N5 Solving Quadratics

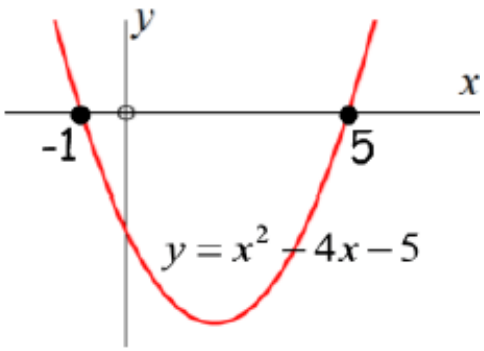
Relationships

SPTA Mathematics – Topic Questions with Notes

Definition: the **roots** of a quadratic equation are another word for its solutions.
The roots of a graph of an equation are the points that the graph crosses the x-axis.

Example 1 – from a graph

Using the graph shown, write down the two solutions of the equation $x^2 - 4x - 5 = 0$



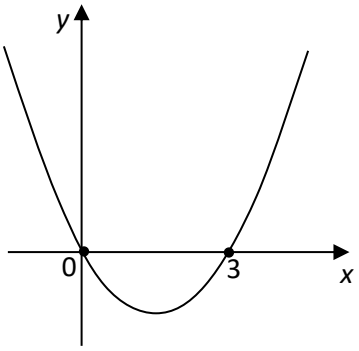
Solution

The roots are $x = -1$ and $x = 5$.

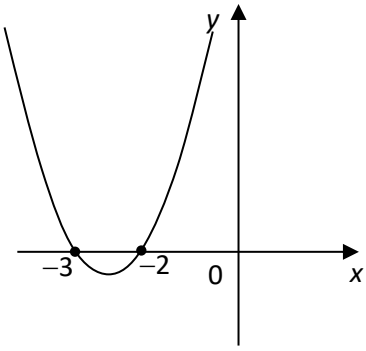
Exercise 1

1. Use the sketches below to solve the quadratic equations.

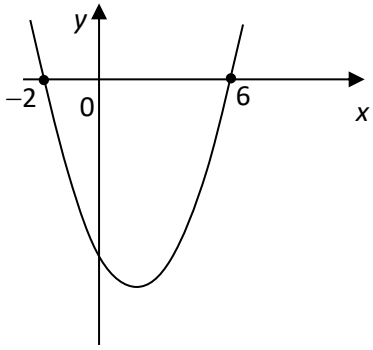
(a) $x^2 - 3x = 0$



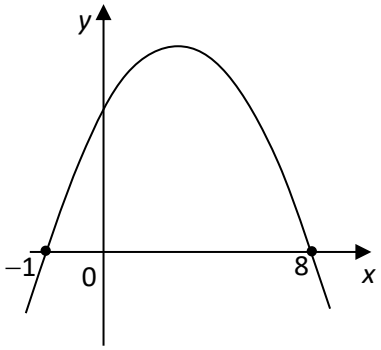
(b) $x^2 + 5x + 6 = 0$



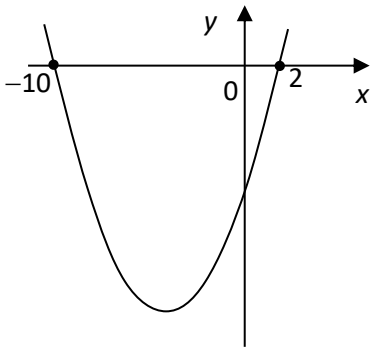
(c) $x^2 - 4x - 12 = 0$



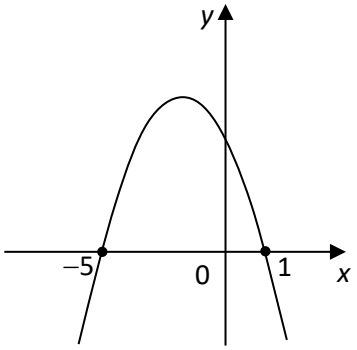
(d) $8 + 7x - x^2 = 0$



(e) $x^2 + 8x - 20 = 0$



(f) $5 - 4x - x^2 = 0$



Factorising is the simplest way of solving a quadratic equation, but you can only use it when the expression can actually be factorised! See page 15 for help on factorising.

Important – you must rearrange the equation so that is has '= 0' on the right-hand side. If you do not do this, you will risk losing all of the marks.

Example 2 - factorising

Use factorising to solve the equation $2x^2 - 6x = 0$

Solution

Step 1 – check that the equation has '= 0' on the right-hand side.
On this occasion, it does, so we do not need to do anything more.

