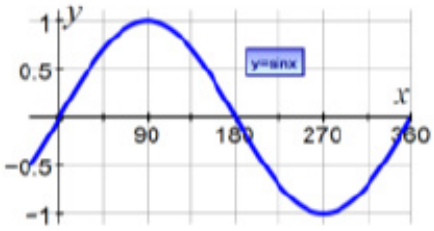
N5 Trig Graphs

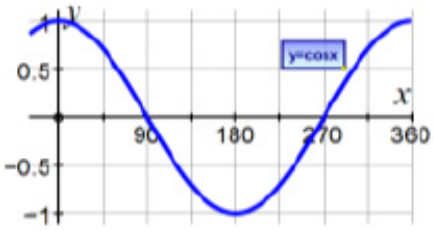
Relationships

SPTA Mathematics – Topic Questions with Notes

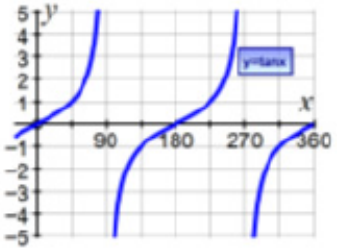
You are should know what the graphs of sin x, cos x and tan x look like between 0° and 360°:



sin



cos



tan

Definition: the **frequency** of a sin or cos graph is how many times the graph repeats itself in 360°. The frequency of a tan graph is how many times it repeats itself in 180°. In the equation of a sin, cos or tan graph, the frequency is the number before x.

Definition: the **amplitude** is a way of describing the height of a sin or cos graph – e.g. the sine and cosine graphs above both have an amplitude of 1. In the equation of a sin or cos graph, the amplitude is the number before sin or cos.

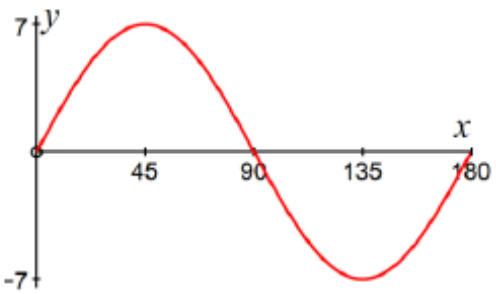
Definition: the **period** of a graph is how many degrees it takes the graph to do one complete cycle. In the graphs above, sin x and cos x have a period of 360° and tan x has a period of 180°.

$$\text{Period of a sin or cos graph} = \frac{360°}{\text{Frequency}} \qquad \text{Period of a tan graph} = \frac{180°}{\text{Frequency}}$$

Equation	Frequency	Amplitude	Period
$y = \cos x$	1	1	360°
$y = 3\sin 4x$	4	3	90°
$y = 6\cos 2x$	2	6	180°
$y = 5\tan 2x$	2	5	90°

Example 1

The graph on the right has an equation of the form $y = a \sin bx$. What are the values of a and b?



Solution

The maximum and minimum are 7 and –7, so the amplitude is 7 – i.e. a = 7.

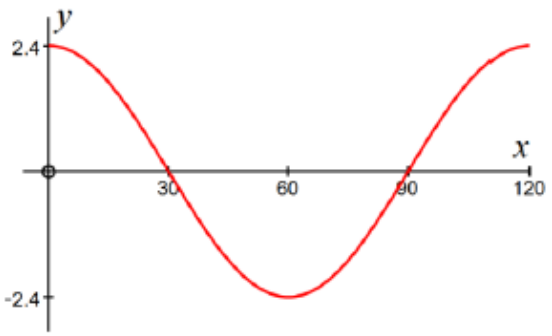
The graph repeats itself once between 0° and 180°. This means that it repeats itself twice between 0° and 360°, so it has a frequency of 2, so $b = 2$.

Answer: the graph is $y = 7 \sin 2x$

Look at the x axis carefully. It is not just a case of counting how many waves you can see, but you have to work out how many there would be in 360°. In exams, the graphs will often stop before 360°.

Example 2

The graph on the right has an equation of the form $y = a \cos bx$. What are the values of a and b?



Solution

The maximum and minimum are 2·4 and –2·4, so the amplitude is 2·4. This means that $a = 2 \cdot 4$.

This graph only goes up to 120°. The graph repeats itself once between 0° and 120°. This means that it repeats itself three times between 0° and 360°, so its frequency is 3, meaning $b = 3$.

Answer: the graph is $y = 2 \cdot 4 \cos 3x$

Example 3

What is the period of the function $3 \cos 4x$?

Solution

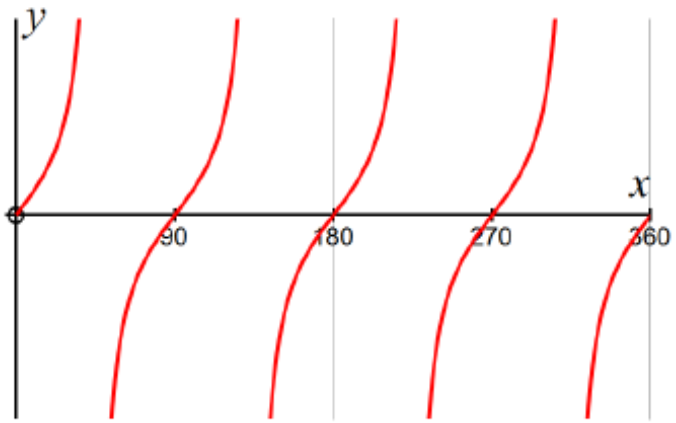
From the equation, the frequency of the graph is 4.

Using the formula for period, the period is $360 \div 4 = 90°$

With a tan graph, the frequency is how many times the graph repeats in 180°, not 360°.

Example 4

The graph on the right has an equation of the form $y = \tan bx$. What is the values of b?



Solution

The frequency of a tan graph is the number of waves in 180°.

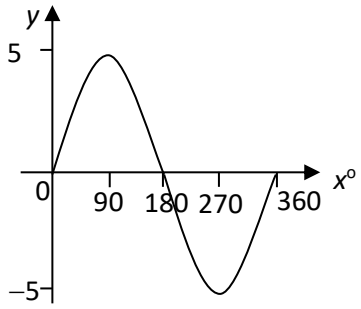
This means the frequency of this graph is 2 (and not 4), so $b = 2$.

Answer: the graph is $y = \tan 2x$

Exercise 1

1. The graphs represent the functions $a \sin x^o$ and $a \cos x^o$. Write down the equation for each.

a)



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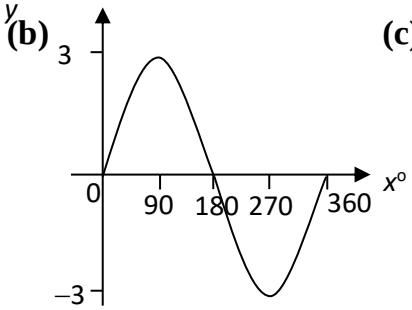

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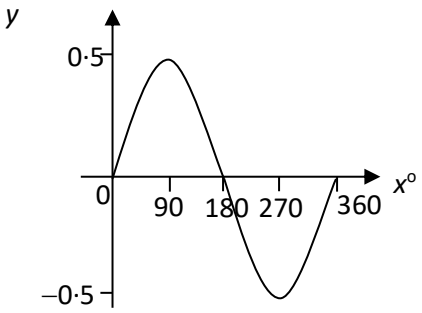

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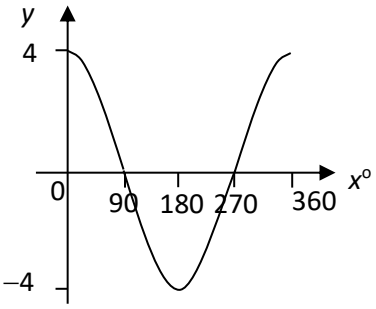

