ZETA MATHS

National 5 +

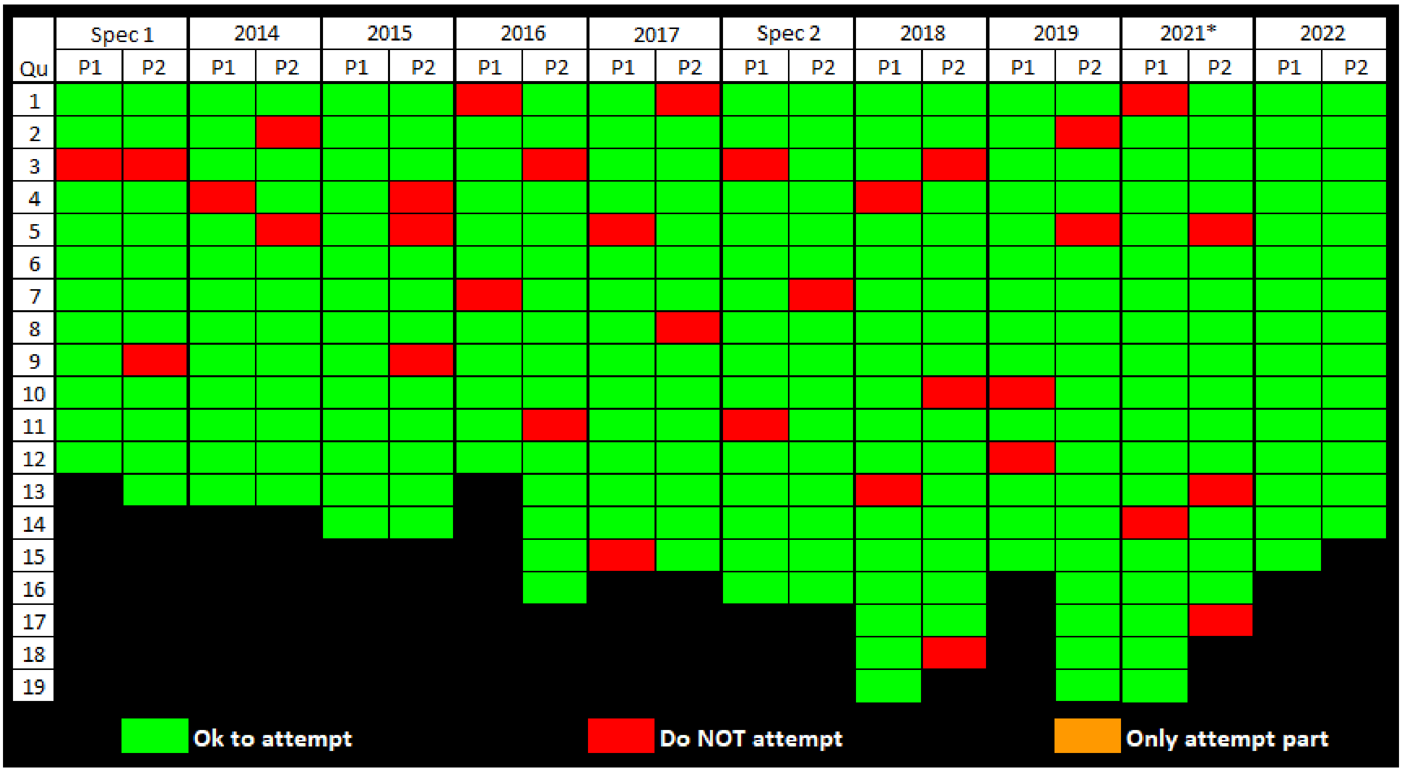
MATHEMATICS

Learning Checklist

This checklist covers every skill that learners need for success at National 5 Mathematics. Each section of this checklist corresponds to the **Zeta Maths National 5+ Mathematics** textbook (available from **www.zetamaths.com** or on Amazon Kindle). The topic names in this document are linked for easy navigation of the checklist and colour coded to correspond with skills: **numerical**, **algebraic**, **geometric**, **trigonometric** and **statistical**.

National 5 Mathematics

Past Paper Grid



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🟩 **Ok to attempt** 🟥 **Do NOT attempt** 🟧 **Only attempt part**

The following topics will not be assessed in the 2023 SQA National 5 exam: Vectors, Similarity & 3D coordinates

