N5 Similarity

Relationships

SPTA Mathematics – Topic Questions with Notes

Similar Shapes

Two shapes are similar if they are exactly the same shape but a different size. All their angles will be identical, and all their lengths will be in proportion to each other. One shape will be an enlargement of the other. The factor of the enlargement is called the **scale factor**.

$$\text{Scale Factor} = \frac{\text{Length in 'new' shape}}{\text{Length in 'old' shape}}$$

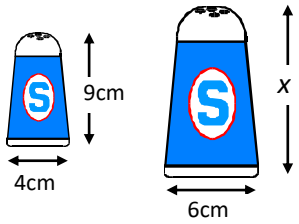
Exercise 1

1. Each diagram below shows a pair of similar shapes or objects. For each pair:

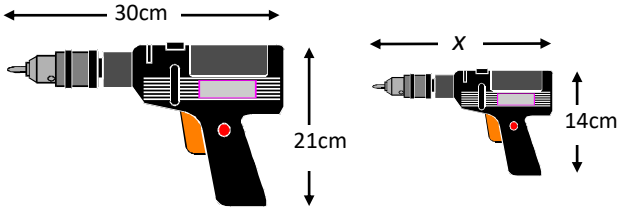
i) state the scale factor (from left to right)

ii) calculate the length marked *x*.

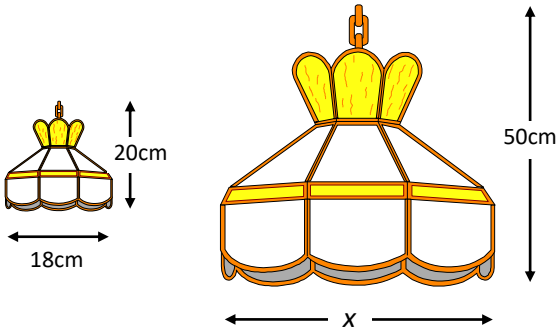
a)



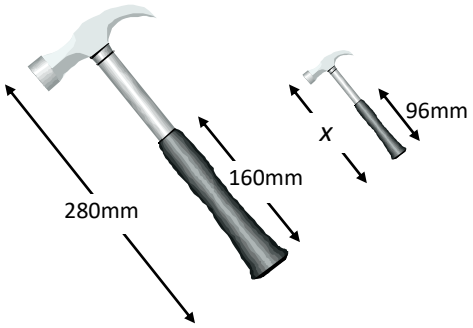
(b)



c)

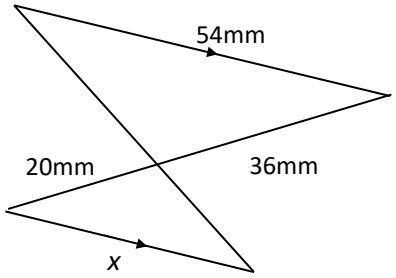


(d)

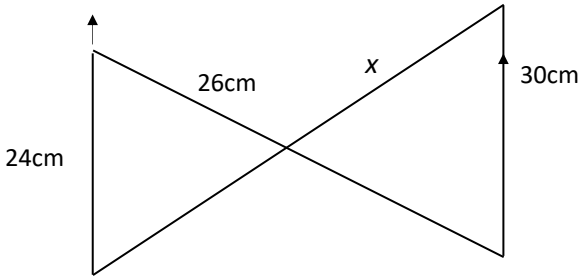


2. Calculate the length of the side marked x in each diagram below.

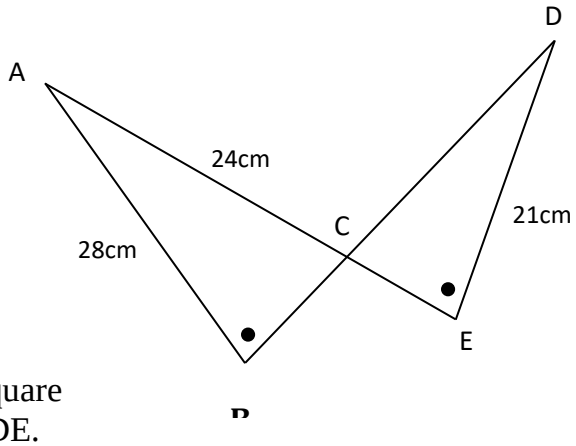
a)