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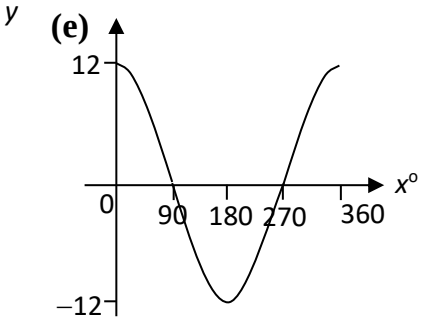

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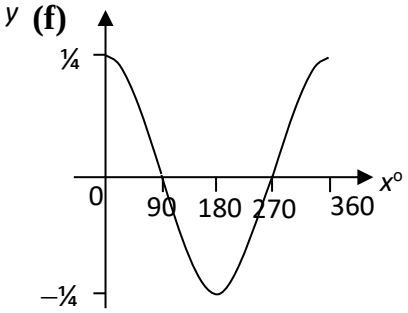

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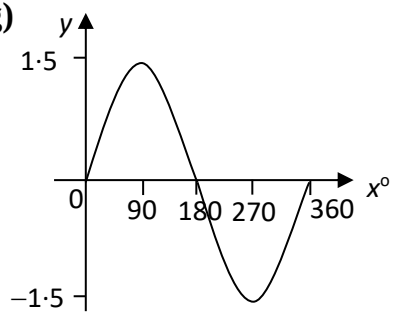

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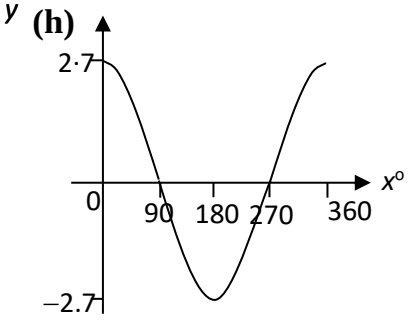

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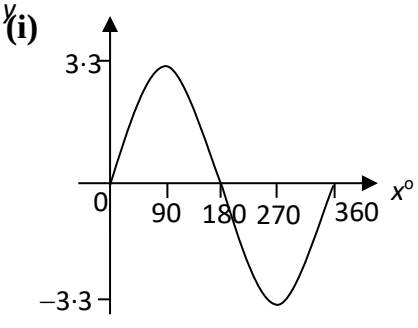

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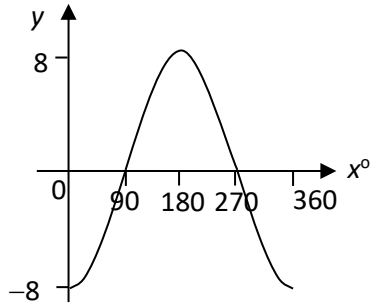

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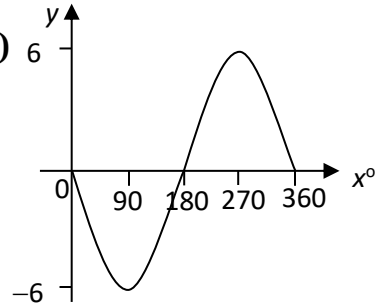

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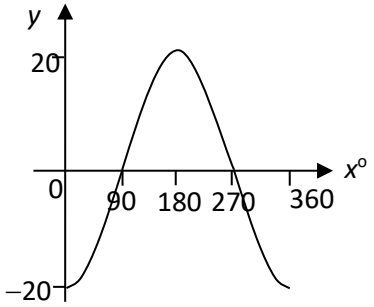

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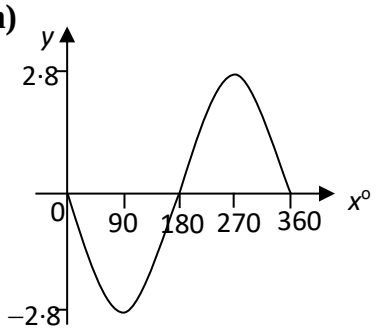

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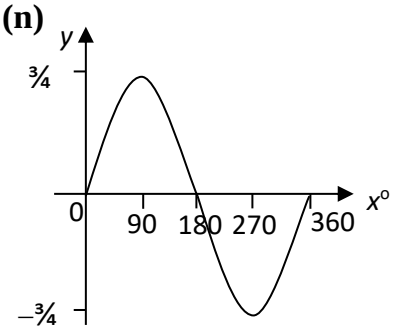

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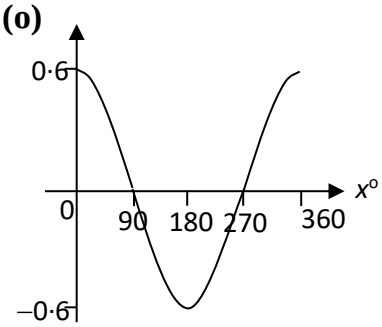
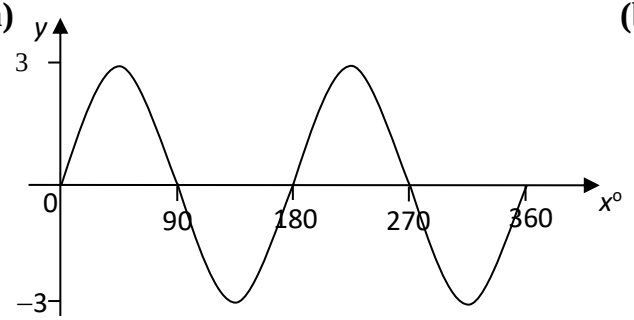


2. The graphs represent trigonometric functions. Write down the equation for each.

a)



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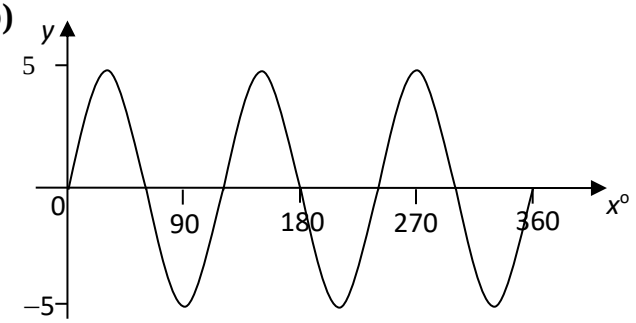

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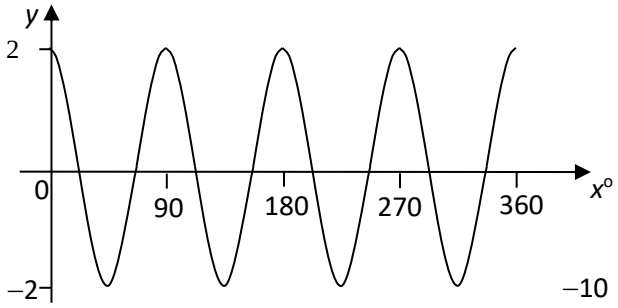
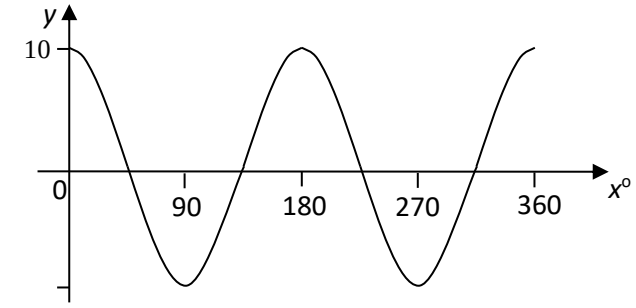


(d)



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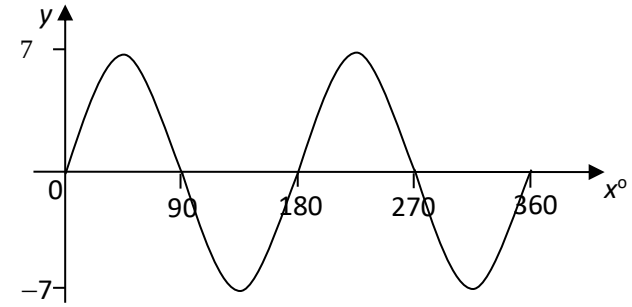

