N5 Straight Line

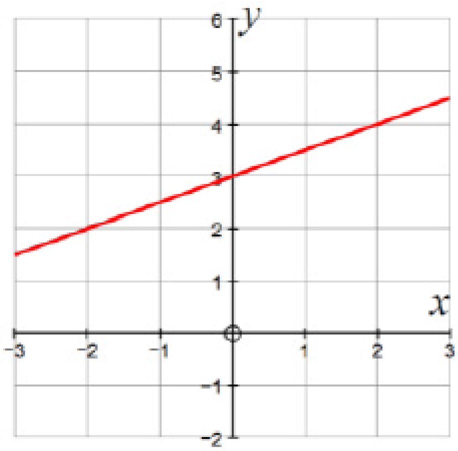
Relationships

SPTA Mathematics – Topic Questions with Notes

The Equation of a Straight Line

The equation of any straight line is linked to the gradient of the line, and the y-intercept of the line:

- In the expressions and formulae unit, it is explained how to work out the **gradient** of a straight line. For example, it is shown that the gradient of the line on the right is ½ (see page 26).
- **Definition:** the **y-intercept** of a straight line is the number on the y-axis that the line passes through. For line on the right, the y-intercept is 3.



Formula
The equation of any straight line can be written $y = mx + c$, where m is the gradient of the line and c is the y-intercept of the line.

In everyday language, this means that:

- the gradient is "the number before x"
- the y-intercept is "the number that is not before x"

Examples 1

Equation	Gradient	y-intercept
$y = 2x - 5$	2	–5
$y = 8 - x$	–1	8
$y = 4 - 3x$	–3	4
$y = 3 + \frac{5}{2}x$	$\frac{5}{2}$	3

Example 2

What is the equation of the straight line shown at the top of the page?

Solution

The line has gradient ½ and y-intercept 3. Therefore it has equation $\underline{y = \frac{1}{2}x + 3}$

Another method for calculating the equation of a straight line from a diagram or sketch is shown on page 35.

The equation of the line must begin $y = \ldots$ (that is, y has to be the **subject** of the equation). If it does not, the equation must be rearranged so that y is the subject.

Example 3

Find the gradient and y-intercept of the straight lines

(a) $3y = 6x - 9$ **(b) $x + y = 5$** **(c) $4y - 8x = 4$**

Solution

(a) In the equation given, $3y$ is the subject. To make y the subject, we divide through by 3:

$$3y = 6x - 9$$
$$y = 2x - 3$$

Therefore the gradient is 2 and the y-intercept is -3.

(b) In the equation given, $x + y$ is the subject. To make y the subject, we move x to the opposite side:

$$x + y = 5$$
$$y = 5 - x$$

Therefore the gradient is -1 and the y-intercept is 5.

(c) For this equation, we must move the x term across, and divide through by 4. It does not matter which order we do this in.

$$4y - 8x = 4$$
$$4y = 8x + 4 \quad \text{(moving } -8x \text{ across to become } +8x\text{)}$$
$$y = 2x + 1 \quad \text{(dividing through by 4)}$$

Therefore the gradient is 2 and the y-intercept is 1.

Drawing a Straight Line from its equation

You need to know how to draw or sketch a line when given its equation. At National 4, you used a table of values to draw a straight line. You can still do this. However there is a quicker way that involves a knowledge of $y = mx + c$.

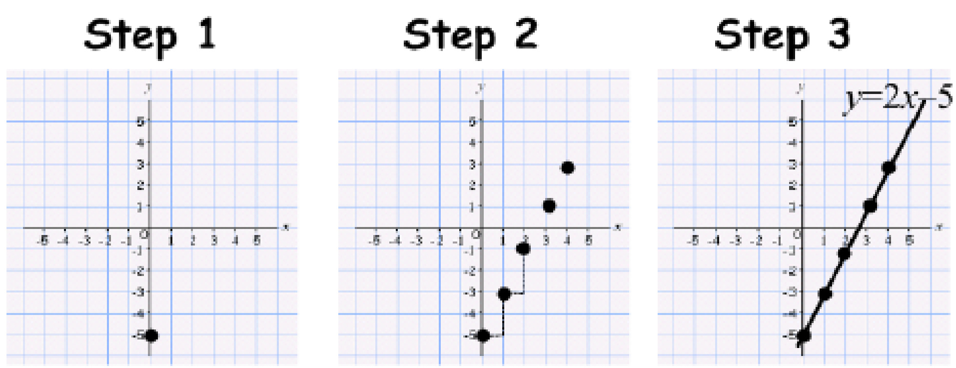
Example 1 – Drawing accurately

Draw the line $y = 2x - 5$

Step 1 – the y-intercept is -5, so the line goes through $(0, -5)$. Plot this point.