(b)



3. In the diagram $\angle ABC = \angle CED$, AB = 28cm, AC = 24cm and ED = 21cm.



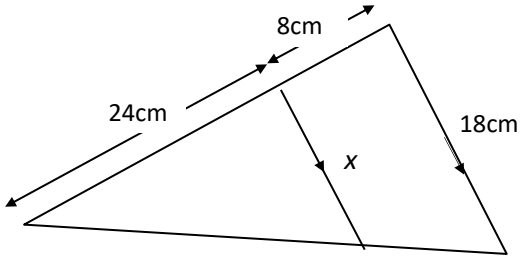
a) Explain why the triangles ABC and CDE are similar.

b) Calculate the length of CD.

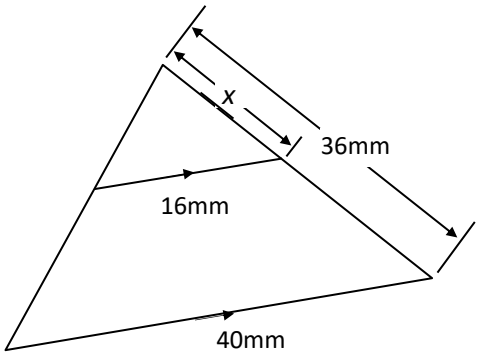
c) Given that the area of triangle ABC is 144 square centimetres, calculate the area of triangle CDE.

4. Calculate the length of the side marked x in each diagram below.

a)



(b)

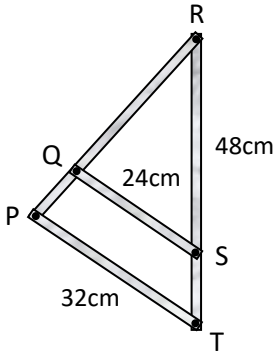


5. The diagram opposite shows an aluminium pipe frame.

The cross members QS and PT are parallel.

RS = 48cm, QS = 24cm and PT = 32cm as shown.

Calculate the length of ST

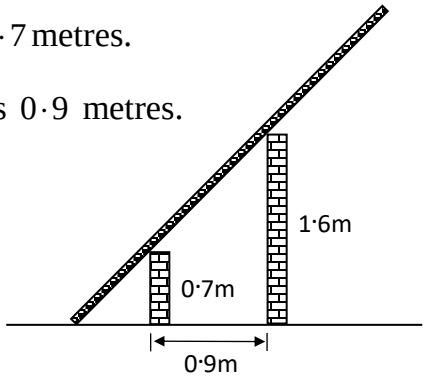


6. In the diagram a ladder is laid against two walls as shown.

The higher wall is $1\cdot6$ metres high, and the lower wall is $0\cdot7$ metres.

The distance between the two left hand faces of the walls is $0\cdot9$ metres.

Calculate the distance between the foot of the ladder and the lower wall.

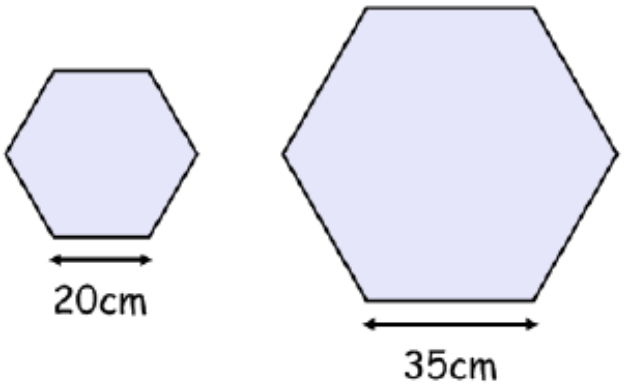


For National 5, you need to understand how the scale factor affects area and volume.