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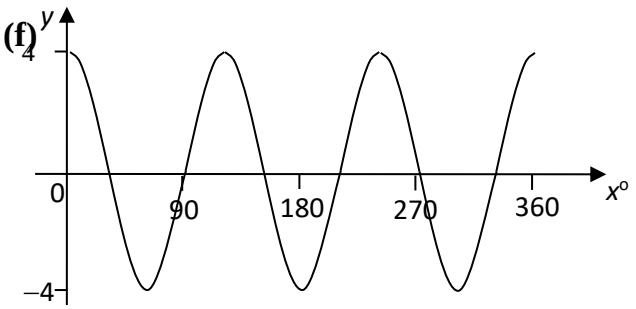
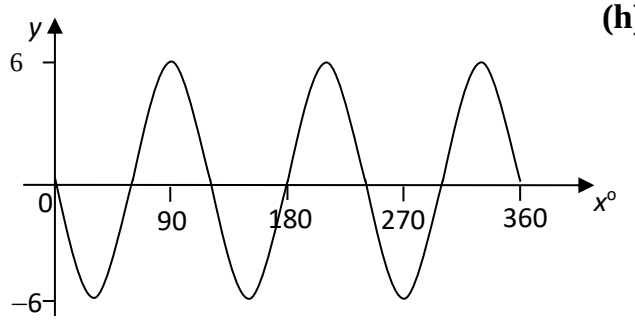
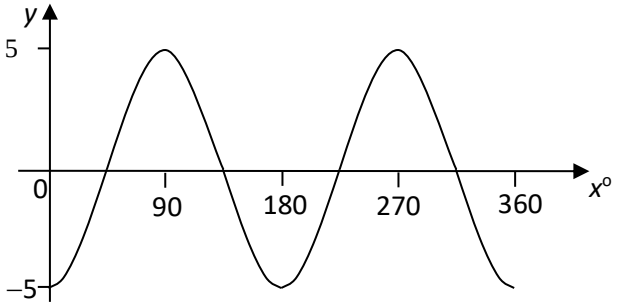


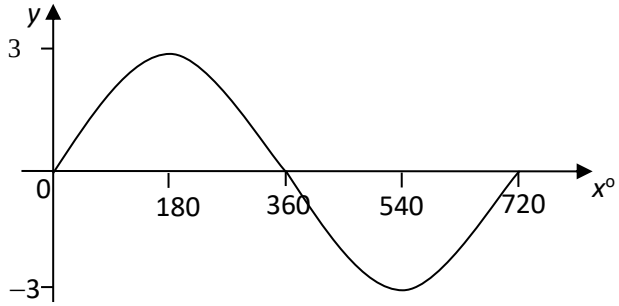
g)


y
6
0
90
180
270
360
x°
–6

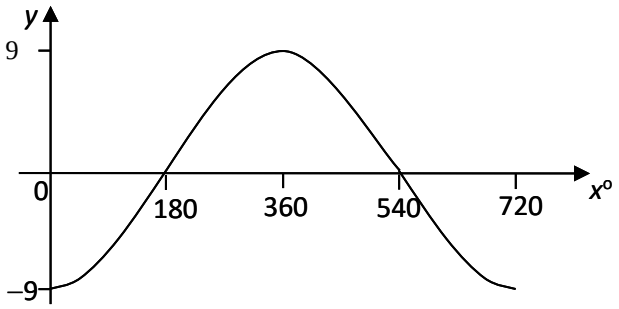
(h)


y
5
0
90
180
270
360
x°
–5

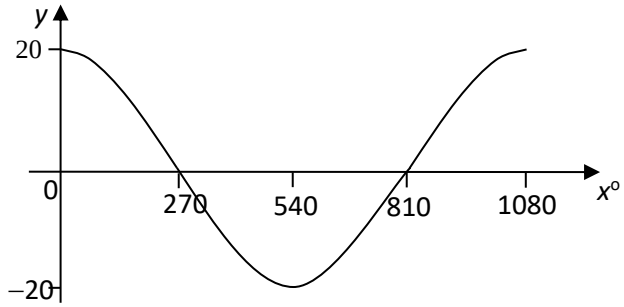
i)


y
3
0
180
360
540
720
x°
–3

(j)


y
9
0
180
360
540
720
x°
–9

k)


y
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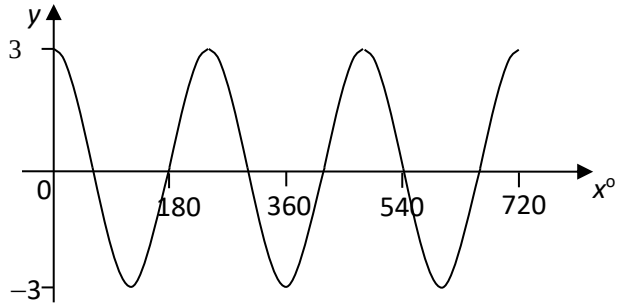
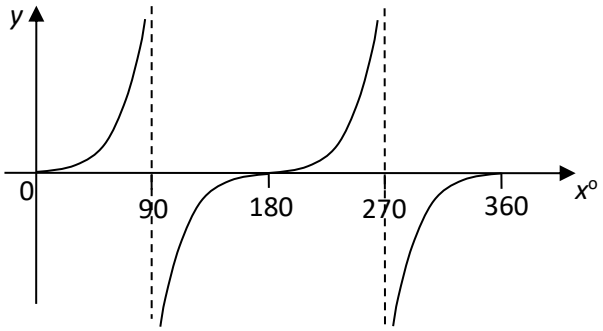
270
540
810
1080
x°
–20

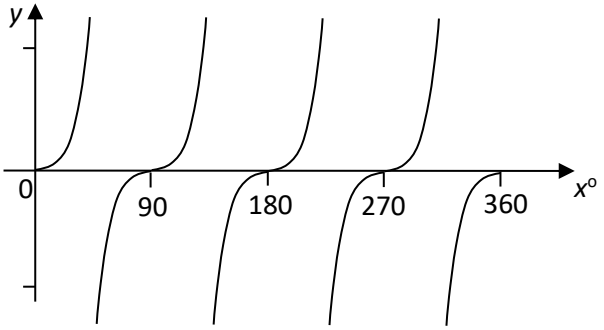
(l)


y
3
0
180
360
540
720
x°
–3

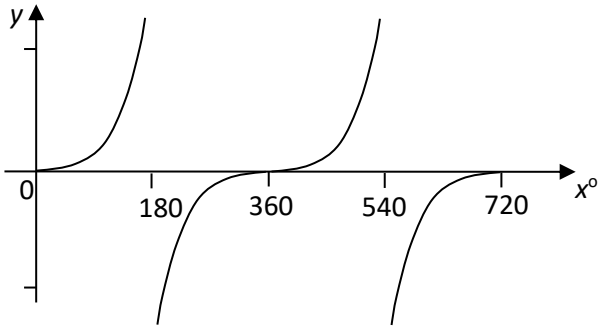
m)


y
0
90
180
270
360
x°

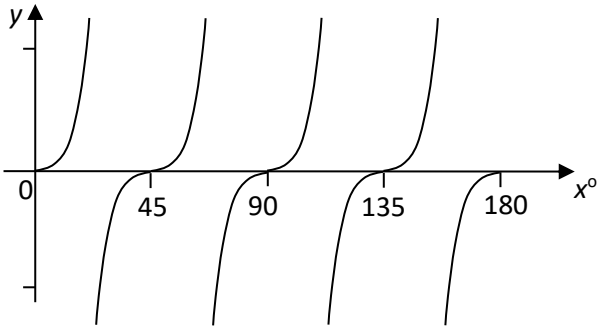
(n)