Contents

Section	Topic	Skills	🟥	🟨	🟩
1	**Rounding**				
1.1	Round to decimal places	**Examples:** $25.1241 \rightarrow 25.1$ **(1 d.p.)** $34.678 \rightarrow 34.68$ **(2 d.p.)**			
1.2	Round to significant figures	**Examples:** $1276 \rightarrow 1300$ **(2 s.f.)** $0.06356 \rightarrow 0.064$ **(2 s.f.)** $37{,}684 \rightarrow 37{,}700$ **(3 s.f.)** $0.005832 \rightarrow 0.00583$ **(3 s.f.)**			
2	**Surds**				
2	Surds	A surd is a number expressed in root form that cannot be simplified further. A surd is an irrational number, i.e. a number that cannot be expressed as a fraction, such numbers when expressed in decimal form have an infinite number of decimal places.			
2.2	Basic surd simplification	Learn the Square Numbers: 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169. Identify the largest square number factors that divide into the number being simplified, then take the root of them. **Example:** $\sqrt{50} = \sqrt{25 \times 2} = \sqrt{25} \times \sqrt{2} = 5\sqrt{2}$			
2.3	Multiplyication of surds	$\sqrt{5} \times \sqrt{15} = \sqrt{75} = \sqrt{25} \times \sqrt{3} = 5\sqrt{3}$			
2.4	Division of surds	$\frac{\sqrt{48}}{\sqrt{3}} = \sqrt{\frac{48}{3}} = \sqrt{16} = 4$			
2.5	Addition and subtraction of surds	$\sqrt{50} + \sqrt{8} = \sqrt{25} \times \sqrt{2} + \sqrt{4} \times \sqrt{2} = 5\sqrt{2} + 2\sqrt{2} = 7\sqrt{2}$			
2.6	Rationalise the denominator	Remove the surd from the denominator. **Example:** $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{\sqrt{9}} = \frac{\sqrt{3}}{3}$			
3	**Indices**				
3	Indices	The term **index** or **indices** (plural) is another word for the power or powers of a number. The power of a number is how many times a number or term is multiplied by itself. For example, $3 = 3^1$, so it has a power of 1. This means 3 is only present once. With the number 3^2, there are two 3's multiplied together, **i.e.** $3^2 = 3 \times 3 = 9$. **NB.** $a^0 = 1$			
3.5	The first law of indices	$a^x \times a^y = a^{x+y}$ **Examples:** $a^3 \times a^2 = a^{3+2} = a^5$ $3b^5 \times 4b^{-2} = 12b^{5+(-2)} = 12b^3$			
3.6	The second law of indices	$\frac{a^x}{a^y} = a^{x-y}$ **Example:** $\frac{6a^7}{2a^5} = 3a^{7-2} = 3a^5$			
3.8	The third law of indices – raising powers to powers	$(a^x)^y = a^{x \times y}$ **Example:** $(a^3)^4 = a^{3\times4} = a^{12}$			

Section	**Topic**	**Skills**	🟥	🟨	🟩
3.9	The fourth law of indices – negative indices	$a^{-x} = \frac{1}{a^x}$ **Example:** $2a^{-3} = \frac{2}{a^3}$			
3.10	Fractional indices	$a^{\frac{x}{y}} = \sqrt[y]{a^x}$ **Example:** $x^{-\frac{3}{4}} = \frac{1}{x^{\frac{3}{4}}} = \frac{1}{\sqrt[4]{x^3}}$			
4	**Standard Form**				
4.1	Change from normal to standard form and vice versa	**Standard Form**, also known as scientific notation, is a method of writing either very large or very small numbers in a usable and convenient way. The numbers are standardized in that they are always written with the leading number being **greater than or equal to 1** and **less than 10**. **Examples:** ***a.*** $2350 = 2.35 \times 10^3$ ***b.*** $0.0000058 = 5.8 \times 10^{-6}$			
4.2-3	Calculations using standard form	$(1.3 \times 10^5) \times (8 \times 10^3) = 10.4 \times 10^8 = 1.04 \times 10^9$			
5	**Expanding Brackets**				
5.3	Addition or subtraction of two or more brackets	When two or more brackets are being added or subtracted from one another, expand each bracket, then simplify: ***a.*** $4(y-7)+3(4y+6)$ ‘ ***b.*** $3(3z+4)-2(6z-5)$ $= 4y - 28 + 12y + 18$ $= 9z + 12 - 12z + 10$ $= 16y - 10$ $= -2x + 22$			
5.4-10	Multiplication of two brackets	Use **FOIL** (**F**irsts **O**utsides **I**nsides **L**asts) or another suitable method $(x+3)(x-2) = x^2 + 3x - 2x - 6 = x^2 + x - 6$			
5.11	Multiplication of brackets – two by three	Every term in the first bracket must multiply every term in the second. $(x+2)(x^2-3x-4) = x^3 - 3x^2 - 4x + 2x^2 - 6x - 8$ $= x^3 - x^2 - 10x - 8$			
6	**Factorising**				
6.1-3	Common factor	When factorising any expression, the first thing to look for is a **common factor**. A common factor is a factor that each of the terms share. **Example:** $4x^2 + 8x = 4x(x+2)$			
6.4-5	Difference of two squares	A difference of two squares is when one square number is taken away from another. **Example:** ***a.*** $a^2 - 16 = (a+4)(a-4)$ ***b.*** $4x^2 - 36 = 4(x^2 - 9)$ ***(NB:** common factor first)* $= 4(x-3)(x+3)$			
6.6-9	Trinomial	**Step 1:** Start by considering the First terms in the bracket these will be factors of the first term of the trinomial. **Step 2:** Move to the Last terms in the brackets. These must be factors of the third term in the trinomial. **Step 3:** The Outsides and Insides of the brackets must add to give the middle term. **Example:** $x^2 - x - 6 = (x-3)(x+2)$			