Step 2 – the gradient is 2, so move **'along 1 up 2'**. Do this a few times to obtain four or five points.

Step 3 – draw and label the line.



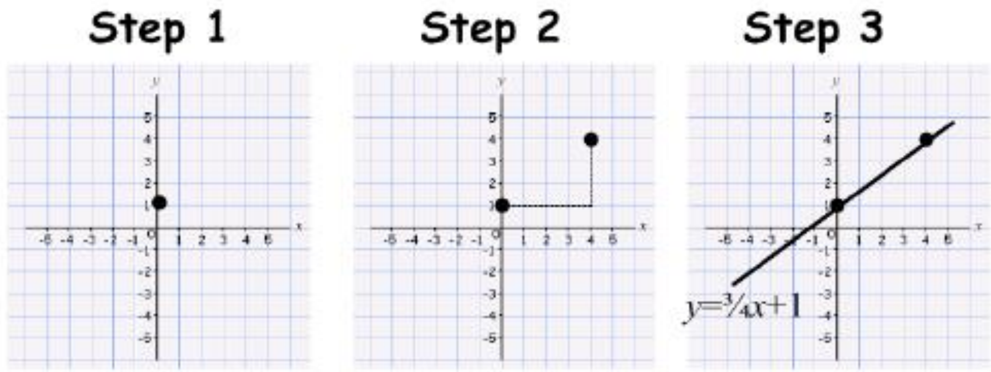
Example 2 – Drawing accurately, fraction gradient

Draw the line $y = \frac{3}{4}x + 1$

Step 1 – the y-intercept is 1, so the line goes through $(0,1)$. Plot this point.

Step 2 – the gradient is $\frac{3}{4}$. This means that you go **'along 4 up 3'** from your first point. Do this a few times to obtain four or five points.

Step 3 – draw and label the line.



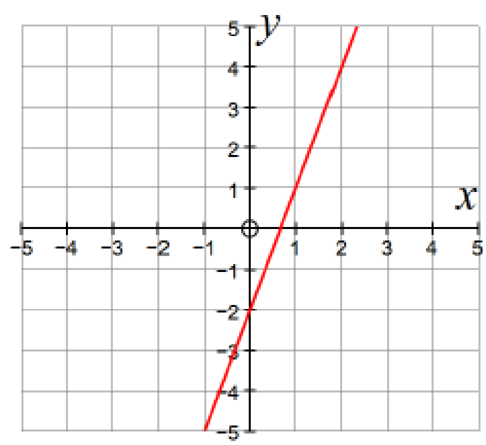
Equation of a graph from a diagram

To find the equation of a straight line from a diagram, you have a choice of two formulae. Which one you use depends on whether or not you are told the y-intercept:

- If you are told the y-intercept, you can use the formula $y = mx + c$.
- If you are not told the y-intercept, there is a second formula $y - b = m(x - a)$

Example 1 – where the y intercept is given

Find the equation of the straight line in this diagram.



Solution

IMPORTANT – show your working to get all the marks. Do not rush straight to step 3.

Step 1 – write down the y-intercept: $c = -2$

Step 2 – calculate the gradient of the line.

Two points on the line are (0,–2) and (1,1). Using the method from page 26:

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1-(-2)}{1-0} = \frac{3}{1} = 3$$

(you may be able to spot this from the diagram without working. This is OK so long as you clearly write down what the gradient is)

Step 3 – put these values into $y = mx + c$

Answer: $y = 3x - 2$

Formula. This formula is **not** given on the National 5 Mathematics exam paper.
The equation of the line with gradient m that goes through the point (a , b) is given by:

$$y - b = m(x - a)$$

The only exception is when m is undefined, when the equation of the line is of the form $x = a$

This formula will give the same final answer (once simplified) no matter which point on the line is used.