y
0
90
180
270
360
x°

o)


y
0
180
360
540
720
x°

(p)


y
0
45
90
135
180
x°

3. Make sketches of the following functions, $0 \le x < 360$, clearly marking any important points.

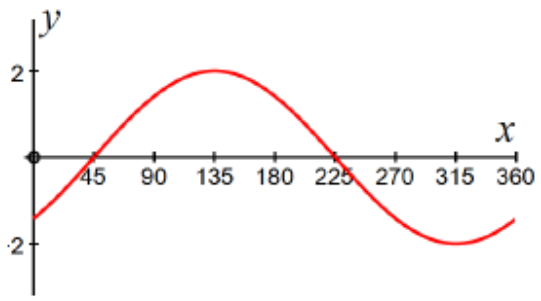
(a)	$y = \cos x^o$	**(b)**	$y = \sin x^o$	**(c)**	$y = \tan x^o$
(d)	$y = 3 \sin x^o$	**(e)**	$y = 2 \cos x^o$	**(f)**	$y = \sin 2x^o$
(g)	$y = \cos 3x^o$	**(h)**	$y = 2 \sin 3x^o$	**(i)**	$y = 3 \cos 2x^o$
(j)	$y = 4 \cos 3x^o$	**(k)**	$y = 3 \sin \frac{1}{2}x^o$	**(l)**	$y = 5 \cos \frac{3}{2} x^o$
(m)	$y = \tan 2x^o$	**(n)**	$y = -2 \sin 3x^o$	**(o)**	$y = -8 \cos 4x^o$

Definition: the **phase angle** is the amount a graph has been shifted to the right. In the equation of a sin, cos or tan graph, the phase angle is the number taken away from x in the brackets.

Equation	Period	Amplitude	Phase Angle
$y = 2\cos(x - 45)°$	1	2	45°
$y = \sin(x - 30)°$	1	1	30°
$y = 4\cos(x - 15)°$	1	4	15°

Example 5

The graph on the right has an equation of the form $y = a \sin(x - b)°$. What are the values of a and b?



Solution

The maximum and minimum are 2 and –2, so the amplitude is 2. So this means that $a = 2$.

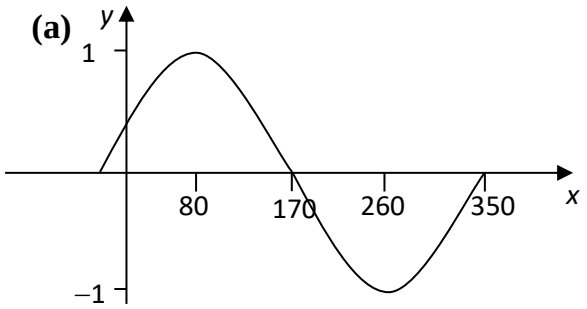
The graph has been shifted 45° to the right, so $b = 45°$.

Answer: the graph is $y = 2\sin(x - 45)°$

Exercise 2

1. The graphs represent the functions $\sin (x \pm a)^o$ and $\cos (x \pm a)^o$. Write down each equation.

(a)



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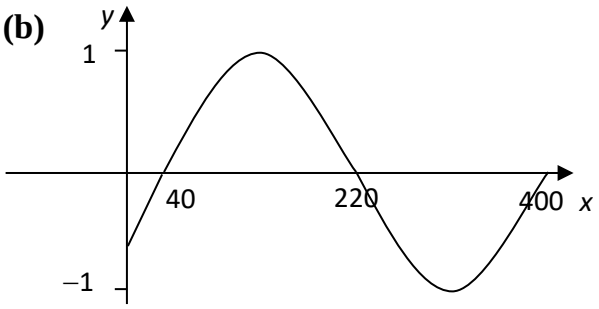

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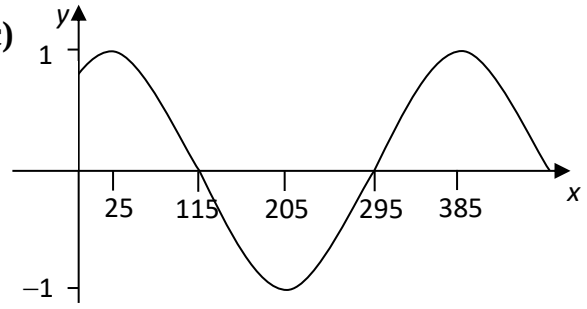
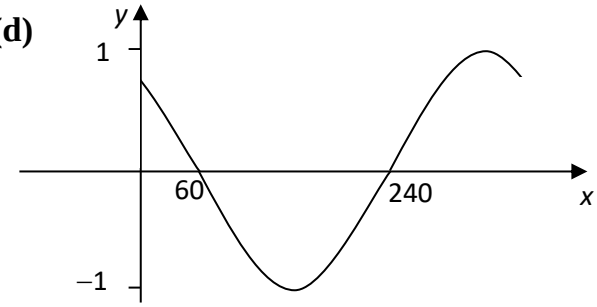

y
1
25
115
205
295
385
x
–1

(d)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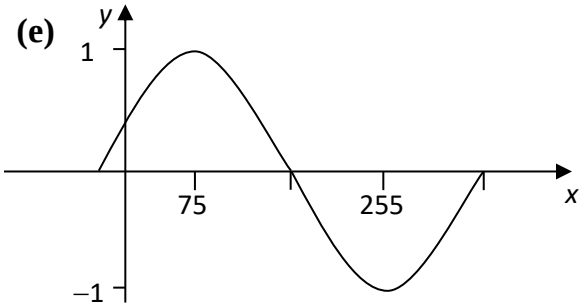
240
x
–1

(e)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x
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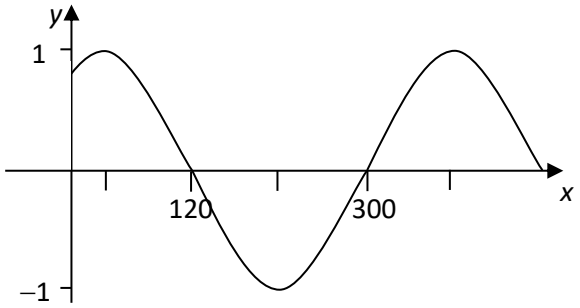
(f)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x
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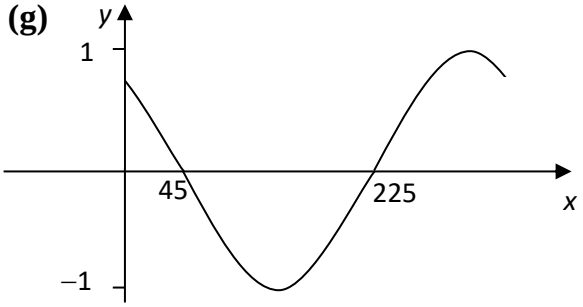
(g)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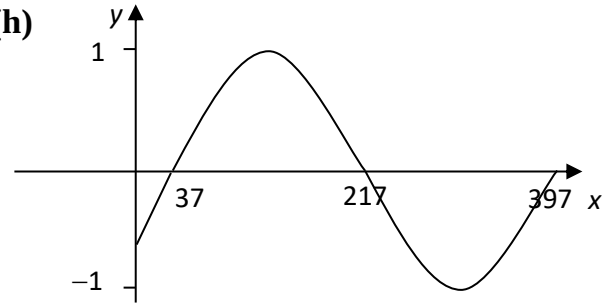
x
–1