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6.10	Trinomials – non-unitary coefficient of x^2	This is more difficult. Same process as 6.6-9. The Outsides add Insides give a check of the correct answer: **Example:** $3x^2 - 13x - 10$ $=$ ~~$(3x - 5)(x + 2)$~~ *Check*: $3x \times 2 + (-5) \times x = 6x - 5x = -x$ ✗ $= (3x + 2)(x - 5)$ *Check:* $3x \times (-5) + 2 \times x = -15x + 2x = -13x$ ✓ **NB:** If the answer is wrong, score out and try alterative factors or positions. Keep a note of the factors you have tried.			
7	**Completing the Square**				
7.1	Completing the square	$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 + c - \left(\frac{b}{2a}\right)^2$ **Example:** $x^2 + 8x + 19 = (x + 4)^2 + 19 - 16$ $= (x + 4)^2 + 3$			
8	**Algebraic Fractions**				
8.1-2	Simplifying algebraic fractions by factorising	**Step 1:** Factorise expression **Step 2:** Look for common factors. **Step 3:** Cancel and simplify $\frac{6x^2 - 12x}{x^2 + x - 6} = \frac{6x\cancel{(x-2)}}{(x+3)\cancel{(x-2)}} = \frac{6x}{x+3}$			
8.3	Multiplying algebraic fractions	Multiply the numerators, then multiply the denominators. **NB:** It is often better to simply before multiplying. $\frac{6ab}{5c} \times \frac{5ac}{2b} = \frac{3a}{1} \times \frac{a}{1} = \frac{3a \times a}{1} = 3a^2$			
8.4	Dividing algebraic fractions	Invert the second fraction, then multiply $\frac{6x^2}{7y} \div \frac{4x}{3z} = \frac{6x^2}{7y} \times \frac{3z}{4x} = \frac{3x}{7y} \times \frac{3z}{2} = \frac{9xz}{14y}$			
8.5	Addition and subtraction of algebraic fractions	Find a common denominator. This can be done either by working out the lowest common denominator, or by using **Smile** and **Kiss**. $\frac{5a}{b} + \frac{3d}{2c} = \frac{10ac}{2bc} + \frac{3bd}{2bc} = \frac{10ac + 3bd}{2bc}$			
9	**Gradients**				
9.1-3	The Gradient Formula	Know that gradient is represented by the letter m **Step 1:** Select two coordinates **Step 2:** Label them (x_1, y_1) (x_2, y_2) **Step 3:** Substitute them into gradient formula **Example:** $(\overset{x_1}{-4}, \overset{y_1}{4}), (\overset{x_2}{12}, \overset{y_2}{-28})$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-28 - 4}{12 - (-4)} = \frac{-32}{16} = -2$			
10	**Circles – Arcs & Sectors**				
10.4	The length of an arc of a circle	$\frac{\text{Length of Arc}}{\pi D} = \frac{\text{Angle}}{360}$ or $\text{Length of Arc} = \frac{\text{Angle}}{360} \times \pi D$			

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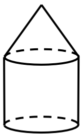
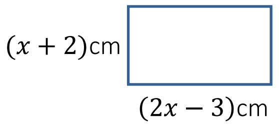
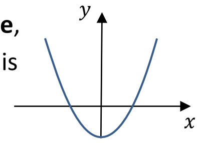
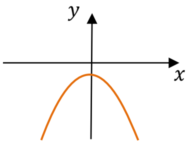
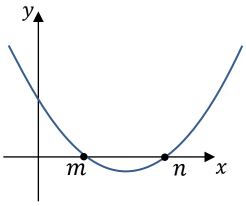
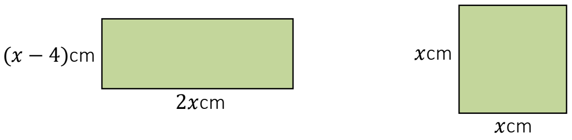
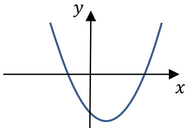
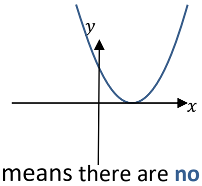
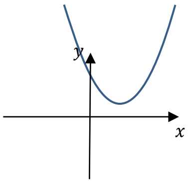
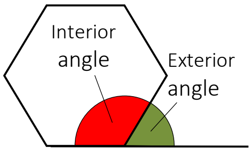
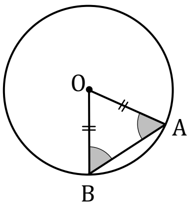
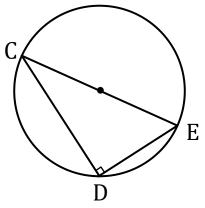
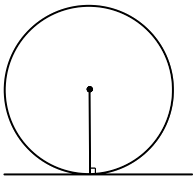
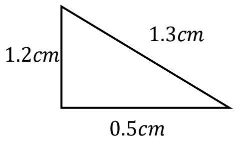
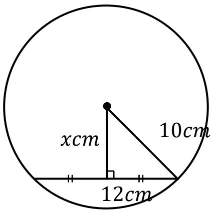
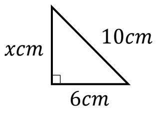
10.5	The area of a sector of a circle	$\frac{\text{Area of Sector}}{\pi \text{r}^2} = \frac{\text{Angle}}{360}$ or $\text{Area of Sector} = \frac{\text{Angle}}{360} \times \pi \text{r}^2$			
10.6	Finding an angle, radius or diameter	Rearrange the formula and used to find other unknowns: $\frac{\text{Length ofArc}}{\pi \text{D}} = \frac{\text{Area of Sector}}{\pi \text{r}^2} = \frac{\text{Angle}}{360}$			
11	**3D Solids - Volume**				
11.2	Volume of a cylinder	$V = \pi r^2 h$			
11.3	Volume of a pyramid	$V = \frac{1}{3} \times Area\ of\ base \times height$			
11.4	Volume of a cone	$V = \frac{1}{3}\pi r^2 h$			
11.5	Volume of a sphere	$V = \frac{4}{3}\pi r^3$			
11.6	Calculating a height or radius using volume formulae	Rearrange the formulae, then substitute in the given values. **Example:** Cylinder has volume 400cm^3 and radius 6cm, find the height. $V = \pi r^2 h \qquad h = \frac{\pi r^2}{V} \qquad h = \frac{\pi \times 6^2}{400}$			
11.7	Volume of composite solids	Composite solids are made up of two or more solids. To find the volume of composite solids, find the volume of each solid and add them together. 			
12	**The Straight Line**				
12	Gradient	• Represented by m • Measure of steepness of slope • Positive gradient: the line is *increasing* • Negative gradient: the line is *decreasing*			
12	y-intercept	• Represented by c • Shows where the line crosses the y-axis • Find by making $x = 0$			
12.1	The equation of a line from the gradient and y-intercept	**Step 1:** Find gradient m (section 9.1-3) **Step 2:** Find y-intercept c **Step 3:** Substitute into $y = mx + c$ (see above for definitions)			
12.2	Sketch a line from its equation	**Step 1:** Rearrange equation to the form $y = mx + c$ **Step 2:** Draw a table of points, determine x and y intercepts, or use gradient to step along from y-intercept. **Step 3:** Plot points on coordinate axes			
12.3	The equation of a line from two points	Use this when there are only two points (**i.e.** no y-intercept) **Step 1:** Find gradient m **Step 2:** Substitue into $y - b = m(x - a)$ where (a, b) are taken from either one of the points			
12.4	The gradient and intercepts from an equation	To find the gradient and y-intercept of a line, rearrange the line into the form $y = mx + c$, then simply read the values for m and c. **Example:** $3y + 6x = 12$ $3y = -6x + 12$ $y = -2x + 4 \qquad m = -2,\ c = 4$			
12.5	Equations of parallel lines	Parallel lines have the same gradient.			

