National Qualifications 2016

2016 Mathematics Paper 1 (Non-calculator)

National 5

Finalised Marking Instructions

Detailed Marking Instructions for each question

Question	Generic Scheme	Illustrative Scheme	Max Mark
1.	**Ans:** $\begin{pmatrix}-3\\-4\end{pmatrix}$ $\bullet^1$ calculate $\frac{1}{2}\mathbf{p}$ $\bullet^2$ solution	 $\bullet^1$ $\begin{pmatrix}2\\-3\end{pmatrix}$ $\bullet^2$ $\begin{pmatrix}-3\\-4\end{pmatrix}$	**2**

Notes:

1. Correct answer without working award 2/2
2. Treat $\begin{pmatrix}\frac{-3}{-4}\end{pmatrix}$ as bad form award 2/2
3. Where there are no brackets ie $\begin{matrix}-3\\-4\end{matrix}$ award 1/2
4. For $\frac{-3}{-4}$ award 1/2
5. Where there is invalid subsequent working $\bullet^2$ is not available
   eg award 1/2 ✓× for the following:

   $\begin{pmatrix}-3\\-4\end{pmatrix}\rightarrow$ (a) $(-3,-4)$

   (b) $-3+(-4)=-7$

   (c) $\sqrt{(-3)^2+(-4)^2}=5$

Commonly Observed Responses:

1. $\begin{pmatrix}4\\-6\end{pmatrix}+\begin{pmatrix}-5\\-1\end{pmatrix}=\begin{pmatrix}-1\\-7\end{pmatrix}$ award 1/2

2. $\frac{1}{2}\left(\begin{pmatrix}4\\-6\end{pmatrix}+\begin{pmatrix}-5\\-1\end{pmatrix}\right)=\begin{pmatrix}-0{\cdot}5\\-3{\cdot}5\end{pmatrix}$ award 1/2

3. $\begin{pmatrix}4\\-6\end{pmatrix}+\frac{1}{2}\begin{pmatrix}-5\\-1\end{pmatrix}=\begin{pmatrix}1{\cdot}5\\-6{\cdot}5\end{pmatrix}$ award 1/2

Question			Generic Scheme	Illustrative Scheme	Max Mark
2.			**Ans:** $\frac{13}{28}$ **Method 1** $\bullet^1$ start the calculation correctly $\bullet^2$ consistent answer in simplest form **Method 2** $\bullet^1$ start the calculation correctly $\bullet^2$ consistent answer in simplest form	 $\bullet^1$ $\frac{7}{21}+\frac{6}{21}$ $\bullet^2$ $\frac{13}{28}$ $\bullet^1$ $\frac{3}{12}+\frac{6}{28}$ or equivalent $\bullet^2$ $\frac{13}{28}$	**2**

Notes:

1. Correct answer without working award 0/2.
2. Final answer must be in simplest form eg for $\frac{39}{84}$ award 1/2 ✓×
3. $\bullet^2$ is only available where simplifying is required.
4. For subsequent incorrect working, $\bullet^2$ is not available eg for $\frac{13}{28}=2\frac{2}{28}=2\frac{1}{14}$ award 1/2 ✓×

Commonly Observed Responses:

1. For an answer of $\frac{9}{40}$ obtained from

   (a) Method 1: $\frac{3}{4}\left(\frac{1}{3}+\frac{2}{7}\right)=\frac{3}{4}\times\frac{3}{10}=\frac{9}{40}$ award 0/2

   (b) Method 2: $\frac{3}{12}+\frac{6}{28}=\frac{9}{40}$ award 1/2 ✓×

Question			Generic Scheme	Illustrative Scheme	Max Mark
3.			**Ans: 157 cm^2** **Method 1** •^{1} appropriate fraction •^{2} correct substitution into area of sector formula •^{3} calculate area of sector **Method 2** •^{1} appropriate fraction •^{2} correct substitution into area of sector formula •^{3} calculate area of sector	 •^{1} $\frac{45}{360}$ or equivalent •^{2} $\frac{45}{360}\times 3\cdot 14\times 20^2$ •^{3} 157 (cm^2) •^{1} $\frac{360}{45}$ or equivalent •^{2} $3\cdot 14\times 20^2\div\frac{360}{45}$ •^{3} 157 (cm^2)	**3**