Example 2 – no y intercept given

Find the equation of the line with gradient 4 that goes through the point (5, –1). Rearrange your line into the form $y = mx + c$

Solution

Using $m = 4$ and $(a,b) = (5,-1)$, we have:

$$y - b = m(x - a)$$

$$y - (-1) = 4(x - 5)$$

$$y + 1 = 4x - 20 \qquad \text{(multiplying out the bracket)}$$

$$y = 4x - 20 - 1 \qquad \text{(moving the } +1 \text{ over to the right-hand side)}$$

$$y = 4x - 21 \qquad \text{(rearranging into the form } y = mx + c\text{)}$$

Example 3 – from coordinates

(a) Find the equation of the straight line AB that goes through the points A(1, –1) and B(3, 5) in the form $y = mx + c$

(b) Hence state the y-intercept of the line AB

Solution

(a) First, find the gradient.

$$m_{AB} = \frac{5 - (-1)}{3 - 1} = \frac{6}{2} = 3$$

Then using the formula $y - b = m(x - a)$, with $m = 3$ and the point (3, 5) [this is a random choice, we could just have easily used (1, –1)], we obtain:

$$y - b = m(x - a)$$

$$y - 5 = 3(x - 3)$$

$$y - 5 = 3x - 9$$

$$y = 3x - 9 + 5 \qquad \text{(moving} -5 \text{ over)}$$

$$y = 3x - 4 \qquad \text{(simplifying)}$$

(b) Using the answer from part (a), the y-intercept is –4.

