (calculation must involve a power of the scale factor)	 • [1] $\frac{60}{100}$ • [2] $13\cdot75\times\left(\frac{60}{100}\right)^2$ • [3] (£)4·95 • [1] $\frac{100}{60}$ • [2] $13\cdot75\div\left(\frac{100}{60}\right)^2$ • [3] (£)4·95	

Notes:
1. Correct answer without working award 3/3
2. Disregard incorrect units or omission of units
3. Answer must be rounded to nearest penny if required.

Commonly Observed Responses:

1. $13\cdot75\times\frac{60}{100}=8\cdot25$ award 1/3 ✓××
2. $13\cdot75\times\left(\frac{60}{100}\right)^3=2\cdot97$ award 2/3 ✓×✓
3. $(13\cdot75)^2\times\frac{60}{100}=113\cdot44$ award 1/3 ✓××
4. $13\cdot75\times\left(\frac{100}{60}\right)^2=38\cdot19$ award 2/3 ✓×✓
5. $13\cdot75\div\left(\frac{100}{60}\right)^2=13\cdot75\div1\cdot67^2=4\cdot93$ award 2/3 ✓✓×
   (Premature rounding leads to inaccurate answer)
6. $13\cdot75\times\left(\frac{100}{60}\right)^2=13\cdot75\times1\cdot67^2=38\cdot35$ award 1/3 ✓××
   (Premature rounding leads to inaccurate answer)

Question			Generic Scheme	Illustrative Scheme	Max Mark
12.			**Ans:** $k = \dfrac{L^2 + p}{4t}$ • [1] square • [2] add p • [3] divide by $4t$	 • [1] $L^2 = 4kt - p$ • [2] $4kt = L^2 + p$ • [3] $k = \dfrac{L^2 + p}{4t}$	**3**

Notes:

1. Correct answer without working award 3/3.
2. Final answer should be in simplest form

   (a) $\dfrac{1}{4}\left(\dfrac{L^2 + p}{t}\right)$ award 3/3

   (b) $\dfrac{\left(\dfrac{L^2 + p}{t}\right)}{4}$ award 2/3 ✓✓×
3. For subsequent incorrect working, •[3] is not available.

Commonly Observed Responses:

1. For the response below award 1/3

- add p $\quad L + p = \sqrt{4kt}$ ×
- divide by 4t $\quad \dfrac{L + p}{4t} = \sqrt{k}$ ×
- square $\quad k = \left(\dfrac{L + p}{4t}\right)^2$ ✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
13.	**Ans:** $\dfrac{8x-7}{(x-2)(x+1)}$		**3**
	• [1] correct denominator	• [1] $(x-2)(x+1)$	
	• [2] correct numerator	• [2] $3(x+1)+5(x-2)$	
	• [3] remove brackets and collect like terms in numerator	• [3] $\dfrac{8x-7}{(x-2)(x+1)}$	

Notes:

1. Correct answer without working award 3/3.
2. Accept $\dfrac{3(x+1)}{(x-2)(x+1)}+\dfrac{5(x-2)}{(x-2)(x+1)}$ for the award of •[1] and •[2].
3. Do **not** accept $x-2(x+1)$ or $x+1(x-2)$ for the award of •[1] unless the correct expansion appears in the final answer.
4. Where a candidate chooses to expand the brackets in the denominator, then •[1] is only available for a correct expansion.

   eg (a) $\dfrac{3(x+1)}{(x-2)(x+1)}+\dfrac{5(x-2)}{(x-2)(x+1)}=\dfrac{8x-7}{x^2-x-2}$ award 3/3

   (b) $\dfrac{3(x+1)}{(x-2)(x+1)}+\dfrac{5(x-2)}{(x-2)(x+1)}=\dfrac{8x-7}{x^2-2}$ award 2/3 ✓✓×

   (c) $\dfrac{3(x+1)}{x^2-2}+\dfrac{5(x-2)}{x^2-2}=\dfrac{8x-7}{x^2-2}$ award 2/3 ×✓✓

Commonly Observed Responses:

1. $\dfrac{3x+1}{(x-2)(x+1)}+\dfrac{5x-2}{(x-2)(x+1)}=\dfrac{8x-1}{(x-2)(x+1)}$ award 1/3 ✓××

Question			Generic Scheme	Illustrative Scheme	Max Mark
14.			**Ans: x = 102·5, 282·5** •[1] rearrange equation •[2] find one value of x •[3] find another value of x	•[1] $\tan x = -\frac{9}{2}$ •[2] x = 102·5 •[3] x = 282·5	**3**

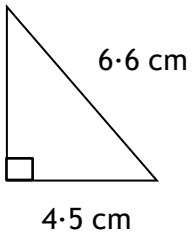
Notes:
1. Correct answer without working award 2/3
2. For x = 178·6, 358·6 (uses RAD), award 3/3 (with working), 2/3 (without working)
3. For x = 93·9, 273·9 (uses GRAD), award 3/3 (with working), 2/3 (without working)
4. Do not penalise omission of degree signs throughout the question

Commonly Observed Responses:

1. If $\tan x^\circ < 0$ then award •[2] and •[3] for correct 2nd and 4th quadrant angles
   eg $\tan x = -\frac{9}{2} \rightarrow$ (a) $x = 77\cdot5, 102\cdot5$ award 2/3 ✓×✓
   (b) $x = 77\cdot5, 282\cdot5$ award 2/3 ✓×✓
   (c) $x = 77\cdot5, 257\cdot5$ award 1/3 ✓××

2. If $\tan x > 0$ then •[2] is not available (working eased) but award •[3] for correct 3rd quadrant angle eg $\tan x^\circ = \frac{9}{2} \rightarrow$ (a) $x = 77\cdot5, 257\cdot5$ award 1/3 ××✓
   (b) $x = 77\cdot5, 102\cdot5$ award 0/3
   (c) $x = 77\cdot5, 282\cdot5$ award 0/3
   (d) $\tan x^\circ = \frac{1}{2} \rightarrow x = 26\cdot6, 206\cdot6$ award 1/3 ××✓

3. $\tan x^\circ = -\frac{9}{2} \rightarrow x = -77\cdot5$
   (a) $x = 257\cdot5\ \left[180 - (-77\cdot5)\right],\ 437\cdot5\ \left[360 - (-77\cdot5)\right]$ award 1/3 ✓××
   (incorrect application of CAST diagram and $437\cdot5 > 360$)
   (b) $x = 102\cdot5\ \left[-77\cdot5 + 180\right],\ 282\cdot5\ \left[102\cdot5 + 180\right]$ award 3/3
   (correct application of periodicity of $\tan x^\circ$)

Question			Generic Scheme	Illustrative Scheme	Max Mark
15.			**Ans: 11·4… (cm)** •[1] marshal facts and recognise right-angled triangle	•[1] 	**4**
			•[2] correct Pythagoras statement	•[2] $x^2 = 6 \cdot 6^2 - 4 \cdot 5^2$	
			•[3] correct calculation of x	•[3] 4·8…	
			•[4] find height of label	•[4] 11·4… (cm)	

Notes:

1. For correct answer without working award 0/4
2. •[4] is for adding 6·6 to a previously calculated value
3. In the absence of a diagram accept $x^2 = 6 \cdot 6^2 - 4 \cdot 5^2$ as evidence for the award of •[1] and •[2].
4. Where a candidate assumes an angle of 45° in the right-angled triangle, only •[1] and •[4] are available.