Notes:

1. Correct answer without working award 0/3.

2. Accept "$\div 8$" in working as evidence of $\frac{45}{360}$.

3. Accept "$\times$ 3·14" in working as evidence of substitution into formula.

Commonly Observed Responses:

1. $\frac{45}{360}\times\pi r^2 = 8\times 3\cdot 14\times 20^2 = 10048\,(\text{cm}^2)$ award 2/3 ✓✓×

2. $\frac{360}{45}\times\pi r^2 = 8\times 3\cdot 14\times 20^2 = 10048\,(\text{cm}^2)$ award 2/3 ✓×✓

3. $\frac{45}{360}\times 3\cdot 14\times 20^2\left(=\frac{45}{360}\times 3\cdot 14\times 40\right) = 15\cdot 7\,(\text{cm}^2)$ award 2/3 ✓✓×

4. $\frac{45}{360}\times 3\cdot 14\times 40 = 15\cdot 7\,(\text{cm}^2)$ award 2/3 ✓×✓

5. $\frac{45}{360}\times\pi\times 20^2$ award 1/3 ✓××

6. $3\cdot 14\times 20^2 = 1256\,(\text{cm}^2)$ award 0/

Question			Generic Scheme	Illustrative Scheme	Max Mark
4.	(a)		**Ans:** $2c+3d=9\cdot6$ $\bullet^1$ construct equation	 $\bullet^1$ $2c+3d=9\cdot6$	**1**
Notes:					
Commonly Observed Responses:					
	(b)		**Ans:** $3c+4d=13\cdot3$ $\bullet^1$ construct equation	 $\bullet^1$ $3c+4d=13\cdot3$	**1**
Notes:					
Commonly Observed Responses:					
	(c)		**Ans: A cloak requires 1·5 m^2 of material** **A dress requires 2·2 m^2 of material** $\bullet^1$ evidence of scaling $\bullet^2$ follow a valid strategy through to produce values for c and d $\bullet^3$ calculate correct values for c and d $\bullet^4$ communicate answers in **square metres**	 $\bullet^1$ eg $6c+9d=28\cdot8$ $6c+8d=26\cdot6$ $\bullet^2$ values for c and d $\bullet^3$ $c=1\cdot5$ and $d=2\cdot2$ $\bullet^4$ cloak 1·5 m^2 dress 2·2 m^2	**4**
Notes: 1. Correct answer without working award 0/4**.** 2. $\bullet^4$ is not available if either c or d is negative. 3. (a) where a candidate calculates values for c and d, $\bullet^4$ can only be awarded for a **conclusion** containing the words 'cloak' and 'dress' along with the **correct units** in **both** cases (b) where a candidate only calculates a value for **either** c **or** d, $\bullet^4$ can only be awarded If the **conclusion** contains the word 'cloak' or 'dress' along with the **correct units**					
Commonly Observed Responses:					

Question			Generic Scheme	Illustrative Scheme	Max Mark
5.	(a)		**Ans:** $W = 20A + 40$		**3**
			$\bullet^1$ gradient	$\bullet^1$ $\frac{240}{12}$ or equivalent	
			$\bullet^2$ substitute gradient and a point into $y = mx + c$ or $y - b = m(x - a)$	$\bullet^2$ $y - 100 = \frac{240}{12}(x - 3)$ or $y - 340 = \frac{240}{12}(x - 15)$ or $100 = \frac{240}{12} \times 3 + c$ or $340 = \frac{240}{12} \times 15 + c$	
			$\bullet^3$ state equation in terms of W and A **and** in simplest form (remove any brackets and collect constants)	$\bullet^3$ $W = 20A + 40$ or equivalent	

Notes:

1. Correct answer without working award 3/3.
2. $\bullet^3$ is not available for invalid subsequent working
   eg $W = 20A + 40 \rightarrow W = 2A + 4$ award 2/3 ✓✓×
3. Where $\frac{240}{12}$ is simplified incorrectly $\bullet^2$ is still available
   eg $m = \frac{240}{12} = \frac{20}{3} \rightarrow y - 100 = \frac{20}{3}(x - 3) \rightarrow W = \frac{20}{3}A + 80$ award 2/3 ✓✓×