Facts
For two similar shapes connected by a scale factor s:
- Lengths in the 'new' shape are found by multiplying lengths in the 'old' shape by s
- The area of the 'new' shape is found by multiplying the area of the 'old' shape by s^2
- The 'new' volume is found by multiplying the volume of the 'old' shape by s^3

Example 1 – area

Two hexagons are similar in shape as shown in the diagram. The smaller hexagon has an area of 2500cm². Calculate the area of the larger hexagon.



Solution

Step one – calculate the scale factor.

$$\text{s.f.} = \frac{35}{20} = 1\cdot75$$

Step two – calculate the area, remembering to *square* the scale factor for area.

$$\text{Area} = 2500 \times 1\cdot75^2 = \underline{\underline{7656\cdot25\,\text{cm}^2}}$$

Exercise 2

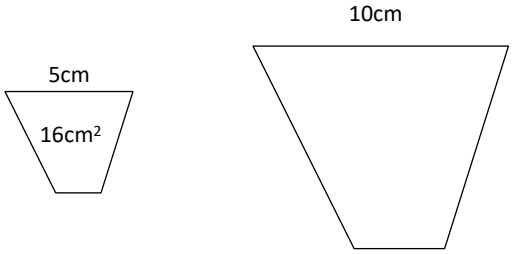
1. For each pair of pictures below

i) State the enlargement scale factor for the lengths

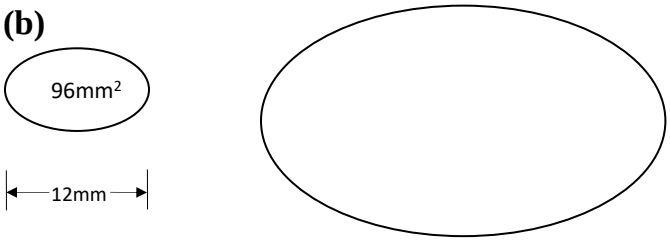
ii) State the scale factor for the areas.

iii) Calculate the area of the larger shape.

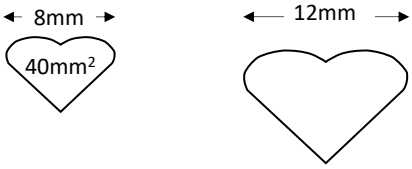
a)



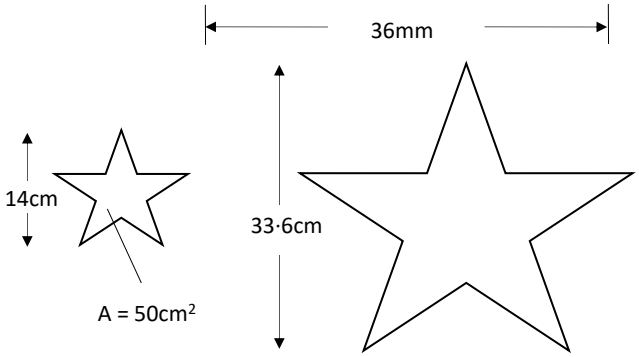
(b)



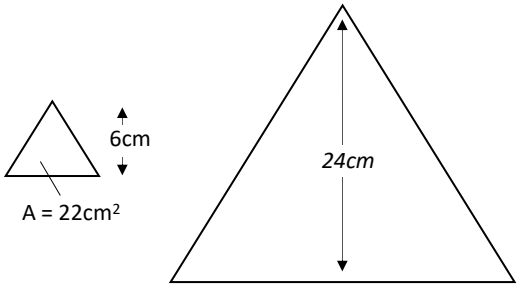
c)



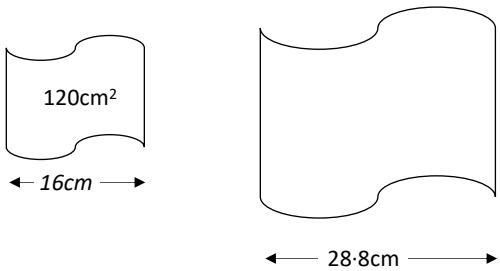
(d)



e)



(f)



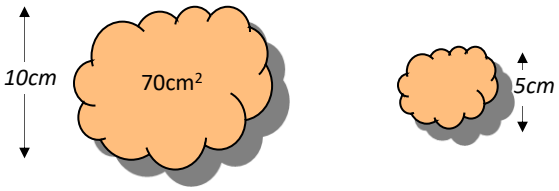
2. For each pair of pictures below

i) State the reduction scale factor for the lengths.

