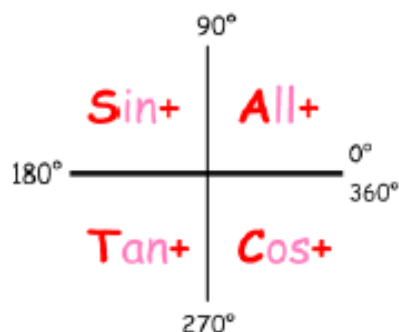


You can recognise these questions because they will ask you to **solve each equation for** $0 \leq x < 360^\circ$. (This just means x must be between 0° and 360° .)

These questions will usually have TWO answers:

- Your calculator will give you the first using $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$
- To get the other, you need either a CAST diagram or a sketch of the graph.

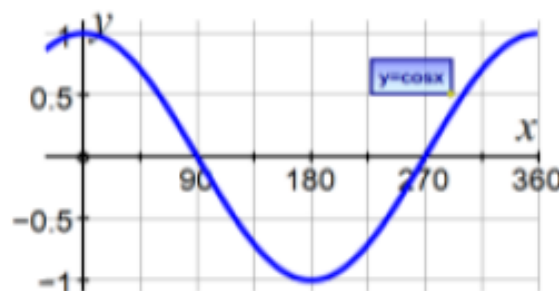
$$180 - x$$



$$180 + x$$

$$360 - x$$

or



Example 1 (Positive values of sin, cos and tan)

Solve the equation $5 \sin x - 2 = 1$ for $0 \leq x < 360^\circ$:

Solution:

Step One – rearrange the equation

$$5 \sin x - 2 = 1$$

$$5 \sin x = 1 + 2$$

$$5 \sin x = 3$$

$$\sin x = \frac{3}{5}$$

Step Two – find the first solution using $\sin^{-1}$

$$x = \sin^{-1}(3 \div 5)$$

$$x = 36.9^\circ$$

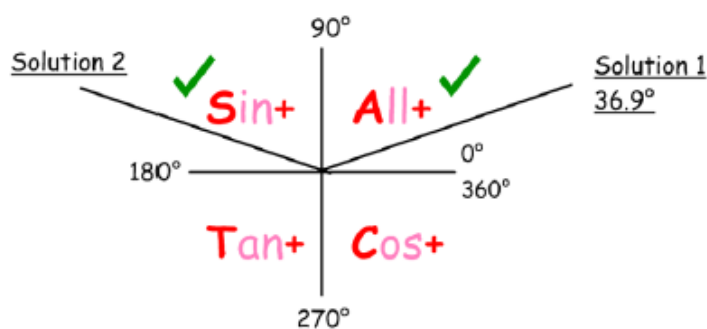
Step Three – find the second solution using CAST

This question involves **sin**. The number on the right is $\frac{3}{5}$, which is **positive**. This means we tick the ALL and SIN quadrants.



Putting both answers into the diagram shows using symmetry that solution 2 is given by $180 - 36.9 = 143.1^\circ$.

Answer: $x = 36.9^\circ, x = 143.1^\circ$



Example 2 (Negative values of sin, cos and tan)

Solve the equation $3 \cos x + 3 = 1$ for $0 \leq x < 360^\circ$:

Solution:

Step One – rearrange the equation

$$3 \cos x + 3 = 1$$

$$3 \cos x = 1 - 3$$

$$3 \cos x = -2$$

$$\cos x = \frac{-2}{3}$$

$$x = \cos^{-1}(-2 \div 3)$$

$$x = 131.8^\circ$$

Step Two – find the first solution using $\cos^{-1}$

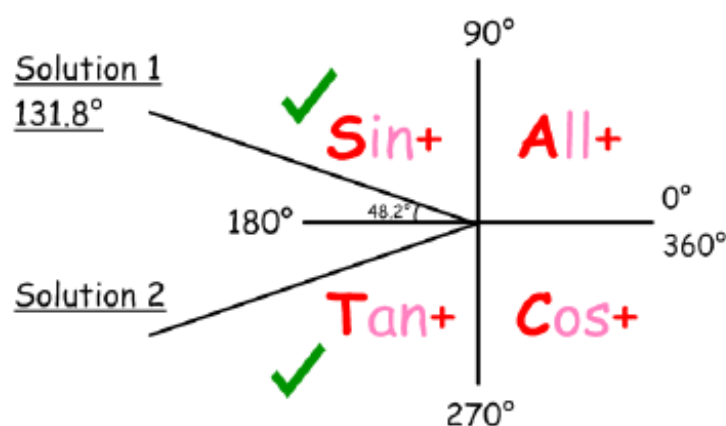
Step Three – find the second solution using CAST

This question involves **cos**. The number on the right is $-\frac{2}{3}$, which is **negative**. This means that cos is NOT positive, so we do NOT tick ALL and COS, instead we tick the SIN and TAN quadrants.