Step 2 – factorise the expression

Step 3 – split up into two separate equations and solve

$$2x^2 - 6x = 0$$
$$2x(x-3) = 0$$

$$2x = 0 \qquad x - 3 = 0$$
$$x = 0, \qquad x = 3$$

Exercise 2

1. Solve these quadratic equations by factorising first.

(a)	$x^2 + 4x = 0$	**(b)**	$c^2 - 2c = 0$	**(c)**	$y^2 + 8y = 0$
(d)	$p^2 - p = 0$	**(e)**	$z^2 + z = 0$	**(f)**	$n^2 + 7n = 0$
(g)	$2t^2 + 4t = 0$	**(h)**	$5x^2 - 20x = 0$	**(i)**	$6b^2 - 18b = 0$
(j)	$4y^2 - 6y = 0$	**(k)**	$6a^2 + 9a = 0$	**(l)**	$14x^2 + 21x = 0$
(m)	$5x - x^2 = 0$	**(n)**	$9b - b^2 = 0$	**(o)**	$2m - m^2 = 0$
(p)	$6w - 4w^2 = 0$	**(q)**	$9c - 12c^2 = 0$	**(r)**	$4y - 10y^2 = 0$

Example 3 - Difference of Two Squares

Use factorising to solve the equation $y^2 - 49 = 0$

Solution

Step 1 – check that the equation has '= 0' on the right-hand side.
On this occasion, it does, so we do not need to do anything more.

Step 2 – factorise the expression

$$y^2 - 49 = 0$$
$$(y + 7)(y - 7) = 0$$

Step 3 – split up into two separate equations and solve

$$y + 7 = 0 \qquad y - 7 = 0$$
$$y = -7, \qquad y = 7$$

Exercise 3

1. Solve these quadratic equations by factorising first.

(a)	$x^2 - 25 = 0$	**(b)**	$b^2 - 1 = 0$	**(c)**	$y^2 - 4 = 0$
(d)	$a^2 - 36 = 0$	**(e)**	$z^2 - 9 = 0$	**(f)**	$k^2 - 64 = 0$
(g)	$x^2 - 16 = 0$	**(h)**	$p^2 - 144 = 0$	**(i)**	$m^2 - 100 = 0$
(j)	$t^2 - 49 = 0$	**(k)**	$a^2 - 81 = 0$	**(l)**	$s^2 - 121 = 0$
(m)	$2a^2 - 18 = 0$	**(n)**	$5c^2 - 80 = 0$	**(o)**	$4y^2 - 64 = 0$

Example 4 – factorising with a coefficient of x^2

Use factorising to solve the equation $2x^2 + 9x - 5 = 0$

Solution

Step 1 – check that the equation has '= 0' on the right-hand side.
On this occasion it does, so we do not need to do anything more.

Step 2 – factorise the expression

$$2x^2 + 9x - 5 = 0$$
$$(2x - 1)(x + 5) = 0$$

Step 3 – split up into two separate equations and solve

$$2x - 1 = 0 \qquad x + 5 = 0$$
$$2x = 1$$
$$x = \tfrac{1}{2}, \qquad x = -5$$

Example 5 – right-hand side is not equal to zero)

Use factorising to solve the equation $x^2 - 2x - 10 = 5$

Solution

Step 1 – check that the equation has '= 0' on the right-hand side.
It **does not**, so we need to rearrange by moving the 5 over to the left hand side.

$$x^2 - 2x - 10 = 5$$
$$x^2 - 2x - 10 - 5 = 0$$
$$x^2 - 2x - 15 = 0$$

Step 2 – factorise the rearranged expression

$$x^2 - 2x - 15 = 0$$
$$(x + 3)(x - 5) = 0$$

Step 3 – split up into two separate equations and solve

$$x + 3 = 0 \qquad x - 5 = 0$$
$$x = -3, \qquad x = 5$$

Exercise 4

1. Solve these quadratic equations by factorising first.

(a)	$x^2 + 4x + 3 = 0$	**(b)**	$y^2 + 6y + 5 = 0$	**(c)**	$a^2 + 8a + 7 = 0$
(d)	$m^2 + 5m + 6 = 0$	**(e)**	$c^2 + 6c + 8 = 0$	**(f)**	$z^2 + 7z + 12 = 0$
(g)	$15 - 2x - x^2 = 0$	**(h)**	$b^2 - 8b + 16 = 0$	**(i)**	$x^2 - 7x + 10 = 0$
(j)	$w^2 - 12w + 27 = 0$	**(k)**	$18 + 7y - y^2 = 0$	**(l)**	$k^2 - 10k + 24 = 0$
(m)	$8 - 2x - x^2 = 0$	**(n)**	$6 + m - m^2 = 0$	**(o)**	$t^2 - 7t - 30 = 0$
(p)	$a^2 + 5a - 14 = 0$	**(q)**	$c^2 - 2c - 15 = 0$	**(r)**	$12 - 4p - p^2 = 0$