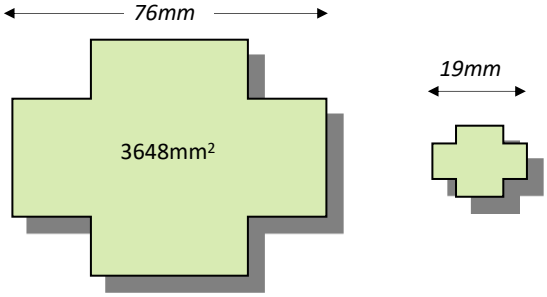
ii) State the scale factor for the areas.

iii) Calculate the area of the smaller shape.

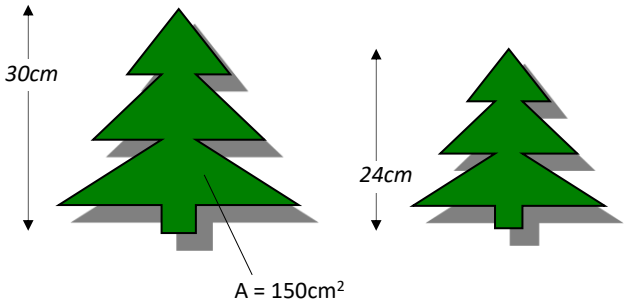
a)



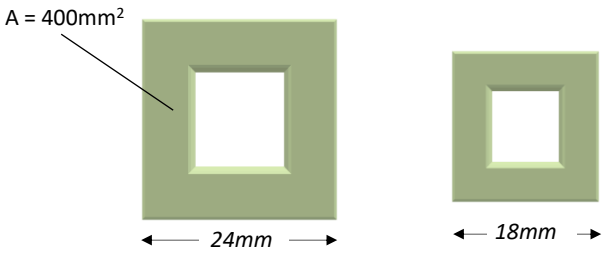
(b)



c)

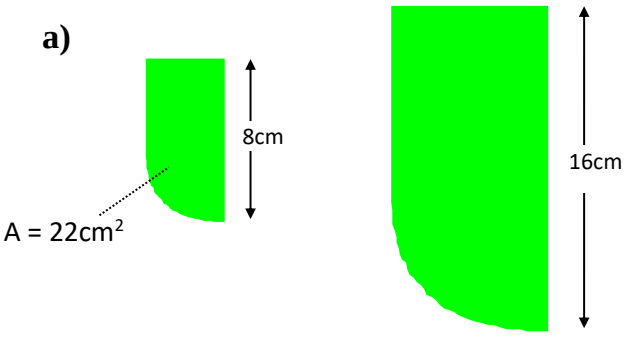


(d)

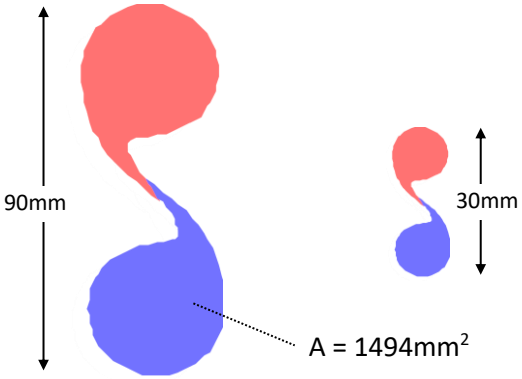


3. Each pair of shapes below is mathematically similar.
Calculate the **area** of each right-hand shape.

a)