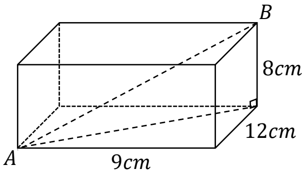
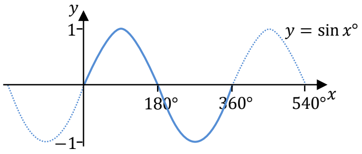
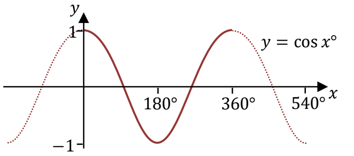
Section	**Topic**	**Skills**	🟥	🟨	🟩
13	**Functions**				
13.1	Basic functions	A function in mathematics takes an input value, then applies a rule to it and produces an output value or image. **a.** If $f(x) = 2x - 1$, evaluate $f(x)$ when $x = 3$. $f(x) = 2x - 1$ $f(3) = 2(3) - 1$ $f(3) = 6 - 1$ $f(3) = 5$ **b.** If $g(x) = 4x - 3$. Calculate a when $g(a) = 29$ $4a - 3 = 29$ $4a = 32$ $a = 8$			
14	**Solving Equations 1 – Linear Equations**				
14.6	Equations with brackets	When solving equations with brackets, first expand the brackets and simplify, then solve the equations. **Example:** $2(x + 4) = 10$ $2x + 8 = 10$ $2x = 2$ $x = 1$			
14.7	Forming equations	The perimeter of the rectangle is $40cm$. Calculate the value of x.  $2(x + 2) + 2(2x - 3) = 40$ $6x - 2 = 40$ $6x = 42$ $x = 7cm$			
14.9	Equations with fractions	To solve equations involving more than one fraction, multiply everything by the **lowest common denominator** of the fractions. **Example:** Solve $\frac{1}{5}(b - 2) = \frac{3}{2}$ $(\times \mathbf{10}) \quad (\times \mathbf{10})$ $\frac{10}{5}(b - 2) = \frac{30}{2}$ $2(b - 2) = 15$ $2b - 4 = 15$ $2b = 19$ $b = \frac{19}{2}$			
14.8,10	Solving inequations	Inequations are solved *in the same way as equations*, with two exceptions. Firstly, if multiplying or dividing by a negative value, the inequality sign is reversed. Secondly, when the unknown is on the right of the inequality, the sign must be reversed to move the unknown to the left. **a.** $-c \geq 5$ **b.** $4 \geq d$ $c \leq -5$ $d \leq 4$			
15	**Solving Equations 2 – Simultaneous Linear Equations**				
15.1	Simultanoues equations by substitution	Simultaneous equations can be solved by substitution if one of the equations has an unknown with a coefficient equal to one, or if the unknown is of equal value in both equations.			

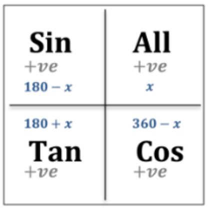
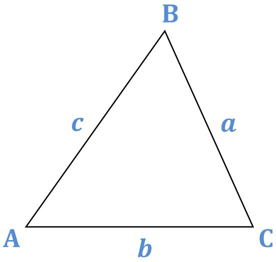
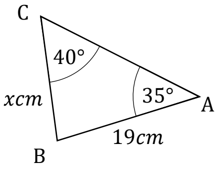
Section	Topic	Skills	🟥	🟨	🟩
		Example: Solve the system of equations $y = x + 2$ and $y = 3x - 6$. *Substitute* $y = x + 2$ into $y = 3x - 6$ $x + 2 = 3x - 6$ $8 = 2x$ $2x = 8$ $x = 4$ *Substitute* $x = 4$ into $y = x + 2$ $y = 4 + 2$ $y = 6$ Solution: $x = 4$ and $y = 6$.			
15.2	Simultaneous equations by elimination	**Step 1:** Scale equations to make one unknown equal with opposite sign. **Step 2:** Add Equations to eliminate equal term and solve. **Step 3:** Substitute number to find second term. **Example:** $2x + \mathbf{4y} = 2$ **(A)** $x + 4y = 5$ **(B)** **(B)** $\times -1$ $-x - \mathbf{4y} = -5$ **(C)** **(A)+(C)** $x \quad = -3$ $x = -3$ *Substitute* $x = -3$ into **(A)**. $2(-3) + 4y = 2$ $-6 + 4y = 2$ $y = 2$ Solution: $x = -3$ and $y = 2$.			
15.3	Forming equations to solve simultaneously	Form equations from a variety of contexts to solve for unknowns.			
15.4	Solve simultanous equations graphically	Sketch the lines on squared paper (see **12.2**), then find the point of intersection of the lines.			
16	**Changing the Subject of a Formula**				
16.3-6	Changing the subject of a formula	The subject of a formula is the value that is on its own on one side of the formula (usually the left). Change the subject in each of the following questions to $\boldsymbol{x}$. **1.** $x = \frac{\boldsymbol{a}^2-3}{4}$ $4x = \boldsymbol{a}^2 - 3$ $4x + 3 = \boldsymbol{a}^2$ $\boldsymbol{a} = \sqrt{4x+3}$ **2.** $y = \sqrt{\frac{\boldsymbol{a}+2j}{b}}$ $y^2 = \frac{\boldsymbol{a}+2j}{b}$ $by^2 = \boldsymbol{a} + 2j$ $\boldsymbol{a} = by^2 - 2j$			
17	**Quadratic Functions & Graphs**				
17	Quadratic Graphs	The graph of a quadratic function is called a **parabola**. Parabolas are either ‘n’ or ‘u’ shaped, depending on whether the coefficient of the x^2 term is positive or negative. When the coefficient of the x^2 term is **positive**, the graph has a **minimum turning point**. This produces a ‘u’ shape graph (as in the blue graph on the right). 			