2. Solve these quadratic equations by factorising first.

(a)	$2x^2 + 7x + 5 = 0$	**(b)**	$2p^2 + 11p + 5 = 0$	**(c)**	$3t^2 + 10t + 3 = 0$
(d)	$3k^2 + 7k + 2 = 0$	**(e)**	$3y^2 + 8y + 5 = 0$	**(f)**	$6 - 7a - 5a^2 = 0$
(g)	$3 - 5w - 2w^2 = 0$	**(h)**	$3d^2 - 5d + 2 = 0$	**(i)**	$5x^2 - 16x + 3 = 0$
(j)	$3m^2 - 14m + 8 = 0$	**(k)**	$7 + 5c - 2c^2 = 0$	**(l)**	$1 - 5y - 6y^2 = 0$
(m)	$3x^2 - 2x = 1$	**(n)**	$4q^2 + 5q = 6$	**(o)**	$4t(t - 1) - 3 = 0$
(p)	$3m^2 + 2m = 5$	**(q)**	$36v^2 = -v + 2$	**(r)**	$7s^2 = 4 + 27s$

Answers

Exercise 1

1. **a)** $x = 0$ or 3 **(b)** $x = -3$ or -2 **(c)** $x = -2$ or 6 **(d)** $x = -1$ or 8

e) $x = -10$ or 2 **(f)** $x = -5$ or 1

Exercise 2

1. **a)** 0 and –4 **(b)** 0 and 2 **(c)** 0 and –8 **(d)** 0 and 1

e) 0 and –1 **(f)** 0 and –7 **(g)** 0 and –2 **(h)** 0 and 4

i) 0 and 3 **(j)** 0 and $\frac{3}{2}$ **(k)** 0 or $-\frac{3}{2}$ **(l)** 0 or $-\frac{3}{2}$

m) 0 and 5 **(n)** 0 and 9 **(o)** 0 and 2 **(p)** 0 and $\frac{3}{2}$

q) 0 and $\frac{3}{4}$ **(r)** 0 and $\frac{2}{5}$

Exercise 3

1. **a)** –5 and 5 **(b)** –1 and 1 **(c)** –2 and 2 **(d)** –6 and 6

e) –3 and 3 **(f)** –8 and 8 **(g)** –4 and 4 **(h)** –12 and 12

i) –10 and 10 **(j)** –7 and 7 **(k)** –9 and 9 **(l)** –11 and 11

m) –3 and 3 **(n)** –4 and 4 **(o)** –4 and 4

Exercise 4

1. **a)** –3 and –1 **(b)** –5 and –1 **(c)** –7 and –1 **(d)** –3 and –2

e) –4 and –2 **(f)** –3 and –4 **(g)** –5 and 3 **(h)** 4 (twice)

i) 5 and 2 **(j)** 3 and 9 **(k)** 9 and –2 **(l)** 4 and 6

m) –4 and 2 **(n)** –2 and 3 **(o)** –3 and 10 **(p)** –7 and 2

q) –3 and 5 **(r)** –6 and 2

2. **a)** $-\frac{5}{2}$ and -1 **(b)** $-\frac{1}{2}$ and -5 **(c)** $-\frac{1}{3}$ and -3 **(d)** $-\frac{1}{3}$ and -2

e) $-\frac{5}{3}$ and -1 **(f)** $\frac{3}{5}$ and -2 **(g)** $\frac{1}{2}$ and -3 **(h)** $\frac{2}{3}$ and 1

i) $\frac{1}{5}$ and 3 **(j)** $\frac{2}{3}$ and 4 **(k)** $\frac{7}{2}$ and -1 **(l)** $\frac{1}{6}$ and -1

m) $-\frac{1}{3}$ and 1 **(n)** $\frac{3}{4}$ and -2 **(o)** $-\frac{1}{2}$ and $\frac{3}{2}$ **(p)** $-\frac{5}{3}$ and 1

q) $\frac{2}{9}$ and $-\frac{1}{4}$ **(r)** $-\frac{1}{7}$ and 4