Commonly Observed Responses:

1. $y = 20x + 40$ award 2/3 ✓✓×
2. $y = 20x$ award 1/3 ✓××
3. $W = \frac{20}{1}A + 40$ award 2/3 ✓✓×
4. $y - 100 = 20x - 3 \rightarrow W = 20A + 97$ award 2/3 ✓×✓

	(b)		**Ans:** $20 \times 12 + 40 = 280$ kg		**1**
			$\bullet^1$ calculate weight using equation from part (a)	$\bullet^1$ $20 \times 12 + 40 = 280$ **(kg)** stated explicitly	

Notes:

1. Correct answer without working award 0/1
2. Follow through mark from part (a) is only available if 12 is multiplied or divided by a whole number greater than 10 or a non-integer value followed by an addition or subtraction.

Commonly Observed Responses:

Question			Generic Scheme	Illustrative Scheme	Max Mark
6.			**Ans: real and distinct** $\bullet^1$ find discriminant $\bullet^2$ state nature of roots	 $\bullet^1$ 53 $\left[5^2-4\times7\times(-1)\right]$ $\bullet^2$ real and distinct (or equivalent)	**2**

Notes:

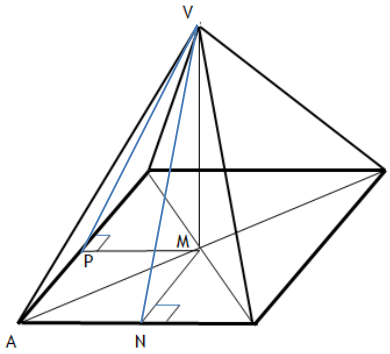
1. Correct answer without working award 0/2
2. 25 + 28 $\rightarrow$ real and distinct award 2/2
3. eg 25 + 28 = 52 $\rightarrow$ real and distinct award 1/2 ×✓
4. Accept '**real** roots'
5. Do not accept 'two distinct roots'
6. Do not award $\bullet^2$ where conclusion is ambiguous
   eg 53 $\rightarrow$ roots are real and even award 1/2 ✓×

Commonly Observed Responses:

1. $\dfrac{-5\pm\sqrt{5^2-4\times7\times(-1)}}{2\times7}=\dfrac{-5\pm\sqrt{53}}{2\times7}$ award 1/2 ✓×
2. $-3\rightarrow$ no real roots award 1/2 ×✓
3. $-3\rightarrow$ no roots award 0/2

Question			Generic Scheme	Illustrative Scheme	Max Mark
7.	(a)		**Ans: (8, 4, 0)** $\bullet^1$ state coordinates of B	$\bullet^1$ (8, 4, 0)	**1**
Notes: 1. Brackets must be shown.					
Commonly Observed Responses:					
	(b)		**Ans: 7** $\bullet^1$ know how to find AM^2 $\bullet^2$ know how to find AV $\bullet^3$ find length of AV	$\bullet^1$ 3^2+2^2 $\bullet^2$ $\sqrt{6^2+(3^2+2^2)}$ $\bullet^3$ 7	**3**

Notes:
1. Correct answer without working award 0/3
2. Alternative methods:



(a)[know how to find AM^2 ....]

$\bullet^1$ $\frac{1}{4}\left(6^2+4^2\right)$

$\bullet^2$ $\sqrt{6^2+\frac{1}{4}\left(6^2+4^2\right)}$

$\bullet^3$ 7

(b)[know how to find VN^2 ....]

$\bullet^1$ 6^2+2^2

$\bullet^2$ $\sqrt{3^2+(6^2+2^2)}$

$\bullet^3$ 7

(c)[know how to find VP^2....]