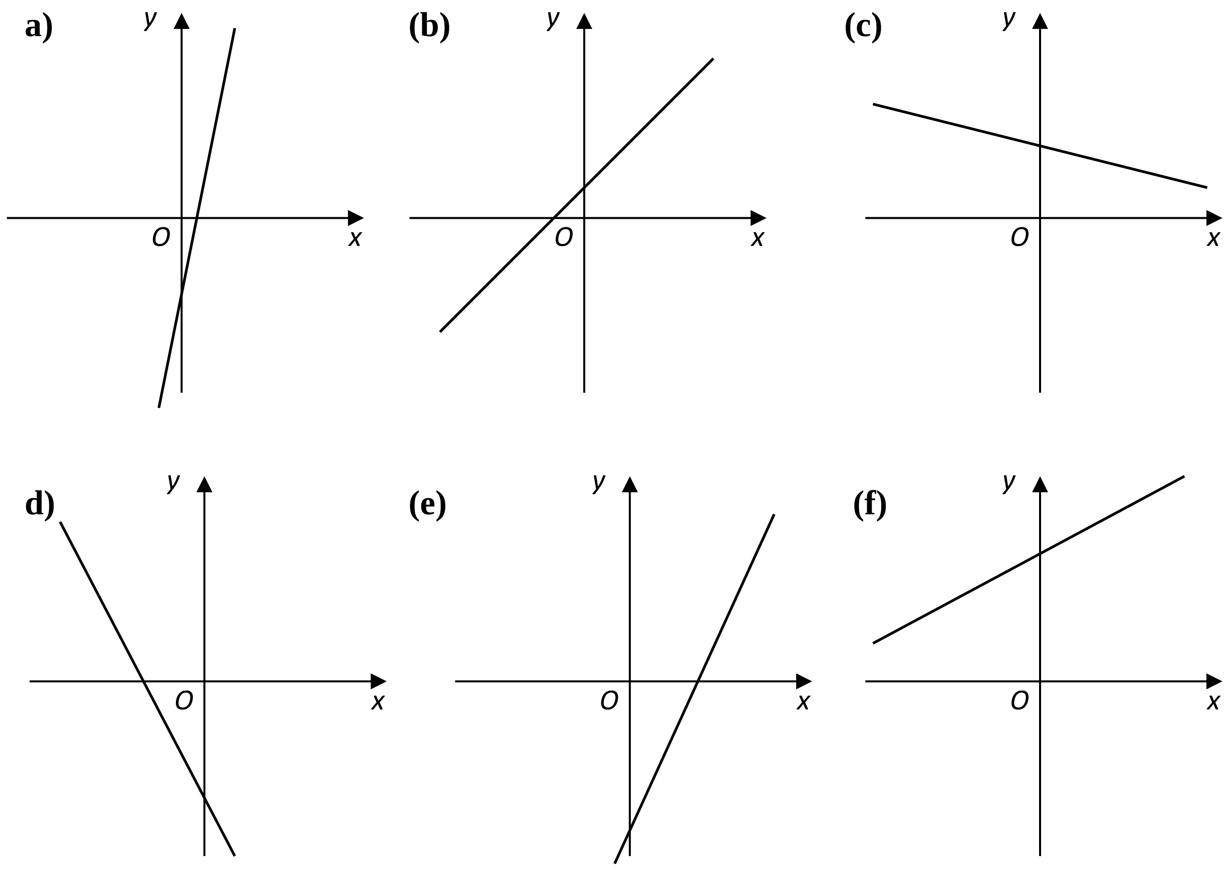
Exercise 1

1. For each line, write down the gradient and the coordinates of the point where it crosses the y – axis.

a)	$y = 3x + 1$	**(b)**	$y = \frac{1}{2}x - 5$	**(c)**	$y = -2x + 3$
d)	$y = -\frac{1}{4}x - 2$	**(e)**	$y = 8x - \frac{1}{2}$	**(f)**	$y = -x + 4$

2. Match these equations with the graphs shown below.

1.	$y = x + 1$	**2**.	$y = -2x - 3$	**3**.	$y = \frac{1}{2}x + 4$
4.	$y = -\frac{1}{4}x + 2$	**5**.	$y = 6x - 2$	**6**.	$y = 3x - 5$



Exercise 2

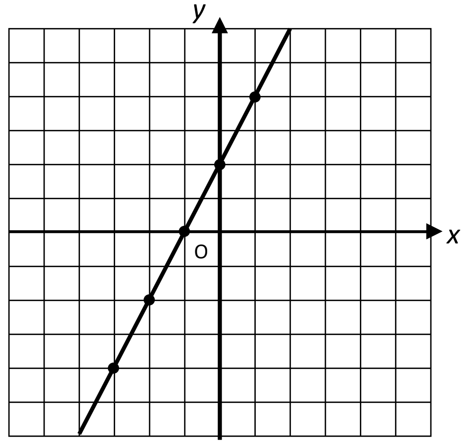
1. Sketch the graphs of lines with equations:

a)	$y = \frac{1}{2}x - 2$	**(b)**	$y = -2x - 1$	**(c)**	$y = -3x + 2$
d)	$y = -x + 3$	**(e)**	$y = 2x + 3$	**(f)**	$y = 4x + 1$

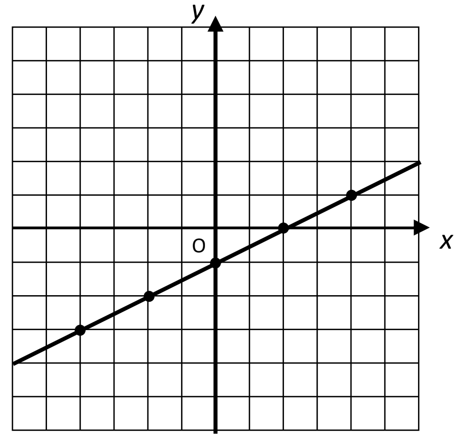
Exercise 3

1. Write down the equation of the lines drawn in the diagrams below.

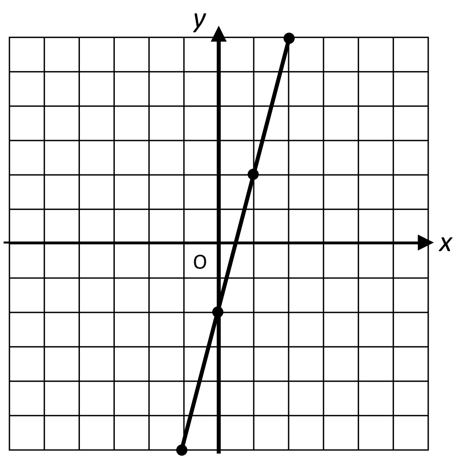
a)