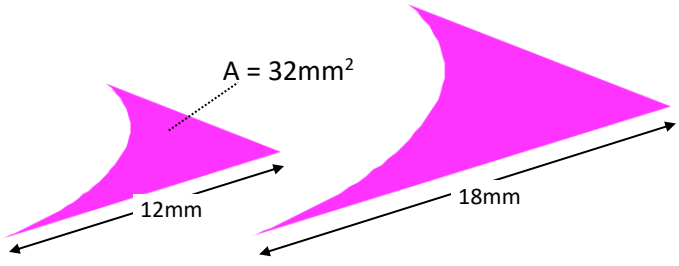


(b)



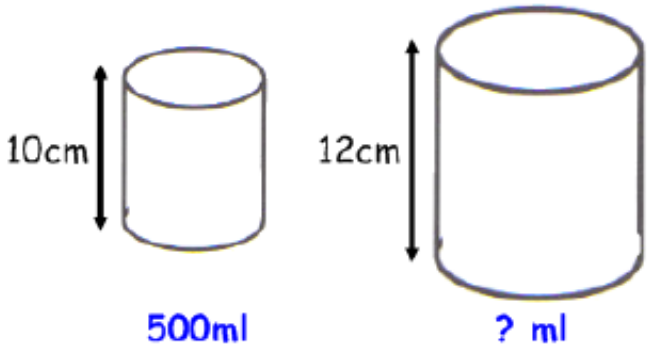
c)

(d)



Example 2 – volume

Two cylindrical drinks cans are mathematically similar. The smaller can holds 500ml of juice. How much will the larger can hold?



Solution

Step one – calculate the scale factor.

$$\text{s.f.} = \frac{12}{10} = 1 \cdot 2$$

Step two – calculate the volume, remembering to *cube* the scale factor for volume.

$$\text{Area} = 500 \times 1 \cdot 2^3 = \underline{\underline{864\text{ml}}}$$

Exercise 3

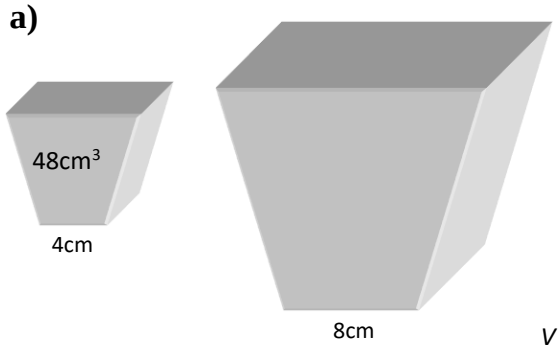
1. For each pair of similar pictures below

i) State the enlargement scale factor for the length.

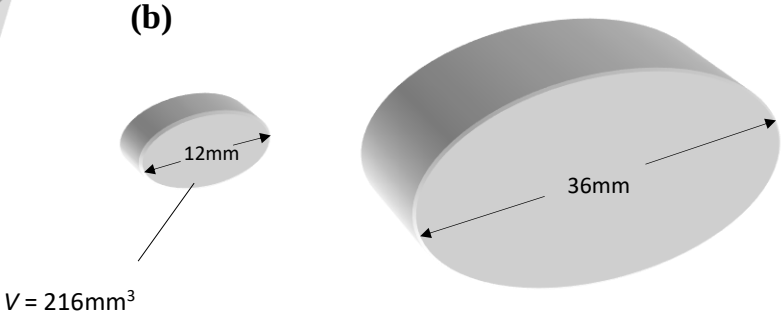
ii) State the scale factor for the volumes.

iii) Calculate the volume of the larger solid.

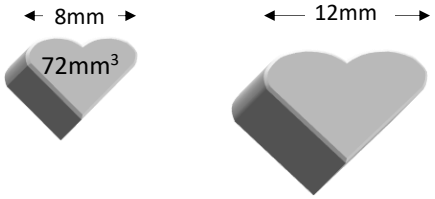
a)