$\bullet^1$ 6^2+3^2

$\bullet^2$ $\sqrt{2^2+(6^2+3^2)}$

$\bullet^3$ 7

Commonly Observed Responses:

1. $\bullet^1$ $\begin{pmatrix}3\\2\\6\end{pmatrix}$ $\rightarrow$ $\bullet^2$ $\sqrt{3^2+2^2+6^2}$ $\rightarrow$ $\bullet^3$ = 7 award 3/3

2. $\begin{pmatrix}7\\2\\6\end{pmatrix}$ $\rightarrow$ $\sqrt{7^2+2^2+6^2}=\sqrt{89}$ award 1/3 ×✓×

Question	Generic Scheme	Illustrative Scheme	Max Mark
8.	**Ans:** $x=-\frac{5}{8}$ **Method 1** $\bullet^1$ multiply throughout by 6 $\bullet^2$ rearrange $\bullet^3$ solve for x **Method 2** $\bullet^1$ rearrange $\bullet^2$ start to solve for x $\bullet^3$ solve for x	 $\bullet^1$ $4x-5=12x$ $\bullet^2$ $-8x=5$ or $-5=8x$ $\bullet^3$ $x=-\frac{5}{8}$ or $x=-0\cdot625$ $\bullet^1$ $\frac{4}{3}x=-\frac{5}{6}$ $\bullet^2$ $x=-\frac{5}{6}\times\frac{3}{4}$ or $24x=-15$ or equivalent $\bullet^3$ $x=-\frac{5}{8}$ or $x=-0\cdot625$	**3**

Notes:
1. Correct answer without working award 0/3
2. $\bullet^1$ is available for multiplying throughout by any common multiple of 3 and 6
3. $\bullet^1$ is not available for $\frac{4x-5}{6}=2x, \frac{12x-15}{18}=2x$ etc.
4. For the award of $\bullet^3$, the answer must be a non-integer value

Commonly Observed Responses:

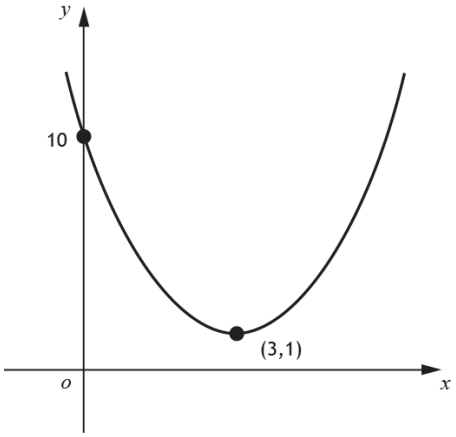
9.	**Ans:** $\frac{2\sqrt{5}}{5}$ $\bullet^1$ correct substitution $\bullet^2$ consistent answer	 $\bullet^1$ $\frac{2}{\sqrt{5}}$ $\bullet^2$ $\frac{2\sqrt{5}}{5}$	

Notes:
1. Correct answer without working award 0/2.

Commonly Observed Responses:
1. $\bullet^2$ is not available where there is invalid subsequent working

eg $\frac{2\sqrt{5}}{5}=2\sqrt{5}$ award 1/2 ✓×

2. $\frac{2}{\sqrt{x}}\times\frac{\sqrt{x}}{\sqrt{x}}=\frac{2\sqrt{x}}{x}$ award 1/2 ×✓

Question	Generic Scheme	Illustrative Scheme	Max Mark
10.	**Ans:**  		**3**
	•[1] coordinates of turning point correct	•[1] (3,1)	
	•[2] sketch parabola with **minimum** turning point consistent with •[1]	•[2] parabola with **minimum** turning point consistent with •[1]	
	•[3] y-intercept correct	•[3] (0,10) or 10	

Notes:

1. Correct answer without working award 3/3.
2. Where the coordinates of the turning point are not stated elsewhere, then for a sketch of a parabola with minimum turning point $(3,-1)$, $(-3,\pm1)$ or $(\pm1,\pm3)$ award •[2] but not •[1]. Otherwise •[2] is only available where the minimum turning point indicated on the sketch is consistent with that stated elsewhere.
3. The sketch of the parabola need not meet or cut the y-axis for the award of •[2].
4. •[2] is not available if the parabola has a maximum turning point.
5. •[3] is not available if the minimum turning point is on the y-axis.
6. Award •[3] where the y-intercept is calculated to be at $y = 10$ and is plotted on the diagram at (0,10) but annotated as (10,0). Treat this special case as bad form.