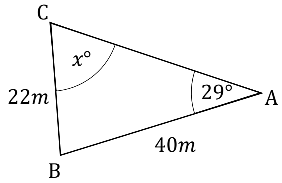
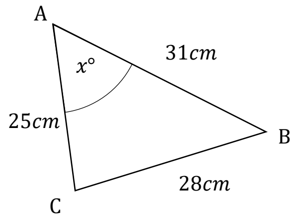
Section	Topic	Skills	🟥	🟨	🟩
		When the coefficient of the x^2 term is **negative**, the graph has a **maximum turning point**. This produces an 'n' shape graph (as in the orange graph on the right).  			
17.1	Finding the equation of a quadratic graph from a turning point	**Step 1:** Substitute turning point $(-p, q)$ into the completed square form $y = k(x+p)^2 + q$, (the value in the bracket is the negative of the x-coordinate of the turning point). **Step 2:** Substitute the other point into the equation to find the value of k. **NB:** For National 5, if the turning point is not the origin, $k = \pm 1$. If graph is positive, 'u' shaped, $k = 1$ and if graph is negative, 'n' shaped, $k = -1$.			
17.2	Finding the equation of a quadratic graph from roots	The **roots** of a quadratic function are the x-coordinates at which the graph cuts the x-axis. These values are also known as the **solutions**, the **zeros** or the **x-intercepts** of a quadratic function. When we have the roots, the equation may be found in the following way: **Step 1:** Substitute the roots into $y = k(x-m)(x-n)$. **Step 2:** Substitute the other point into the equation to find the value of k (as in **17.1**, $k = \pm 1$).  			
17.3	Sketching quadratics from completed square form	**Step 1:** Identify the turning point and shape of the graph from values of k, p and q from the form $y = k(x+p)^2 + q$. **Step 2:** Find y-intercept by making $x = 0$. **Step 3:** Sketch graph, noting turning point, y-intercept and axis of symmetry. (The axis of symmetry is a vertical line, taken from the x-coordinate of the turning point.) **NB:** For National 5, $k = \pm 1$.			
17.4-5	Solving quadratic equations by factorising	To solve a quadratic function is to find the x-coordinates at which the graph of the quadratic function is equal to zero, **i.e.** where the graph intersects the x-axis. **a.** $x^2 + 2x = 0$ **b.** $x^2 - 16 = 0$ $x(x+2) = 0$ $(x+4)(x-4) = 0$ $x = 0$ *or* $x + 2 = 0$ $x + 4 = 0$ *or* $x - 4 = 0$ $x = 0$ *or* $x = -2$ $x = -4$ *or* $x = 4$			
17.7	Solving quadratics using the Quadratic Formula	This method is used when a quadratic cannot be factorised. These questions usually ask for decimal place accuracy. **Example:** Solve the quadratic equation $x^2 + 4x - 2 = 0$ to 1 decimal place. Since $ax^2 + bx + c$, then $a = 1, b = 4, c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $x = \frac{-(4) \pm \sqrt{(4)^2 - 4(1)(-2)}}{2(1)}$ $x = \frac{-(4) \pm \sqrt{24}}{2}$ $x = -2 - \sqrt{6}$ *or* $x = -2 + \sqrt{6}$ $x = -4.4$ (to 1 d.p.) $x = 0.4$ (to 1 d.p.)			

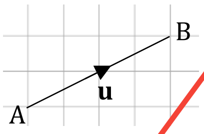
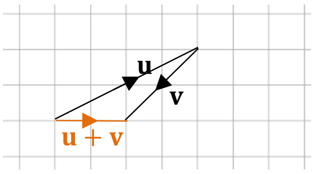
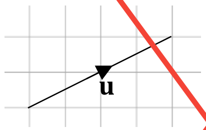
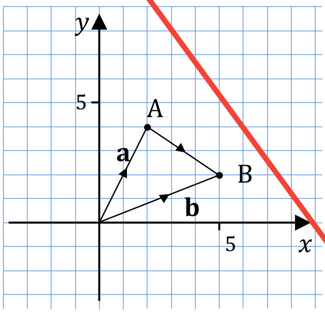
Section	Topic	Skills	🟥	🟨	🟩
17.8	Sketching quadratics from factorised form	**Step 1:** Identify the roots and shape from values of k, m and n from the form $\boldsymbol{y = k(x-m)(x-n)}$. **Step 2:** Find y-intercept by making $x = 0$. **Step 3:** Sketch graph, noting turning point, y-intercept and axis of symmetry. **NB:** For National 5, $k = \pm 1$.			
17.9	Forming quadratic equations	**Example:** The rectangle and the square have the same area. By forming an equation, find the dimensions of the rectangle.  $2x(x-4) = x^2$ $2x^2 - 8x = x^2$ $x^2 - 8x = 0$ $x(x-8) = 0$ $x = 0$ *or* $x - 8 = 0$ $\cancel{x = 0}$ *or* $x = 8$ ∴ Length = $2(8) = 16$cm Breadth = $8 - 4 = 4$cm			
17.10	Determining the nature of roots (The Discriminant)	The formula $b^2 - 4ac$, where $ax^2 + bx + c$ is known as the **discriminant**. The discriminant is used to determine the *nature of the roots*, or the points of intersection between a quadratic curve and the x-axis. There are three results that matter when calculating the discriminant: $\boldsymbol{b^2 - 4ac > 0}$ this means there are **two real and distinct roots**, **i.e.** the curve cuts the x-axis in two different places.  $\boldsymbol{b^2 - 4ac = 0}$ this means there are **two real and equal roots**, **i.e.** the x-axis is a tangent to the curve.  $\boldsymbol{b^2 - 4ac < 0}$ this means there are **no real roots**, **i.e.** the curve does not cut the x-axis. 			