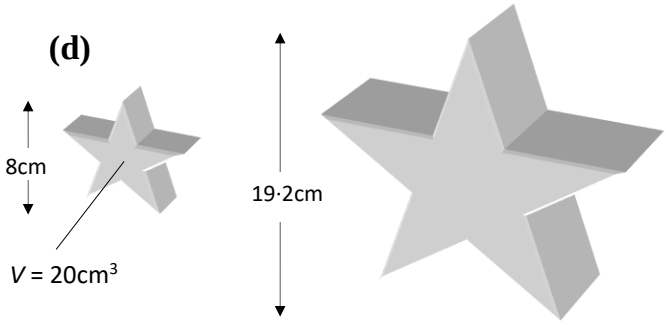
(b)



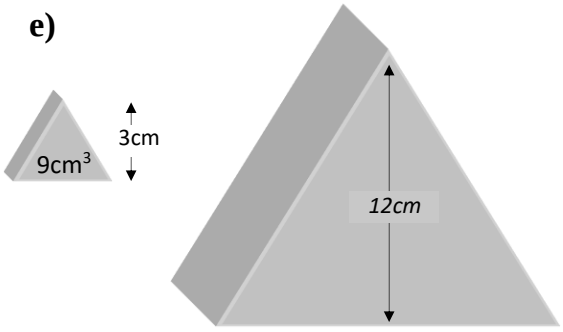
c)



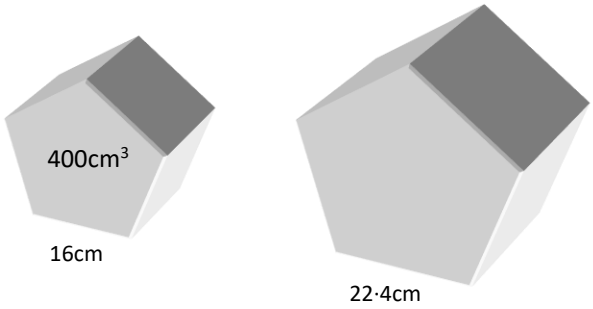
(d)



e)



(f)



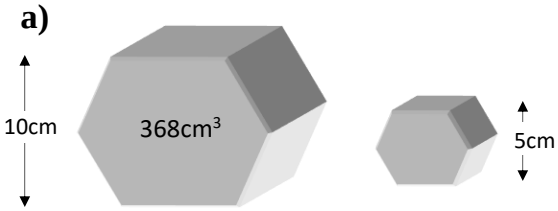
2. For each pair of similar pictures below

i) State the reduction scale factor for the lengths.

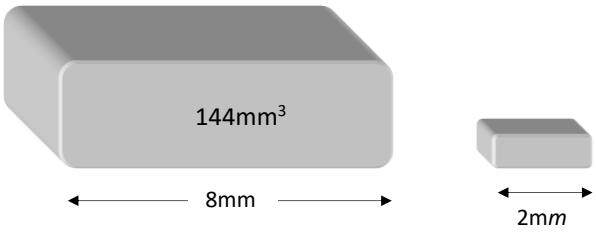
ii) State the scale factor for the volumes.

iii) Calculate the volume of the smaller solid.

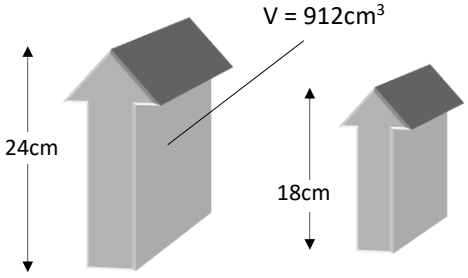
a)



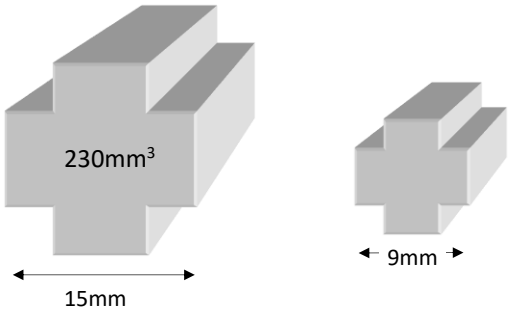
(b)



c)



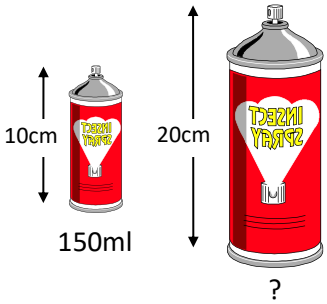
(d)