Commonly Observed Responses:

Question	Generic Scheme	Illustrative Scheme	Max Mark
11.	**Ans:** $\sin^2 x°$		**2**
	$\bullet^1$ identify correct trigonometric identity to be used	$\bullet^1$ $\dfrac{\sin x}{\cos x}$ **or** $\dfrac{\sin^2 x}{\cos^2 x}$	
	$\bullet^2$ use correct trigonometric identity to simplify expression	$\bullet^2$ $\dfrac{\sin^2 x}{\cos^2 x} \times \cos^2 x = \sin^2 x$	

Notes:

1. $\sin^2 x$ without working award 0/2
2. Degree signs are not required
3. $\bullet^2$ is not available if there is invalid subsequent working

   eg (a) $\dfrac{\sin^2 x}{\cos^2 x} \times \cos^2 x = \sin^2 x = 1 - \cos x$ award 1/2 ✓×

   (b) $\dfrac{\sin^2 x}{\cos^2 x} \times \cos^2 x = \sin^2 x = 1 - \cos^2 x$ award 2/2

4. $\bullet^1$ is not available if there are no variables e.g. $\dfrac{\sin^2}{\cos^2} \times \cos^2 = \sin^2$ award 1/2 ×✓
5. $\bullet^1$ is not available if candidate simply states $\tan x = \dfrac{\sin x}{\cos x}$ **and** $\sin^2 x + \cos^2 x = 1$ then proceeds no further
6. Alternative acceptable strategies

   (a) $\bullet^1$ $\tan x \cos x = \sin x$

   $\bullet^2$ $\tan^2 x \cos^2 x = \sin^2 x$

   award 2/2

   (b) $\bullet^1$ $\left(\dfrac{o}{a}\right)^2 \left(\dfrac{a}{h}\right)^2$

   $\bullet^2$ $\dfrac{o^2 a^2}{a^2 h^2} = \dfrac{o^2}{h^2} = \sin^2 x$

   award 2/2

Commonly Observed Responses:

1. $\dfrac{\cos^2 x}{\sin^2 x} \times \cos^2 x = \dfrac{\cos^4 x}{\sin^2 x}$ award 0/2
2. $\tan^2 x(1 - \sin^2 x) = \tan^2 x - \tan^2 x \sin^2 x$ award 0/2

Question	Generic Scheme	Illustrative Scheme	Max Mark
12. (a)	**Ans:** $(2x+1)(x+8)$ • [1] find an expression for the area of the rectangle	[1] $(2x+1)(x+8)$ or equivalent	**1**
Notes: 1. If solution to (a) appears in (b) or (c) award 1/1 2. (a) Accept $(2x+1)\times(x+8)$, $2x+1 \times x+8$ (b) Do **not** accept $2x+1(x+8)$, $x+8(2x+1)$ unless correct expansion appears in (a) (b) or (c)			
Commonly Observed Responses:			
12. (b)	**Ans: proof** • [1] find **expanded** expression for area of the rectangle • [2] find **expanded** expression for area of the triangle • [3] equate expanded expressions and rearrange into required form	• [1] $2x^2+16x+x+8$ • [2] $3x^2+15x$ • [3] $2x^2+16x+x+8=3x^2+15x$ $\Rightarrow x^2-2x-8=0$	**3**
Notes: 1. If solution to (b) appears in (a) or (c) then all three marks are available			
Commonly Observed Responses:			

Question			Generic Scheme	Illustrative Scheme	Max Mark
12.	(c)		**Ans: 12 cm and 9 cm** •[1] factorise x^2-2x-8 •[2] solve equation •[3] reject invalid value of x **and** state length and breadth of rectangle	•[1] $(x-4)(x+2)$ •[2] $x=4$ **and** $x=-2$ •[3] 12 (cm) and 9 (cm)	**3**

Notes:

1. Correct answer without working award 0/3.
2. If solution to (c) appears in (a) or (b) then all three marks are available.
3. •[1] is available for $\dfrac{2\pm\sqrt{(-2)^2-4\times1\times(-8)}}{2\times1}$
4. For an answer obtained by guess and check award 0/3

Commonly Observed Responses:

1.(a) $(2x+1)(x+8)=0\rightarrow x=-\dfrac{1}{2}$ and $x=-8$ award 1/3 ×✓×

(b) $x=-\dfrac{1}{2}$ and $x=-8$ without factorised quadratic equation stated award 0/3

[END OF MARKING INSTRUCTIONS]