Putting both answers into the diagram shows using symmetry that solution 2 is given by $180 + 48.2 = 228.2^\circ$.

Answer: $x = 131.8^\circ, x = 228.2^\circ$



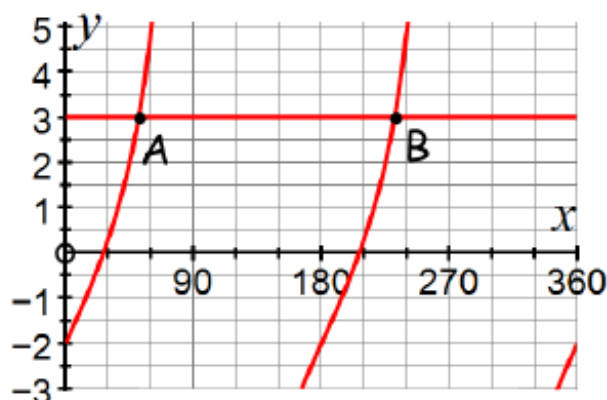
You may be asked to find the point of intersection of two graphs. To do this, we form an equation by making the equations of each graph equal to each other. For example to find the points of intersection of the graphs $5 \sin x$ and $y = 3$, we would solve the equation $5 \sin x = 3$.

Example 3 – from a graph

The diagram shows the graphs of $y = 4 \tan x - 2$ and the graph of $y = 3$ for $0 \leq x < 360^\circ$. Find the x -coordinates of the points of intersection A and B.

Solution

To find the points of intersection, we make the two equations equal to each other: i.e. we solve the equation $4 \tan x - 2 = 3$.



We now solve this using the method from examples 1 and 2:

Step One – rearrange the equation

$$\begin{aligned} 4 \tan x - 2 &= 3 \\ 4 \tan x &= 3 + 2 \\ 4 \tan x &= 5 \end{aligned}$$

Step Two – find the first solution using $\tan^{-1}$

$$\begin{aligned} \tan x &= \frac{5}{4} \\ x &= \tan^{-1}(5 \div 4) \\ x &= 51.3^\circ \end{aligned}$$

Step Three – find the second solution using CAST

This question involves **tan**. The number on the right is $\frac{5}{4}$, which is **positive**. This means we tick the ALL and TAN quadrants.

A CAST diagram shows us that solution 2 is given by $180 + 51.3 = 231.3^\circ$.

Answer: The x-coordinates are $x = 51.3^\circ$ and $x = 231.3^\circ$

Exercise 1

1. Solve the following equations where $0 \leq x \leq 360$

- | | | |
|-----------------------------------|------------------------------------|------------------------------------|
| a) $\sin x^\circ = 0.5$ | (b) $\cos x^\circ = 0.866$ | (c) $\tan x^\circ = 1$ |
| d) $\cos x^\circ = -0.5$ | (e) $\tan x^\circ = -0.577$ | (f) $\sin x^\circ = -0.866$ |
| g) $\tan x^\circ = 1.732$ | (h) $\sin x^\circ = 0.707$ | (i) $\cos x^\circ = 0.707$ |
| j) $\sin x^\circ = -0.707$ | (k) $\cos x^\circ = -0.866$ | (l) $\tan x^\circ = -1.732$ |

2. Solve the following equations where $0 \leq x \leq 360$

- | | | |
|-----------------------------------|------------------------------------|------------------------------------|
| a) $\sin x^\circ = 0.313$ | (b) $\cos x^\circ = 0.425$ | (c) $\tan x^\circ = 5.145$ |
| d) $\cos x^\circ = -0.087$ | (e) $\tan x^\circ = -0.869$ | (f) $\sin x^\circ = -0.191$ |
| g) $\tan x^\circ = 11.43$ | (h) $\sin x^\circ = 0.695$ | (i) $\cos x^\circ = 0.755$ |
| j) $\sin x^\circ = -0.358$ | (k) $\cos x^\circ = -0.682$ | (l) $\tan x^\circ = -0.268$ |

3. Solve the following equations where $0 \leq x \leq 360$

- | | | |
|-----------------------------------|-----------------------------------|-----------------------------------|
| a) $2 \sin x^\circ = 1$ | (b) $3 \cos x^\circ = 2$ | (c) $3 \tan x^\circ = 5$ |
| d) $2 \cos x^\circ = -1$ | (e) $2 \tan x^\circ = -8$ | (f) $4 \sin x^\circ = -3$ |
| g) $5 \tan x^\circ = 23.5$ | (h) $5 \sin x^\circ = 2$ | (i) $6 \cos x^\circ = 1$ |
| j) $8 \sin x^\circ = -3$ | (k) $11 \cos x^\circ = -9$ | (l) $10 \tan x^\circ = -9$ |