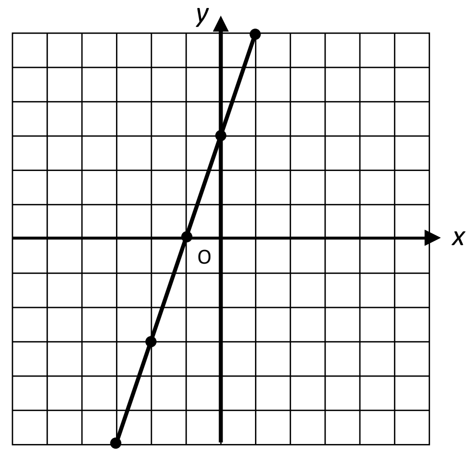
(b)



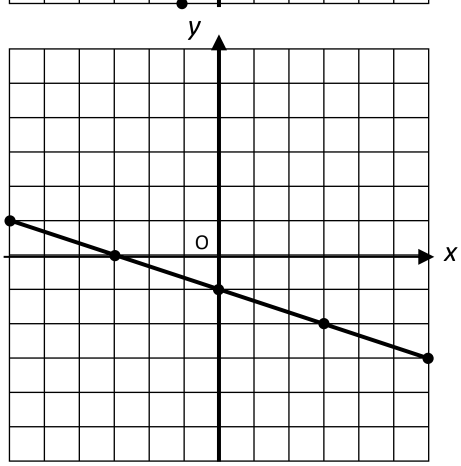
c)



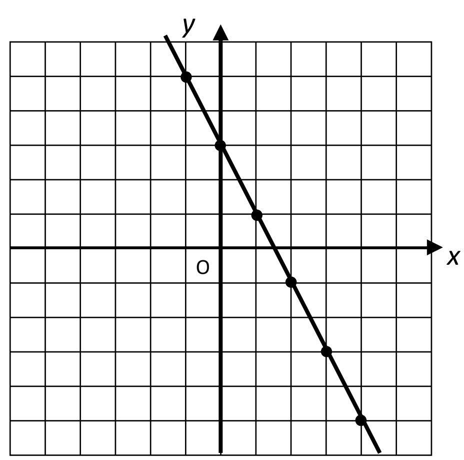
(d)



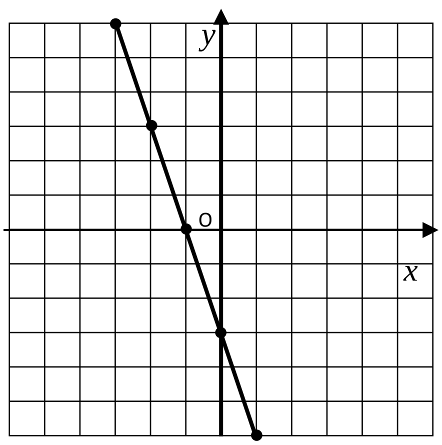
e)



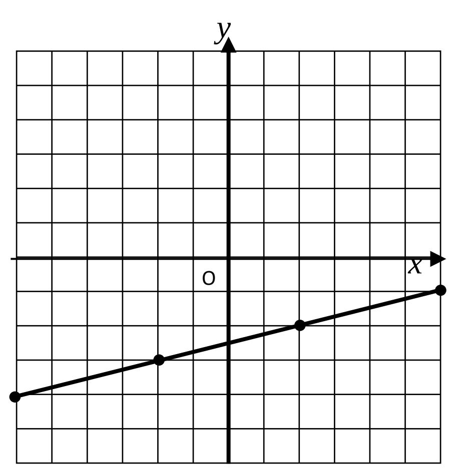
(f)



g)



(h)



Exercise 4

1. Identify the gradient and y – intercept of these lines.

a)	$y = x + 3$	**(b)**	$y = -2x - 1$	**(c)**	$y = \frac{1}{2}x$
d)	$y = -\frac{1}{2}x + 2$	**(e)**	$x + y = 6$	**(f)**	$2y = x - 4$
g)	$3y = x + 12$	**(h)**	$4x + 5y = 20$	**(i)**	$3x - 2y = 12$

2. State the gradient and the y – intercept for each line below.

a)	$y = x - 7$	**(b)**	$y = -5x + 3$	**(c)**	$5y = 3x - 10$
d)	$y = -4x$	**(e)**	$2x + y = 11$	**(f)**	$2y = x - 5$
g)	$3y - x = 18$	**(h)**	$3x + 7y - 21 = 0$	**(i)**	$4x - 5y = 20$