(h)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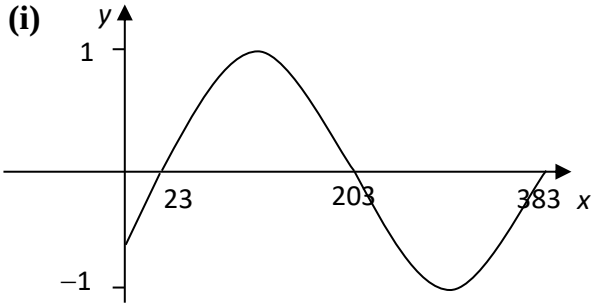
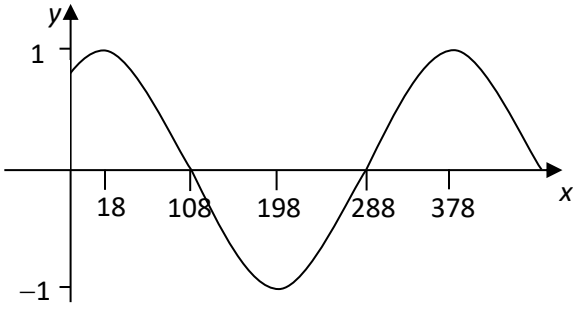
397
x
–1

(i)

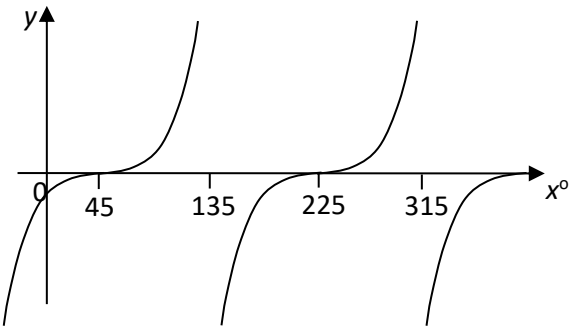

y
1
23
203
383
x
–1

(j)

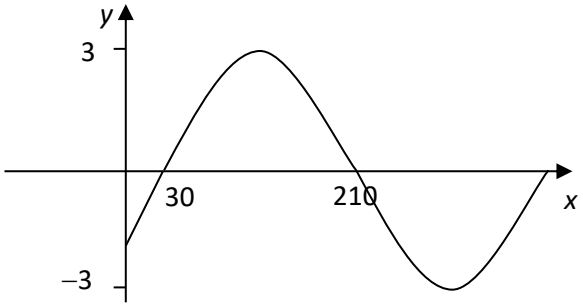

y
1
18
108
198
288
378
x
–1

2. Write down the equation for each graph shown below.

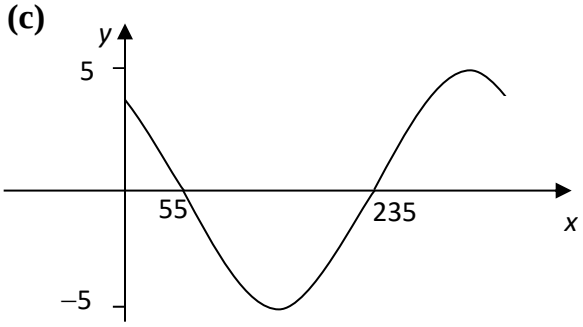
(a)



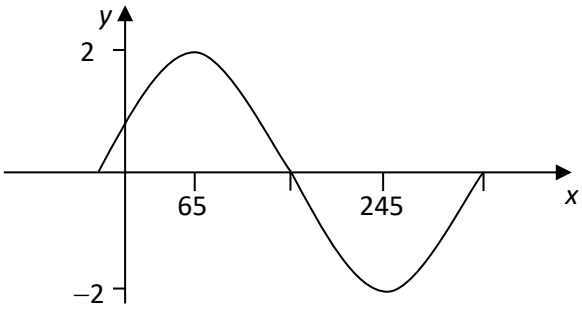
(b)



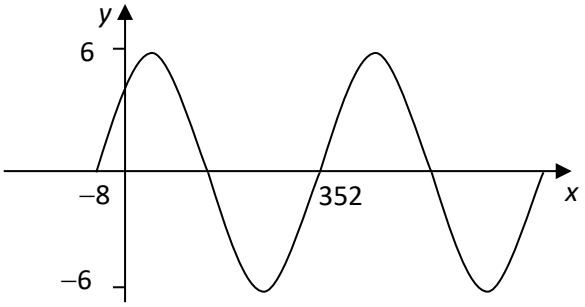
(c)



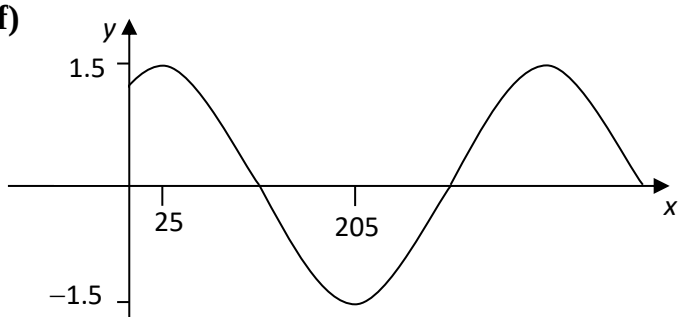
(d)



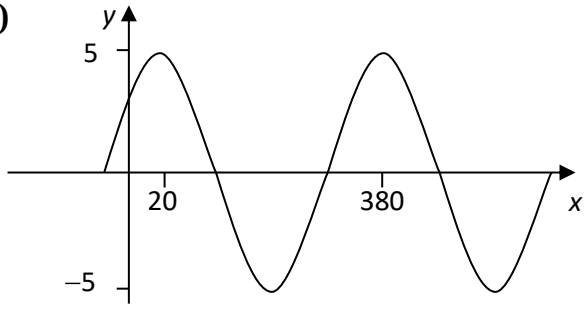
(e)



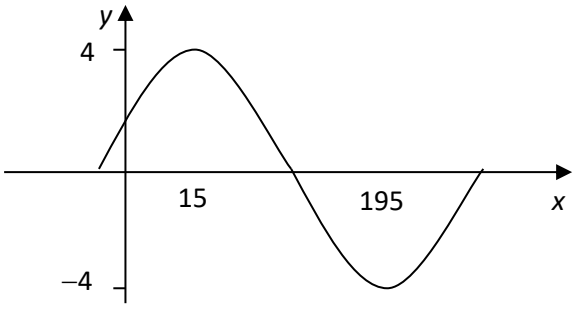
(f)



(g)



(h)



3. Make a neat sketch of these trig. functions showing the important values, $0 \leq x \leq 360$.

(a) $y = \sin (x - 50)^o$ **(b)** $y = \sin (x + 30)^o$

(c) $y = \cos (x - 20)^o$ **(d)** $y = \cos (x + 60)^o$

(e) $y = \tan (x - 30)^o$ **(f)** $y = \sin (x - 45)^o$

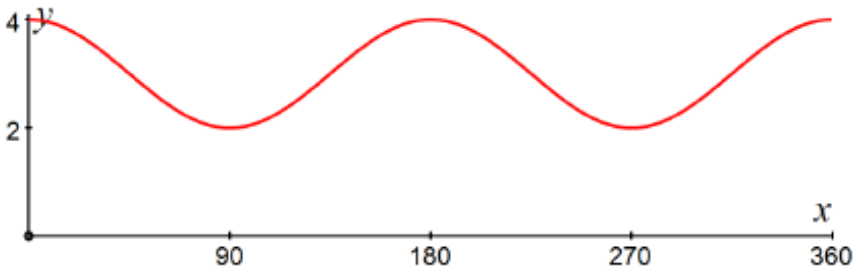
Definition: a **vertical translation** is the amount a graph has been shifted up or down. In the equation of a sin, cos or tan graph, the vertical translation is the number added on at the end of an equation.

Equation	Amplitude	Vertical translation
$y = 2\cos x + 3$	2	3
$y = 5\sin 2x - 1$	5	−1

Example 6

The graph on the right has an equation of the form $y = \cos bx + c$.

What are the values of b and c?



Solution

b is the frequency. The graph repeats twice in 360°, so this means that $b = 2$.