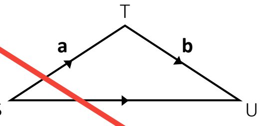
Section	Topic	Skills	🟥	🟨	🟩
18	**Properties of Shape**				
18.1	Angles in polygons	To calculate an **interior angle** of a regular polygon, we can use the formula **$180-(360\div n)$** where n is the number of sides.  To calculate an **exterior angle**, either find the supplement of the interior angle, or simply $360\div n$.			
18.2	Angles in circles	  			
19	**Pythagoras' Theorem**				
19.2	The Converse of Pythagoras	The converse of Pythagoras' Theorem is used to determine whether a triangle is right-angled or not. (The Cosine Rule may also be used, see **section 22**.) **Example:** Determine whether the triangle in the diagram is right-angled.  Longest Side $1.3^2=1.69$ Other sides $1.2^2+0.5^2=1.44+0.25=1.69$ $1.3^2=1.2^2+0.5^2$ ∴ by the converse of Pythagoras' Theorem, the triangle is right-angled. **NB:** If the triangle is not right-angled, there is no need to state 'by the converse of Pythagoras'.			
19.3	Pythagoras' Theorem in circles	Pythagoras' Theorem is used within circles where right-angled triangles can be formed. Most commonly this occurs when a radius and chord intersect at right-angles (this is called a **perpendicular bisector**). **Example:** The circle, centre O, has a radius of $10cm$. The chord in the diagram is $12cm$ long. Calculate the value of x.  Identify the right-angled triangle then use Pythagoras' Theorem.  $x^2=10^2-6^2$ $x^2=100-36$ $x=\sqrt{64}$ $x=8cm$			

Section	Topic	Skills	🟥	🟨	🟩
19.4	The distance between two points	To find the distance between two points or coordinates, Pythagoras' Theorem is used. This often appears in the form of the **Distance Formula:** $\boldsymbol{Distance = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$, but it may be simpler and often more helpful to sketch a right-angled triangle and work out the vertical and horizontal difference, just like when calculating the gradient.			
19.5	Pythagoras' Theorem in three dimensions	**Example:** The diagram on the right represents a cuboid. Calculate the length of the diagonal AB.  $AB^2 = 9^2 + 12^2 + 8^2$ $AB^2 = 81 + 144 + 64$ $AB^2 = 289$ $AB = \sqrt{289}$ $AB = 17cm$			
~~**20**~~	~~**Similarity**~~				
~~**20.1**~~	~~Using linear scale factors~~	~~$\mathbf{LSF} = \frac{\boldsymbol{new\ length}}{\boldsymbol{original\ length}}$~~			
~~**20.2**~~	~~Using area scale factors~~	~~$\mathbf{ASF} = \left(\frac{\boldsymbol{new\ length}}{\boldsymbol{original\ length}}\right)^2$~~			
~~**20.3**~~	~~Using volume scale factors~~	~~$\mathbf{VSF} = \left(\frac{\boldsymbol{new\ length}}{\boldsymbol{original\ length}}\right)^3$~~			
21	**Trigonometric Functions**				
21	Trigonometric Functions	**Trigonometric Graphs** **The Sine Graph** The graph of $y = \sin x°$ has an **amplitude** of 1 and a **period** of 360°.  **The Cosine Graph** The graph of $y = \cos x°$ also has an **amplitude** of 1 and a **period** of 360°. 			
21.1	Finding the equation of a trigonometric function	In trigonometric functions of the form $y = \boldsymbol{a} \sin \boldsymbol{b}x + \boldsymbol{c}$ and $y = \boldsymbol{a} \cos \boldsymbol{b}x + \boldsymbol{c}$, $\boldsymbol{a}$ is the amplitude, $\boldsymbol{b}$ is the number of waves and the value of $\boldsymbol{c}$ translates the graph **vertically**. The amplitude of the graph is the positive value of a. This is half the distance from the maximum to the minimum value. The period of the graph is the length of one full wave. For cos and sin graphs, the period can be derived by dividing 360° by $\boldsymbol{b}$. Alternatively, to find the value of $\boldsymbol{b}$, divide 360° by the period of one wave.			