3. Write down the equation of the lines described below:

a) with gradient 4, passing through the point (0, 5)

b) with gradient –2, passing through the point (0, 1)

c) with gradient $\frac{3}{4}$, passing through the point (0, –3)

d) with gradient 4, passing through the point (3, 1)

e) with gradient –5, passing through the point (–3, 1)

f) with gradient $\frac{1}{2}$, passing through the point (–5, –2)

g) with gradient $\frac{4}{3}$, passing through the point (2, 7)

h) with gradient $-\frac{3}{4}$, passing through the point (–2, –2)

i) with gradient $-\frac{3}{2}$, passing through the point (–5, 3)

4. Find the equation of the line joining each pair of points below.

a)	A(4, 3) and B(8, 11)	**(b)**	C(1, 9) and D(3, 1)	**(c)**	E(–2, 6) and F(8, 8)
d)	G(5, –9) and H(8, –15)	**(e)**	I(0, 6) and J(5, 11)	**(f)**	K(–1, –3) and L(7, –9)
g)	M(–4, 0) and N(–1, 5)	**(h)**	P(2, 2) and Q(–3, 4)	**(i)**	R(5, –1) and S(–2, 10)

5. Find the equations of the lines joining the following pairs of points:

a) (2, 1) and (6, 3) **(b)** (1, 5) and (3, 1) **(c)** (2, 0) and (4, 6)

d) (–2, –3) and (2, 3) **(e)** (–1, 2) and (5, –1) **(f)** (–4, 2) and (4, –4)

g) (–6, –2) and (–5, 3) **(h)** (4, –3) and (6, 5) **(i)** (–2, 3) and (0, –2)

6. Establish the equation of the line passing through each pair of points below.

a) A(2, 1) and B(6, 13) **(b)** C(3, 4) and D(5, –4) **(c)** E(–2, –1) and F(6, 3)

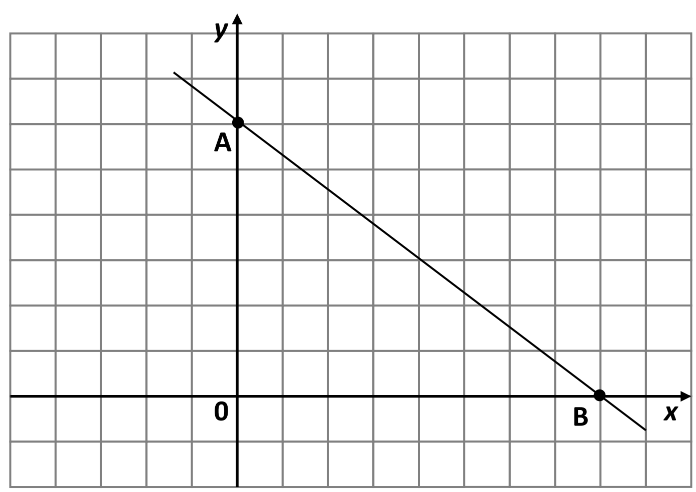
d) G(4, –13) and H(–2, –1) **(e)** I(2, 8) and J(10, 12) **(f)** K(–3, 2) and L(9, –2)

Mixed Exercise

1. A straight line has the equation $3x - 2y = -4$.

Find the gradient and y-intercept of the line.

2. The line AB passes through the points (0, 6) and (8, 0) as shown in the diagram.



Find the equation of the line AB.

3. A straight line has equation $2y + 3x = 8$.

Which line of these gives its gradient and y – intercept? Show working to explain your answer.

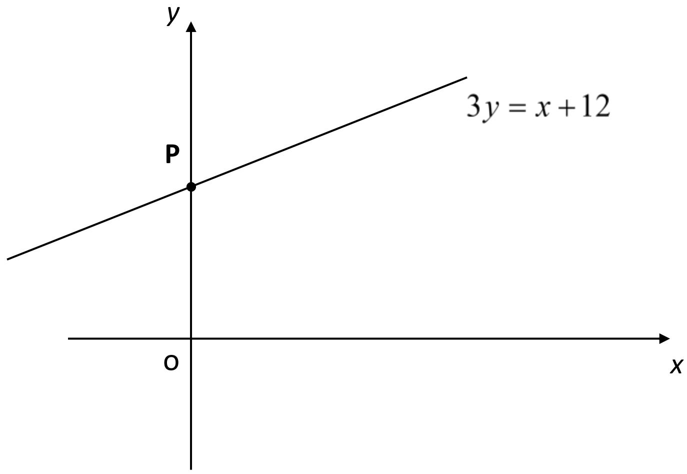
A. 3 and (0, 8) **B.** – 3 and (0. 8)