Commonly Observed Responses:

1. For $x^2 = 6 \cdot 6^2 + 4 \cdot 5^2 \rightarrow x = 7 \cdot 988... \rightarrow$ height $= 14 \cdot 588...$
   (a) with correct diagram award 3/4 ✓×✓✓
   (b) without a diagram award 2/4 ××✓✓

Question			Generic Scheme	Illustrative Scheme	Max Mark
16.			**Ans:** $6\cdot8$ **cm** $\bullet^1$ identify $\cos A$ or angle A $\bullet^2$ substitute into cosine rule (**cos A or angle A must have been found using trigonometry**) $\bullet^3$ calculate BC^2 $\bullet^4$ calculate BC correct to one decimal place	$\bullet^1$ $\cos A = \frac{3}{4}$ or $A = 41\cdot4$ $\bullet^2$ $BC^2 = 6^2 + 10^2 - 2\times6\times10\times\frac{3}{4}$ **or** $BC^2 = 6^2 + 10^2 - 2\times6\times10\times\cos41\cdot4$ $\bullet^3$ $BC = 46$ $\bullet^4$ $BC = 6\cdot8$ (cm)	**4**

Notes:

1. Correct answer without working award 0/4
2. Do not accept the substitution of a length or the value of sin A or tan A in place of angle A in the cosine rule.
3. $\bullet^3$ and $\bullet^4$ are only available for calculations within a valid strategy
4. Alternative valid strategies:

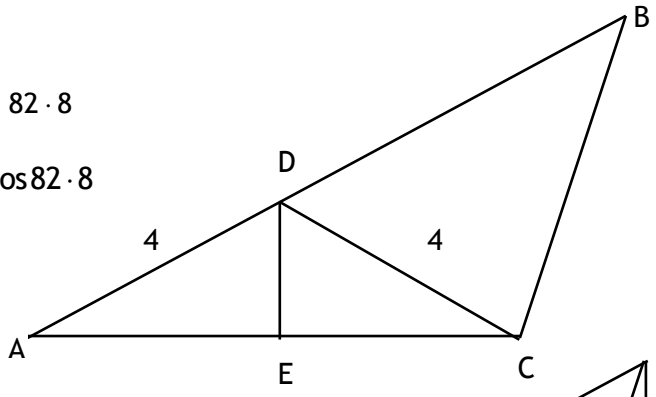
(a) $\bullet^1$ $ADE = \sin^{-1}\left(\frac{3}{4}\right) = 48\cdot6$

$\Rightarrow BDC = 180 - 2\times48\cdot6 = 82\cdot8$

$\bullet^2$ $BC^2 = 6^2 + 4^2 - 2\times6\times4\times\cos82\cdot8$

$\bullet^3$ $BC^2 = 45\cdot984...$

$\bullet^4$ $BC = 6\cdot8$ (cm)

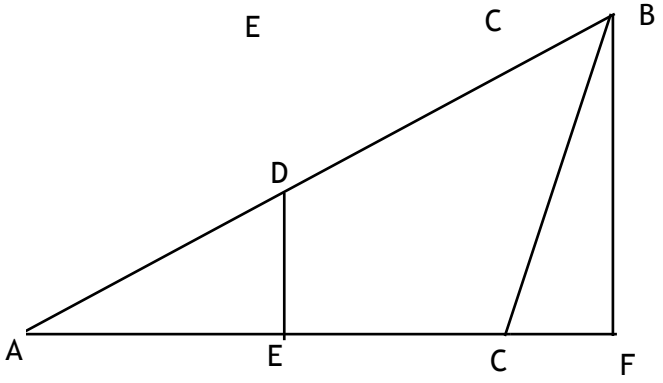


(b) $\bullet^1$ $AF = \frac{10}{4}\times3 = 7\cdot5$

$\bullet^2$ $BF^2 = 10^2 - 7\cdot5^2 = 43\cdot75$

$\bullet^3$ $BC^2 = 43\cdot75 + 1\cdot5^2 = 46$

$\bullet^4$ $BC = 6\cdot8$ (cm)



5. If premature rounding leads to an answer other than 6·8 then $\bullet^4$ is not available.

Commonly Observed Responses:

1. $DE^2 = 4^2 - 3^2 = 7 \rightarrow DE = 2\cdot6$ award 0/4
2. $BC^2 = 6^2 - 4^2 = 20 \rightarrow BC = 4\cdot5$ award 0/4
   [incorrectly assuming that angle BCD = 90° in note 4(a) diagram]

[END OF MARKING INSTRUCTIONS]