Section	**Topic**	**Skills**	🟥	🟨	🟩
		In trigonometric functions of the form $y = a\sin(x + b)$ and $y = a\cos(x + b)$, a is the amplitude, the value of b translates the graph **horizontally**. As with quadratic functions, the value in the bracket is the negative of the direction that the graph shifts.			
21.2	Sketching a trigonometric function	Using the information from **21.1**, sketch required graph between given values. Annotate y-axis with maximum and minimum values. Annotate x-axis.			
21.3	Solving trigonometric equations	Solve the equations $\sin x^\circ = \frac{1}{2}$, for $0 \le x \le 360$. $\sin x^\circ = \frac{1}{2}$ $x^\circ = \sin^{-1}\left(\frac{1}{2}\right)$ $x^\circ = 30^\circ, 180^\circ - 30^\circ$ $x^\circ = 30^\circ, 150^\circ$  			
21.4	Using trigonometric identities	In National 5 Mathematics there are two trigonometric identities to be remembered and used: and $\boldsymbol{\tan x = \frac{\sin x}{\cos x}}$ $\boldsymbol{\sin^2 x + \cos^2 x = 1}$ These identities are equalities that are useful in simplifying trigonometric expressions. **Example:** Show that $\tan x \cos x = \sin x$. $LHS = \tan x \cos x$ $= \frac{\sin x}{\cos x} \times \cos x$ $= \sin x$ $= RHS$			
22	**Triangle Trigonometry**				
22	Triangle Trigonometry	 			
22.1	Finding the area of a triangle	$\boldsymbol{A = \frac{1}{2}ab\sin C}$			
22.2	Using the Sine Rule to find a side	As a general rule, if there is more than one angle in the triangle, either known or to be calculated, the Sine Rule should be used. **Example:** Calculate the value of x.  $\boldsymbol{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$ $\frac{a}{\sin A} = \frac{c}{\sin C}$ $\frac{x}{\sin 35} = \frac{19}{\sin 40}$			

Section	Topic	Skills	🟥	🟨	🟩
		$\frac{x}{\sin 35} = \frac{19}{\sin 40}$ $x = 17.0cm$ (1 d.p.) $x = \frac{19 \sin 35}{\sin 40}$			
22.3	Using the Sine Rule to find an angle	**Example:** Calculate size of angle x.  $\boldsymbol{\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}}$ $\frac{a}{\sin A} = \frac{c}{\sin C}$ $\frac{22}{\sin 29} = \frac{40}{\sin x}$ $22 \sin x = 40 \sin 29$ $\sin x = \frac{40 \sin 29}{22}$ $x = \sin^{-1}\left(\frac{40 \sin 29}{22}\right)$ $x = 61.8°$ (1 d.p.)			
22.4	Using the Cosine Rule to find a side	The Cosine Rule is used when there is only one angle involved in the calculation. **Example:** Calculate the length of side x. $\boldsymbol{a^2 = b^2 + c^2 - 2bc \cos A}$ $x^2 = 15^2 + 19^2 - 2 \times 15 \times 19 \cos 38$ $x = \sqrt{15^2 + 19^2 - 2 \times 15 \times 19 \cos 38}$ $x = 11.7cm$ (1 d.p.)			
21.5	Using the Cosine Rule to find an angle	**Example:** Calculate the length of side x.  $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos x = \frac{25^2 + 31^2 - 28^2}{2 \times 25 \times 31}$ $x = \cos^{-1}\left(\frac{25^2 + 31^2 - 28^2}{2 \times 25 \times 31}\right)$ $x = 58.8°$ (1 d.p.)			
21.7	Bearings	Bearings are angles measured from north in a clockwise direction, written with three figures. They are used for navigation, engineering and many other applications as a standard way of describing direction. **NB:** It can be helpful to extend North arrows vertically and use alternate or corresponding angles to find unknowns.			

Section	Topic	Skills	🟥	🟨	🟩
23	**Vectors**				
23	Vectors	A vector is a quantity that has both a **magnitude** and a **direction**. Vectors can be expressed as line segments with an arrow to indicate direction. These can be named in either of two ways, either using a lower-case bold letter, such as $\mathbf{u}$, or by the letters at the end of each line segment, such as $\overrightarrow{\mathrm{AB}}$, as in the example below.  **NB:** When handwriting lower-case vectors, they must be underlined, i.e. $\mathbf{u} = \underline{u}$			
23.1	Finding a resultant vector from directed line segments	 			
23.2	Expressing directed line segments in component form	Components are then written as $\mathbf{u} = \begin{pmatrix} x \\ y \end{pmatrix}$, where x and y are the difference in each direction. **Example:**  $\mathbf{u} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$			
23.3	Adding and subtracting vectors in component form	**Example:** If $\mathbf{a} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$ *a.* $\mathbf{a} + \mathbf{b}$ $= \begin{pmatrix} 5 \\ 1 \end{pmatrix} + \begin{pmatrix} 6 \\ -5 \end{pmatrix}$ $= \begin{pmatrix} 11 \\ -4 \end{pmatrix}$ *b.* $2\mathbf{a} - 3\mathbf{b}$ $= 2\begin{pmatrix} 5 \\ 1 \end{pmatrix} - 3\begin{pmatrix} 6 \\ -5 \end{pmatrix}$ $= \begin{pmatrix} 10 \\ 2 \end{pmatrix} - \begin{pmatrix} 18 \\ -15 \end{pmatrix}$ $= \begin{pmatrix} -8 \\ 17 \end{pmatrix}$			
23.4	Using position vectors	In mathematics, a **position vector** is a vector from the origin to a point on a coordinate axis, **e.g.** the position vector of coordinate $\mathrm{A}(2, 4)$ is $\mathbf{a} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$. Position vectors are used to find the vector which joins two coordinates. $\overrightarrow{\mathrm{AB}} = -\mathbf{a} + \mathbf{b}$ or more commonly, $\overrightarrow{\mathrm{AB}} = \mathbf{b} - \mathbf{a}$.  			
23.5	Three dimensional coordinates	**Three dimensional coordinates** allow us to reference points in three-dimensional space. In two dimensions we refer to (x, y), but in three dimensions we use (x, y, z). Vector components are written in three dimensions in the same way.			