C. $\frac{3}{2}$ and (0, 4) **D.** $-\frac{3}{2}$ and (0, 4)

4. Find the gradient and y – intercept of the straight line with equation

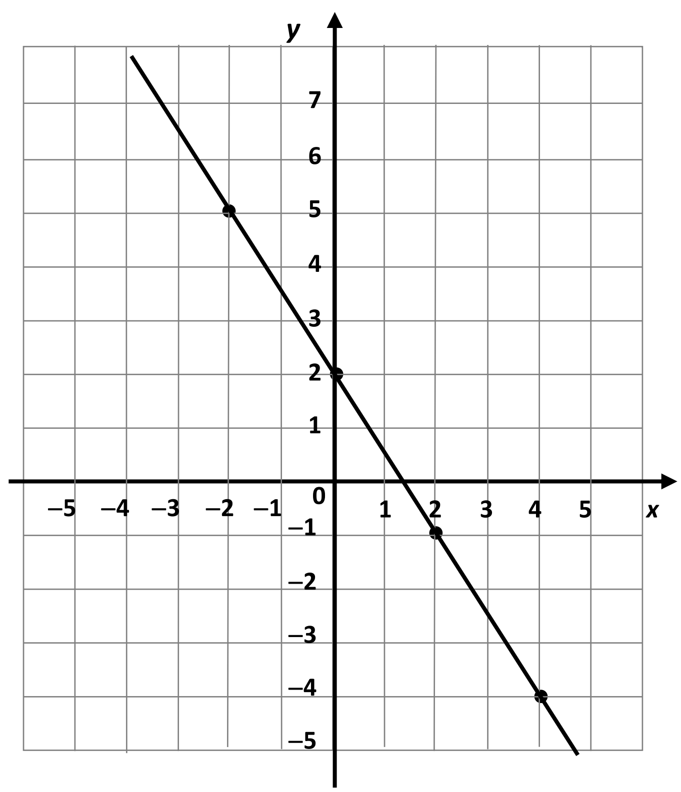
$$3x - 4y = 12.$$

5. The diagram below shows the line with equation $3y = x + 12$.



Find the coordinates of **P**, the point where the line cuts the y-axis.

6. Find the equation of the line shown in the diagram below.



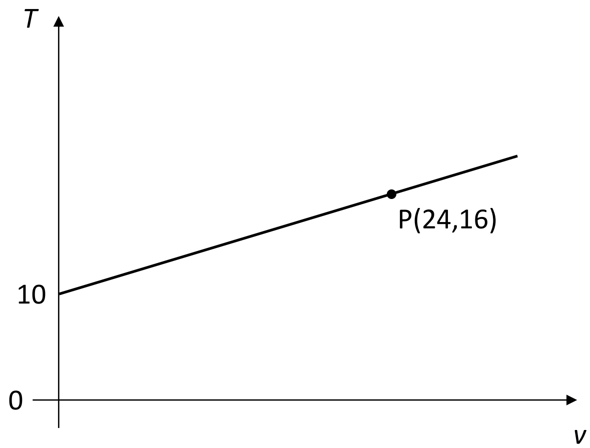
7. A line has equation $2y + 6x = 9$. Find its gradient and y - intercept.

8. A line has equation $3y + 4x = 15$.

Make a sketch of this line on plain paper showing clearly where it crosses the y - axis.

9. The relationship between variables v and T produces a straight line graph as shown below.

The line passes through the point P(24,16) as shown.



a) Find the gradient of the line.

b) Hence, write down the equation of the line in terms of v and T.

10. A straight line has equation $3y - 2x = 6$. Find the gradient and y-intercept of the line.

11. A straight line has equation $3x - 2y = 8$. Find the gradient and y-intercept of the line.

12. Find the equation of the straight line which passes through the point A(3, –2) and is parallel to the line $3y - 2x = 5$

13. **a)** A straight line has equation $4y - 3x = 6$.

State the gradient and the y-intercept point for this line.

b) Write down the equation of the line with gradient $-\frac{1}{2}$ which has the same y – intercept point as the line above.