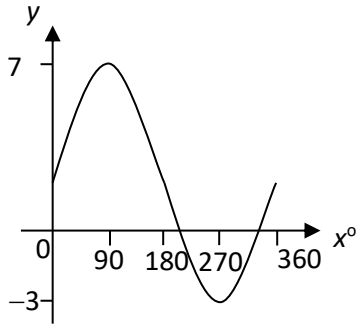
A cos graph would normally have y values between 1 and −1. This graph goes between 4 and 2. This means all the values are 3 higher. Therefore $c = 3$.

Answer: the graph is $y = \cos 2x + 3$

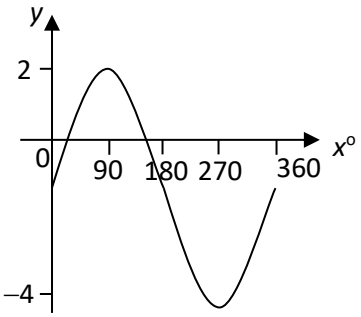
Exercise 3

1. For each graph below, determine the amplitude and the equation of the graph.

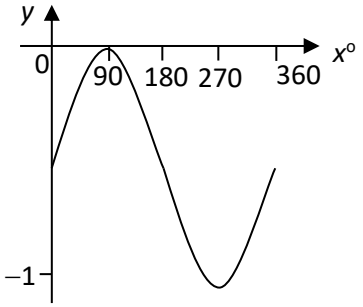
(a)



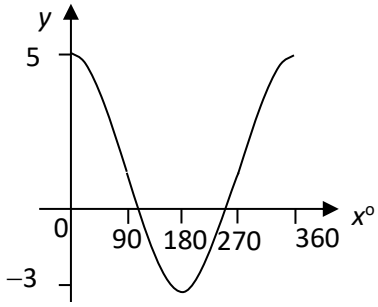
(b)



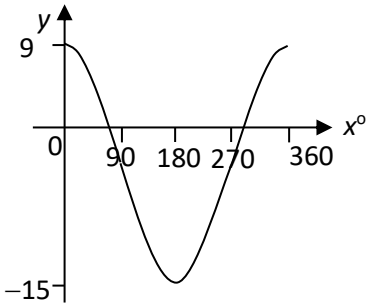
(c)



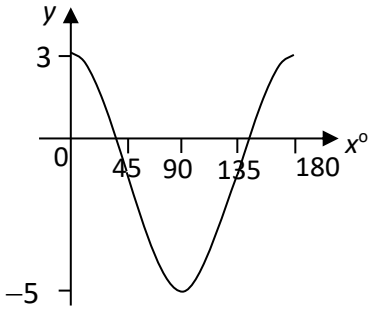
(d)



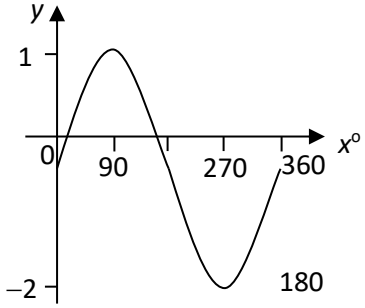
(e)



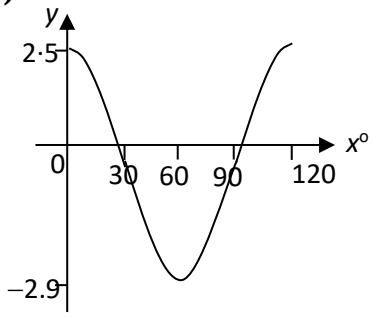
(f)



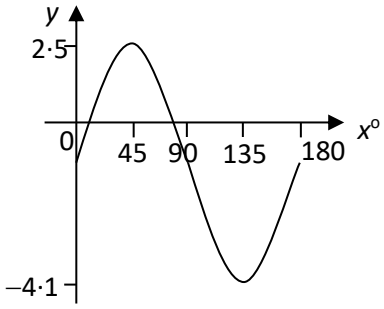
(g)



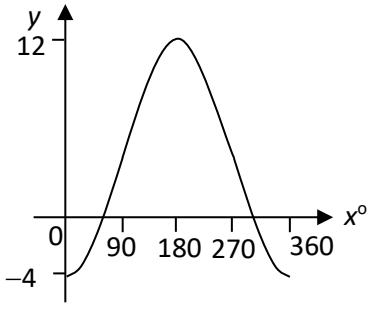
(h)



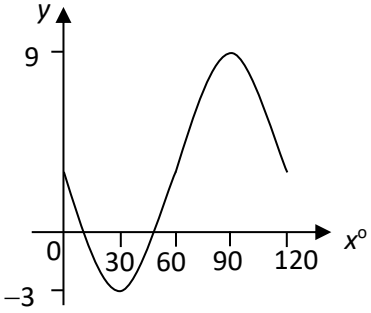
(i)



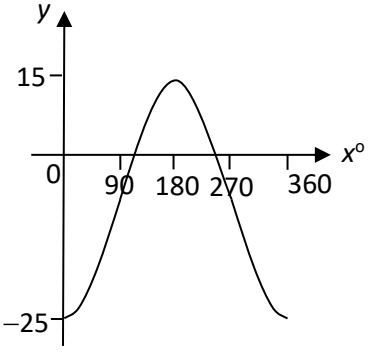
(j)



(k)



(l)



2. Make sketches of the following functions, $0 \leq x < 360$, clearly marking any important points.

(a) $y = 3\cos x^{o} + 1$ **(b)** $y = 2\sin 3x^{o} - 2$ **(c)** $y = 1{\cdot}5\sin 2x^{o} + 3$

(d) $y = 5\sin 2x^{o} + 1$ **(e)** $y = -3\cos 2x^{o} - 1$ (f) $y = -\sin x^{o} - 3$

Answers

Exercise 1

1. **(a)** $y = 5 \sin x^o$ **(b)** $y = 3 \sin x^o$ **(c)** $y = 0{\cdot}5 \sin x^o$ **(d)** $y = 4 \cos x^o$

(e) $y = 12 \cos x^o$ **(f)** $y = \frac{1}{4} \cos x^o$ **(g)** $y = 1{\cdot}5 \sin x^o$ **(h)** $y = 2{\cdot}7 \cos x^o$

(i) $y = 3{\cdot}3 \sin x^o$ **(j)** $y = -8 \cos x^o$ **(k)** $y = -6 \sin x^o$ **(l)** $y = -20 \cos x^o$

(m) $y = -2{\cdot}8 \sin x^o$ **(n)** $y = \frac{3}{4} \sin x^o$ **(o)** $y = 0{\cdot}6 \cos x^o$

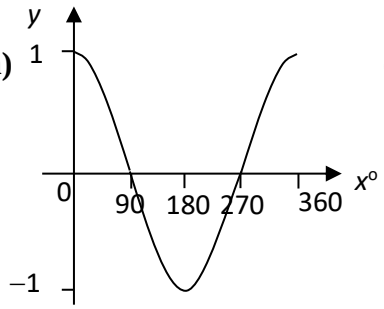
2. **(a)** $y = 3 \sin 2x^o$ **(b)** $y = 5 \sin 3x^o$ **(c)** $y = 2 \cos 4x^o$ **(d)** $y = 10 \cos 2x^o$

(e) $y = 7 \sin 2x^o$ **(f)** $y = 4 \cos 3x^o$ **(g)** $y = -6 \sin 3x^o$ **(h)** $y = -5 \cos 2x^o$

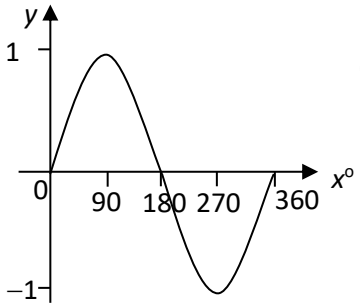
(i) $y = 3 \sin \frac{1}{2} x^o$ **(j)** $y = 9 \cos \frac{1}{2} x^o$ **(k)** $y = 20\cos \frac{1}{3} x^o$ **(l)** $y = 3 \cos \frac{3}{2} x^o$