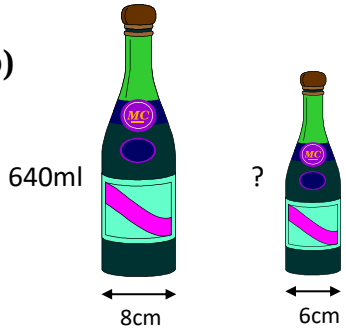
3. Each pair of containers below is mathematically similar.

Calculate the **volume** of each container with a question mark.

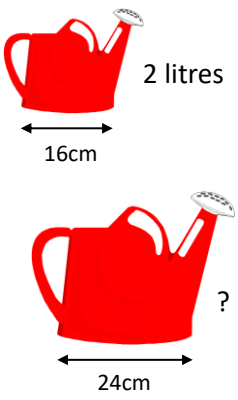
a)



(b)



(c)

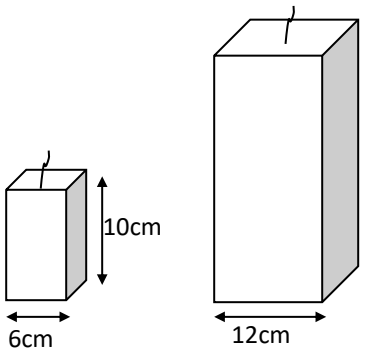


Mixed Exercise

1. The diagram below shows two candles.

Each candle is in the shape of a cuboid with a square base.

The length of time each candle will burn is proportional to its volume.



a) Calculate the volume of the small candle.

b) Bu using scale factors calculate the volume of the large candle.

c) If the small candle burns for 36 hours, calculate the length of time that the large candle will burn for.

2. An international perfume manufacturer prices their bottles of perfume **by volume**.

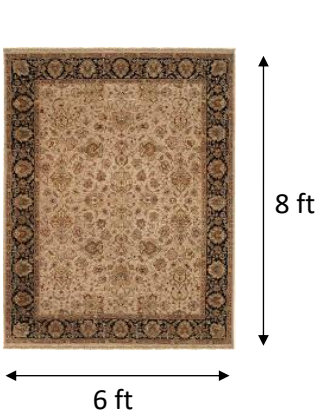
The two bottles below, although containing different volumes, are mathematically similar in shape. Their heights and prices are shown.

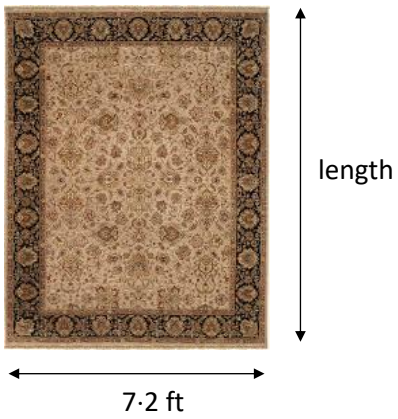
The larger of the two bottles is for sale in France.

Assuming the smaller bottle to be priced correctly, determine whether or not the larger bottle has the correct price tag given that the exchange rate is £1 = 1·10 euros.

3. John is looking to buy a new rug for his main room.

The two rugs below are **mathematically similar** in shape.





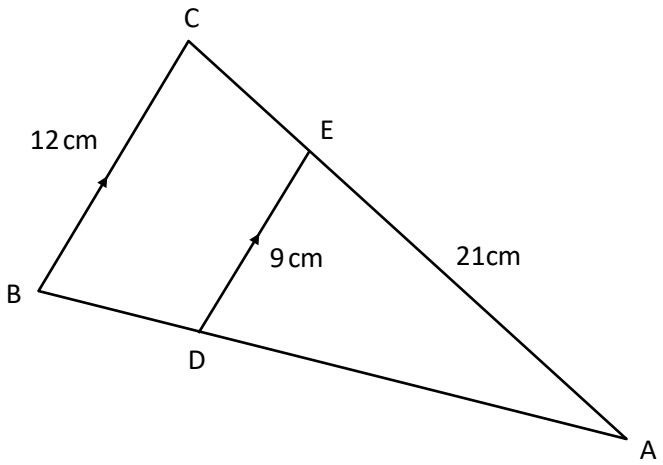
He is hoping that the length of the large rug will be enough to make the **area** of the large rug **at least 72 square feet**.