14. **a)** A straight line has equation $3y - 4x = 12$.

State the gradient and the y-intercept point for this line.

b) Write down the equation of the line with gradient $-\frac{3}{4}$ which has the same y – intercept point as the line above.

Answers

Exercise 1

1. **a)** 3; (0, 1) **(b)** ½; (0, – 5) **(c)** –2; (0, 3)

d) –¼; (0, – 2) **(e)** 8; (0, – ½) **(f)** –1; (0, 4)

2. **1** and **(b)** **2** and **(d)** **3** and **(f)** **4** and **(c)** **5** and **(a)** **6** and **(e)**

Exercise 2

Sketches

Exercise 3

1. **a)** $y = 2x + 2$ **(b)** $y = \frac{1}{2}x - 1$ **(c)** $y = 4x - 2$ **(d)** $y = 3x + 3$

(e) $y = -\frac{1}{3}x - 1$ **(f)** $y = -\frac{1}{2}x + 3$ **(g)** $y = -3x - 3$ **(h)** $y = \frac{1}{4}x - 2\cdot 5$

Exercise 4

1. **a)** 1; (0, 3) **(b)** $-2;\ (0, -1)$ **(c)** $\frac{1}{2};\ (0, 0)$

d) $-\frac{1}{2};\ (0,\ 2)$ **(e)** $-1;\ (0, 6)$ **(f)** $\frac{1}{2}; (0,\ -2)$

g) $\frac{1}{3}; (0,\ 4)$ **(h)** $-\frac{4}{5}; (0,\ 4)$ **(i)** $\frac{3}{2}; (0,\ -6)$

2.	**a)**	$1;\ (0,-7)$	**(b)**	$-5;\ (0,3)$	**(c)**	$\frac{3}{5};(0,-2)$
	d)	$-4;\ (0,0)$	**(e)**	$-2;(0,\ 11)$	**(f)**	$\frac{1}{2};(0,-\frac{5}{2})$
	g)	$\frac{1}{3};(0,\ 6)$	**(h)**	$-\frac{3}{7};(0,\ 3)$	**(i)**	$\frac{4}{5};(0,-4)$
3.	**a)**	$y=4x+5$	**(b)**	$y=-2x+1$	**(c)**	$y=\frac{3}{4}x-3$
	d)	$y=4x-11$	**(e)**	$y=-5x-14$	**(f)**	$2y-x=1$
	g)	$3y-4x=13$	**(h)**	$3x+4y=-14$	**(i)**	$2x+3y=-9$
4.	**a)**	$y=2x-5$	**(b)**	$y+4x=13$	**(c)**	$5y=x+32$
	d)	$y=2x-1$	**(e)**	$y=x+6$	**(f)**	$4x+3y=-15$
	g)	$3y-5x=20$	**(h)**	$2x+5y=14$	**(i)**	$7y+11x=48$
5.	**a)**	$2y-x=0$	**(b)**	$y+2x=7$	**(c)**	$y=3x-6$
	d)	$2x-3y=0$	**(e)**	$2y+x=3$	**(f)**	$4y+3x=-4$
	g)	$y=5x+28$	**(h)**	$y=4x-19$	**(i)**	$2y+5x=-4$
6.	**a)**	$y=3x-5$	**(b)**	$y+4x=16$	**(c)**	$2y-x=0$
	d)	$y+2x=-5$	**(e)**	$2y-x=14$	**(f)**	$3y+x=3$

Mixerd Exercise

1. $\frac{3}{2};\ (0,2)$

2. $y=-\frac{3}{4}x+6$

3. D

4. $\frac{3}{4};\ (0,3)$

5. P(0, 4)

6. $y = -\frac{3}{2}x + 2$

7. -3; $(0, 4\cdot 5)$

8. Line crossing at (0, 5) with gradient $-\frac{4}{3}$

9. **(a)** $\frac{1}{4}$ **(b)** $T = \frac{1}{4}v + 10$

10. $\frac{2}{3}$; $(0, 2)$

11. $\frac{3}{2}$; $(0, -4)$

12. $3y - 2x = -12$

13. **(a)** $\frac{3}{4}$; $(0, 1\cdot 5)$ **(b)** $2y + x = 3$

14. **(a)** $\frac{4}{3}$; $(0, 4)$ **(b)** $4y + 3x = 16$