4. Solve the following equations where $0 \leq x \leq 360$

- | | | |
|------------------------------------|-------------------------------------|-------------------------------------|
| a) $\sin x^\circ - 1 = 0$ | (b) $\cos x^\circ + 1 = 0$ | (c) $\tan x^\circ - 1 = 0$ |
| d) $2 \sin x^\circ + 1 = 0$ | (e) $2 \cos x^\circ - 1 = 0$ | (f) $2 \tan x^\circ - 1 = 0$ |
| g) $4 \cos x^\circ - 3 = 0$ | (h) $3 \sin x^\circ - 2 = 0$ | (i) $5 \cos x^\circ + 2 = 0$ |
| j) $3 \tan x^\circ - 2 = 0$ | (k) $3 \cos x^\circ + 1 = 0$ | (l) $7 \sin x^\circ + 3 = 0$ |

5. Solve the following equations where $0 \leq x \leq 360$

- | | | |
|---|---------------------------------------|--|
| a) $4 \cos x^\circ + 3 = 2$ | (b) $10 \sin x^\circ - 4 = 3$ | (c) $2 \tan x^\circ - 3 = 17$ |
| d) $7 + 10 \cos x^\circ = 12$ | (e) $2 \tan x^\circ + 3 = 5$ | (f) $17 - 5 \cos x^\circ = 20$ |
| g) $5 \sin x^\circ + 3 = 5$ | (h) $21 + 2 \cos x^\circ = 20$ | (i) $2 \sin x^\circ - 1 \cdot 6 = 0$ |
| j) $3 \cos x^\circ + \sqrt{2} = 0$ | (k) $7 \sin x^\circ - 1 = 4$ | (l) $2 \sin x^\circ + \sqrt{3} = 2\sqrt{2}$ |

Answers

Exercise 1

- | | | | | | | |
|----|------------|----------------------------|------------|----------------------------|------------|----------------------------|
| 1. | (a) | $30^\circ, 150^\circ$ | (b) | $30^\circ, 330^\circ$ | (c) | $45^\circ, 225^\circ$ |
| | (d) | $120^\circ, 240^\circ$ | (e) | $150^\circ, 330^\circ$ | (f) | $240^\circ, 300^\circ$ |
| | (g) | $60^\circ, 240^\circ$ | (h) | $45^\circ, 135^\circ$ | (i) | $45^\circ, 315^\circ$ |
| | (j) | $225^\circ, 315^\circ$ | (k) | $150^\circ, 210^\circ$ | (l) | $120^\circ, 300^\circ$ |
| 2. | (a) | $18.2^\circ, 161.8^\circ$ | (b) | $64.8^\circ, 295.2^\circ$ | (c) | $79^\circ, 259^\circ$ |
| | (d) | $95^\circ, 265^\circ$ | (e) | $139^\circ, 319^\circ$ | (f) | $191^\circ, 349^\circ$ |
| | (g) | $85^\circ, 265^\circ$ | (h) | $44^\circ, 136^\circ$ | (i) | $41^\circ, 319^\circ$ |
| | (j) | $201^\circ, 339^\circ$ | (k) | $133^\circ, 227^\circ$ | (l) | $165^\circ, 345^\circ$ |
| 3. | (a) | $30^\circ, 150^\circ$ | (b) | $48.2^\circ, 311.8^\circ$ | (c) | $59^\circ, 239^\circ$ |
| | (d) | $120^\circ, 240^\circ$ | (e) | $104^\circ, 284^\circ$ | (f) | $228.6^\circ, 311.4^\circ$ |
| | (g) | $78^\circ, 258^\circ$ | (h) | $23.6^\circ, 156.4^\circ$ | (i) | $80.4^\circ, 279.6^\circ$ |
| | (j) | $202^\circ, 338^\circ$ | (k) | $144.9^\circ, 215.1^\circ$ | (l) | $138^\circ, 318^\circ$ |
| 4. | (a) | 90° | (b) | 180° | (c) | $45^\circ, 225^\circ$ |
| | (d) | $210^\circ, 330^\circ$ | (e) | $60^\circ, 300^\circ$ | (f) | $26.6^\circ, 206.6^\circ$ |
| | (g) | $41.4^\circ, 318.6^\circ$ | (h) | $41.8^\circ, 138.2^\circ$ | (i) | $113.6^\circ, 246.4^\circ$ |
| | (j) | $33.7^\circ, 213.7^\circ$ | (k) | $109.5^\circ, 250.5^\circ$ | (l) | $205.4^\circ, 334.6^\circ$ |
| 5. | (a) | $104.5^\circ, 255.5^\circ$ | (b) | $44.4^\circ, 135.6^\circ$ | (c) | $84.3^\circ, 264.3^\circ$ |
| | (d) | $60^\circ, 300^\circ$ | (e) | $45^\circ, 225^\circ$ | (f) | $126.9^\circ, 233.1^\circ$ |
| | (g) | $23.6^\circ, 156.4^\circ$ | (h) | $120^\circ, 240^\circ$ | (i) | $53.1^\circ, 126.9^\circ$ |