(m) $y = \tan x^o$ **(n)** $y = \tan 2x^o$ **(o)** $y = \tan \frac{1}{2} x^o$ **(p)** $y = \tan 4x^o$

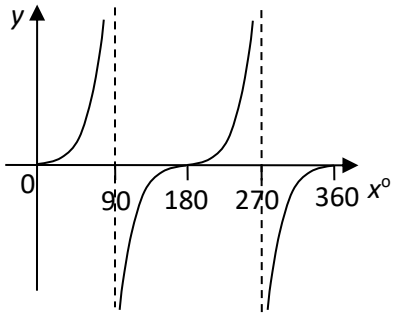
3. **(a)**



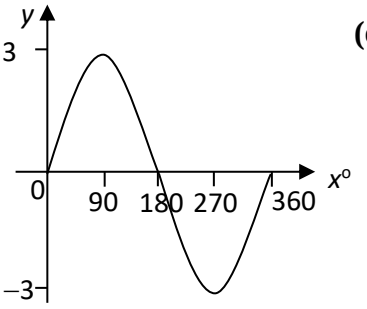
(b)



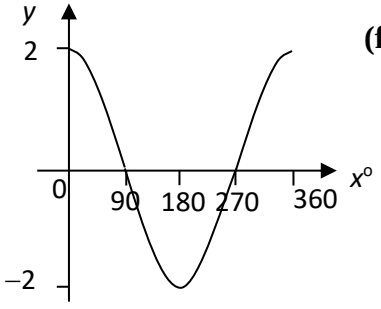
(c)



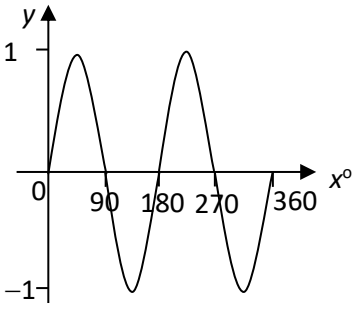
(d)



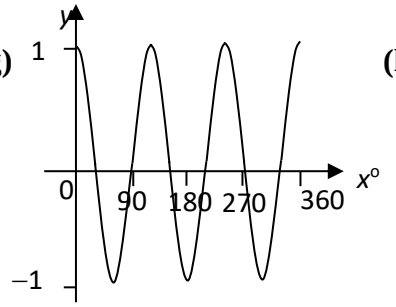
(e)



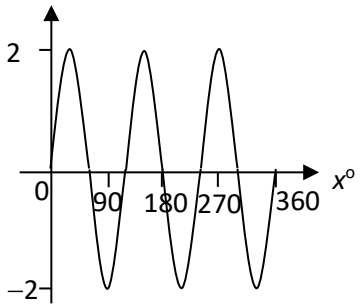
(f)



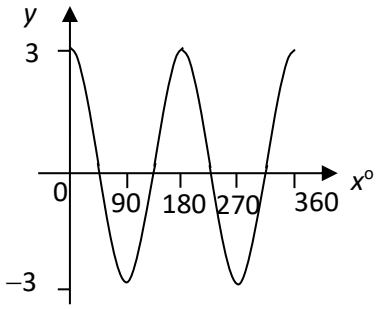
(g)



(h)



(i)



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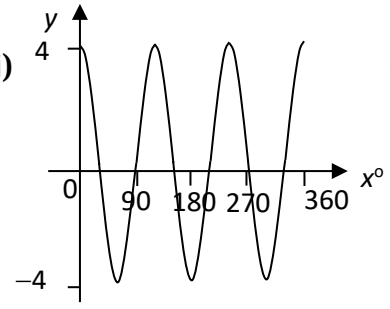
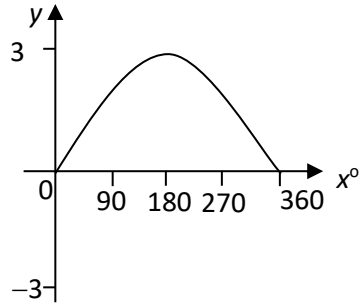
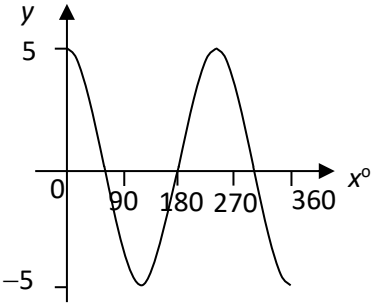


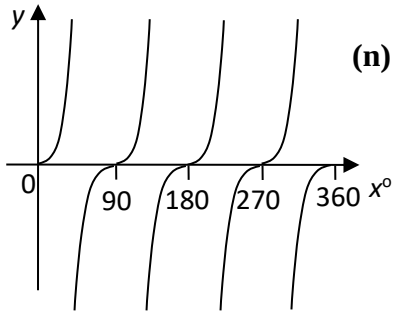
(k)



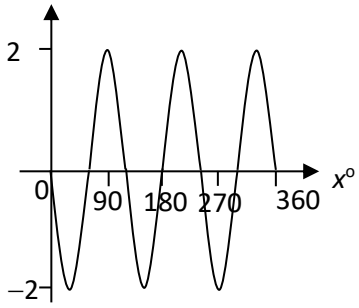
(l)



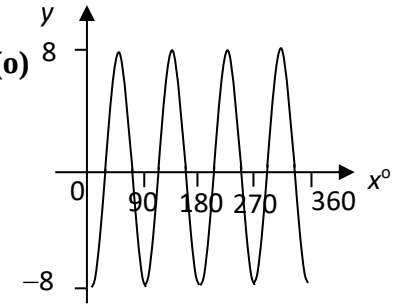
(m)



(n)



(o)



Exercise 2

1. **(a)** $y = \sin (x + 10)^o$ **(b)** $y = \sin (x - 40)^o$ **(c)** $y = \cos (x - 25)^o$

(d) $y = \cos (x + 30)^o$ **(e)** $y = \sin (x + 15)^o$ **(f)** $y = \cos (x - 30)^o$

(g) $y = \cos (x + 45)^o$ **(h)** $y = \sin (x - 37)^o$ **(i)** $y = \sin (x - 23)^o$

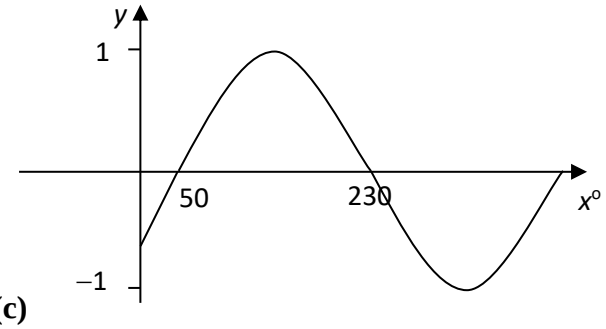
(j) $y = \cos (x - 18)^o$

2. **(a)** $y = \tan (x - 45)^o$ **(b)** $y = 3 \sin (x - 30)^o$ **(c)** $y = 5 \cos (x + 35)^o$

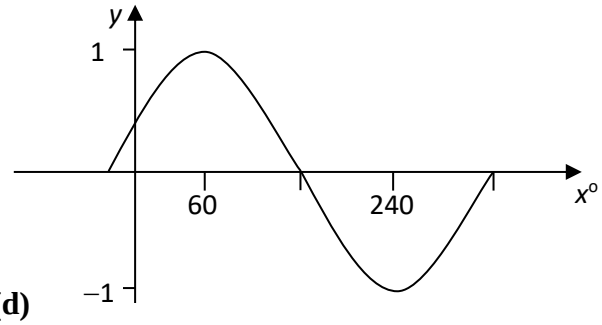
(d) $y = 2 \sin (x + 25)^o$ **(e)** $y = 6 \sin (x + 8)^o$ **(f)** $y = 1.5 \cos (x - 25)^o$