Does the large rug have the required area?

You must show appropriate working with your answer.

4. In the diagram below triangles ABC and ADE are **mathematically similar**.

BC = 12 cm, DE = 9 cm and AE = 21 cm.



Find the length of CE.

Answers

Exercise 1

1. **(a)** **(i)** s.f. = $\frac{3}{2}$ or 1·5 **(ii)** 13·5cm **(b)** **(i)** s.f. = $\frac{2}{3}$ of 0·66... **(ii)** 20cm

(c) **(i)** s.f. = $\frac{5}{2}$ of 2·5 **(ii)** 45cm **(d)** **(i)** s.f. = $\frac{3}{5}$ or 0·6 **(ii)** 168mm

2. **(a)** x = 30mm **(b)** $x = 32 \cdot 5$cm

3. **(a)** Because they are equiangular **(b)** CD = 18cm **(c)** 81 cm^2

4. **(a)** $x = 13 \cdot 5$cm **(b)** $x = 14 \cdot 4$m **5.** ST = 16cm **6.** distance = $0 \cdot 7$m

Exercise 2

1. **(a)** s.f.(L) = 2; s.f. (A) = 4; A = 64cm^2

(b) s.f.(L) = 3; s.f. (A) = 9; A = 864mm^2

(c) s.f.(L) = 1·5; s.f. (A) = 2·25; A = 90mm^2

(d) s.f.(L) = 2·4; s.f. (A) = 5·76; A = 288cm^2

(e) s.f.(L) = 4; s.f. (A) = 16; A = 352cm^2

(f) s.f.(L) = 1·8; s.f. (A) = 3·24; A = 388·8cm^2

2. **(a)** s.f.(L) = 0·5; s.f. (A) = 0·25; A = 17·5cm^2

(b) s.f.(L) = 0·25; s.f. (A) = 0·0625; A = 288mm^2

(c) s.f.(L) = 0·8; s.f. (A) = 0·64; A = 96cm^2

(d) s.f.(L) = 0·75; s.f. (A) = 0·5625; A = 225mm^2

3. **(a)** 88 cm^2 **(b)** 166 mm^2 **(c)** 49cm^2 **(d)** 72 mm^2

Exercise 3

1. **(a)** s.f.(L) = 2; s.f.(V) = 8; $V = 384\text{cm}^3$

(b) s.f.(L) = 3; s.f.(V) = 27; $V = 5832\text{mm}^3$

(c) s.f.(L) = 1·5; s.f.(V) = 3·375; $V = 243\text{mm}^3$

(d) s.f.(L) = 2·4; s.f.(V) = 13·824; $V = 276{\cdot}48\text{cm}^3$

(e) s.f.(L) = 4; s.f.(V) = 64; $V = 576\text{cm}^3$

(f) s.f.(L) = 1·4; s.f.(V) = 2·744; $V = 1097{\cdot}6\text{cm}^3$

2. **(a)** s.f.(L) = 0·5; s.f.(V) = 0·125; $V = 46\text{cm}^3$

(b) s.f.(L) = 0·25; s.f.(V) = 0·015625; $V = 2{\cdot}25\text{mm}^3$

(c) s.f.(L) = 0·75; s.f.(V) = 0·421875; $V = 384{\cdot}75\text{cm}^3$

(d) s.f.(L) = 0·6; s.f.(V) = 0·216; $V = 49{\cdot}68\text{mm}^3$

3. **(a)** 1200 ml **(b)** 270 ml **(c)** 6·75 litres

Mixed Exercise

1. no, will burn for 4 times the time

2. priced correctly

3. rug is too small since $69{\cdot}1 < 72$

4. 7cm