Section	Topic	Skills	🟥	🟨	🟩
		When working with three-dimensional coordinate diagrams, take the x-coordinate first and use the difference in the given coordinates to determine the value. Then repeat for y and z.			
23.6	Magnitude of vectors	The **magnitude** of a vector is the length or size of the vector. The notation for magnitude is two vertical bars, **e.g.** the magnitude of $\mathbf{u}$ is $\lvert\mathbf{u}\rvert$. To calculate the magnitude of a vector, we use the following formula: $\lvert\mathbf{u}\rvert = \sqrt{x^2+y^2}$ or in three dimensions $\lvert\mathbf{u}\rvert = \sqrt{x^2+y^2+z^2}$			
23.7	Vector pathways	A **vector pathway** is a description of the pathway from the beginning to the end of the vector. Vector pathways can be described either in terms of their components or by combinations of known vectors. In the diagram, the pathway from S to U, which we call vector $\overrightarrow{\mathrm{SU}}$ may be described in three ways:  $\overrightarrow{\mathrm{SU}} = \overrightarrow{\mathrm{ST}} + \overrightarrow{\mathrm{TU}} = \mathbf{a}+\mathbf{b}$ Each of these vectors above represent the same journey, each starts at S and ends at U. **NB:** A vector pathway can only be described by combinations of known vectors.			
24	**Percentages**				
24.4	Appreciation/depreciation including compound interest	The terms **appreciation** and **depreciation** usually refer to the value of something either increasing (appreciation) or decreasing (depreciation) over time. This is usually described as a percentage. **Example:** Bethany invests £4000 in a savings account with an interest rate of 4%. How much will her investment be worth after 3 years? Calculate the multiplier as a decimal: $100\% + 4\% = 104\% = 1.04$ Use the multiplier three times: $4000 \times 1.04 \times 1.04 \times 1.04 = 4000 \times 1.04^3 = £4499.46$			
24.5	Using reverse percentages	Using reverse percentages is often called **working backwards** or **reversing the change** or **finding an initial value**. **Example:** A restaurant bill cost £170.50 after a 10% service charge was added on. How much was the bill before the service charge was added? $110\% = 170.5$ $1\% = 170.5 \div 110 = 1.55$ $100\% = 1.55 \times 100 = £155$			
25	**Fractions**				
25.4	Addition of mixed numbers	$2\frac{1}{3} + 3\frac{1}{2} = 5\frac{2}{6} + \frac{3}{6} = 5\frac{5}{6}$			

<table>
<tr><th>Section</th><th>Topic</th><th>Skills</th><th>🟥</th><th>🟨</th><th>🟩</th></tr>
<tr><td>**25.5**</td><td>Subtraction of mixed numbers</td><td>$4\frac{2}{3}-1\frac{1}{4} = 3\frac{8}{12}-\frac{3}{12} = 5\frac{5}{12}$</td><td></td><td></td><td></td></tr>
<tr><td>**25.7**</td><td>Multiplication of mixed numbers</td><td>$3\frac{1}{2}\times 2\frac{1}{5} = \frac{7}{2}\times\frac{11}{5} = \frac{77}{10} = 7\frac{7}{10}$</td><td></td><td></td><td></td></tr>
<tr><td>**25.8**</td><td>Division of mixed numbers</td><td>$\frac{3}{4}\div 1\frac{2}{5} = \frac{3}{4}\div\frac{7}{5} = \frac{3}{4}\times\frac{5}{7} = \frac{15}{28}$</td><td></td><td></td><td></td></tr>
<tr><td>**26**</td><td colspan="5">**Statistics**</td></tr>
<tr><td>**26.2**</td><td>Quartiles and interquartile range</td><td>**Example:** Find the interquartile range of the following data:
12 18 9 19 13 25 11 15
9 11 | 12 13 | 15 18 | 19 25
Q_1 Q_2 Q_3
$Q_1 = \frac{11+12}{2} = 11.5$
$Q_2 = \frac{13+15}{2} = 14$
$Q_1 = \frac{18+19}{2} = 18.5$
$IQR = Q_3 - Q_1$
$IQR = 18.5 - 11.5$
$IQR = 7$</td><td></td><td></td><td></td></tr>
<tr><td>**26.3**</td><td>Mean & standard deviation</td><td>**Example:** Calculate the mean and standard deviation of the following data.:
22 38 19 29 13 25
When using either formula, calculate the mean $\overline{x}$, then draw a table to calculate the values. Finally substitute into the formula.
$\overline{x} = 24.3$
<table>
<tr><th>x</th><th>$(x-\overline{x})$</th><th>$(x-\overline{x})^2$</th></tr>
<tr><td>22</td><td>-2.3</td><td>5.29</td></tr>
<tr><td>38</td><td>13.7</td><td>187.69</td></tr>
<tr><td>19</td><td>-5.3</td><td>28.09</td></tr>
<tr><td>29</td><td>4.7</td><td>22.09</td></tr>
<tr><td>13</td><td>-11.3</td><td>127.69</td></tr>
<tr><td>25</td><td>0.7</td><td>0.49</td></tr>
<tr><td colspan="3">$\Sigma(x-\overline{x})^2 = \mathbf{371.34}$</td></tr>
</table>
$s = \sqrt{\frac{\Sigma(x-\overline{x})^2}{n-1}}$
$s = \sqrt{\frac{371.34}{5}}$
$s = 8.6$ (1 d.p.)</td><td></td><td></td><td></td></tr>
<tr><td>**26.4**</td><td>Comparing data</td><td>In National 5 Mathematics there are two things to compare when comparing two or more sets of data: the **average** and the **spread** of the data. The average is either the mean or the median and the spread is either the interquartile range or the standard deviation.</td><td></td><td></td><td></td></tr>
</table>