(g) $y = 5 \cos (x - 20)^o$ **(h)** $y = 4 \sin (x + 75)^o$

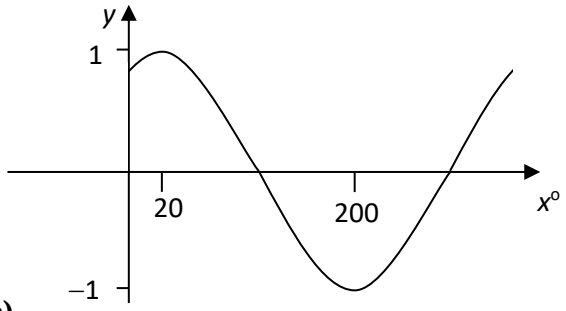
3.. **(a)**



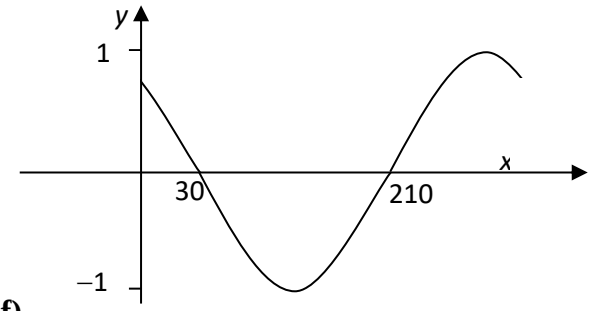
(b)



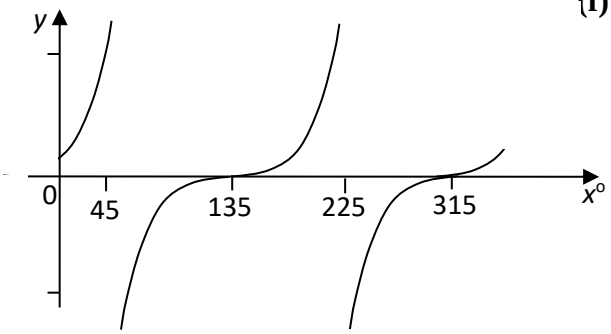
(c)



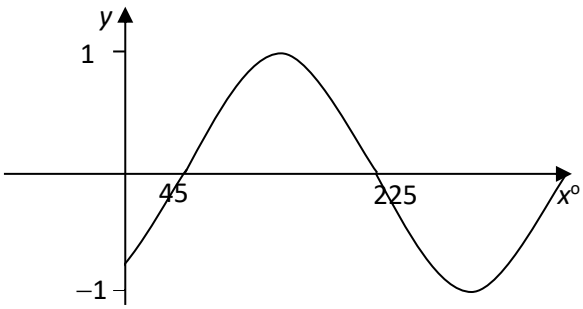
(d)



(e)



(f)



Exercise 3

1. **(a)** 5; $y = 5\sin x^\circ + 2$ **(b)** 3; $y = 3\sin x^\circ - 1$ **(c)** 0·5; $y = 0{\cdot}5\sin x^\circ - 0{\cdot}5$

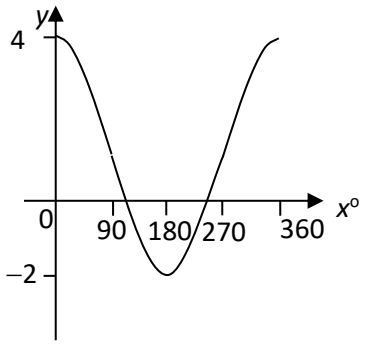
(d) 4; $y = 4\cos x^\circ + 1$ **(e)** 12; $y = 12\cos x^\circ - 3$ **(f)** 4; $y = 4\cos 2x^\circ - 1$

(g) 1·5; $y = 1{\cdot}5\sin x^\circ - 0{\cdot}5$ **(h)** 2·7; $y = 2{\cdot}7\cos 3x^\circ - 0{\cdot}2$

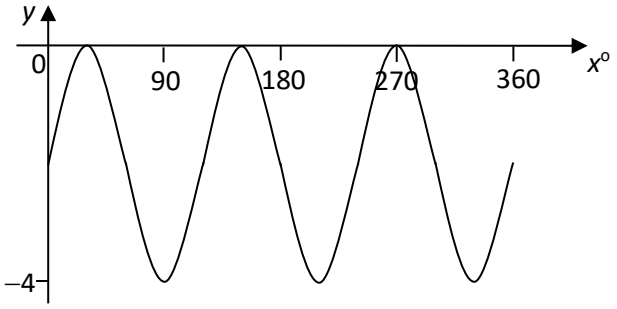
(i) 3·3; $y = 3{\cdot}3\sin 2x^\circ - 0{\cdot}8$ **(j)** 8; $y = -8\cos x^\circ + 4$

(k) 6; $y = -6\sin 3x^\circ + 3$ **(l)** 20; $y = -20\cos x^\circ - 5$

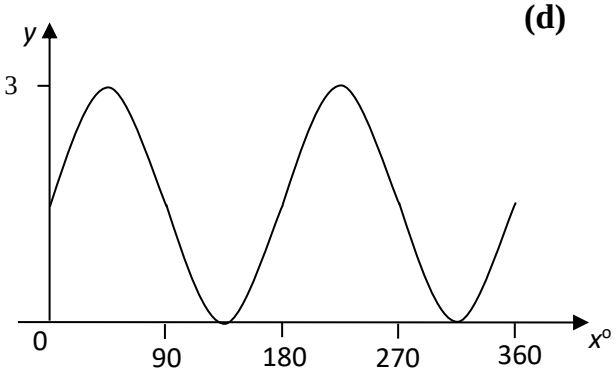
2. **(a)**


y
4
0
90
180
270
360
x°
−2

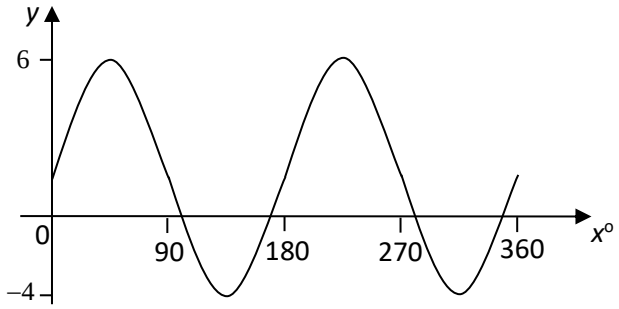
(b)


y
0
90
180
270
360
x°
−4

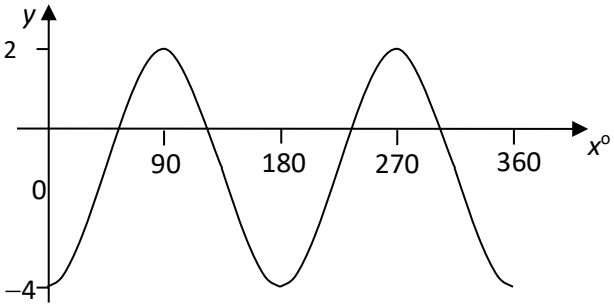
(c)


y
3
0
90
180
270
360
x°

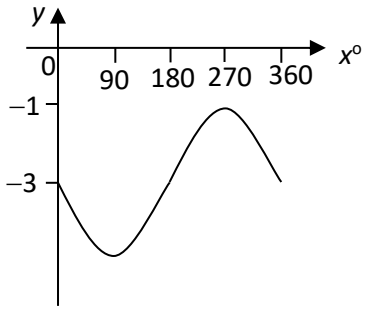
(d)


y
6
0
90
180
270
360
x°
−4

(e)


y
2
0
90
180
270
360
x°
−4

(f)


y
0
90
180
270
360
x°
−1
−3