



When making a sketch of a parabola we need to indicate the coordinates of:

- the **roots**. We would usually find the roots by factorising the equation.
- the **y intercept**. We can always find this by substituting $x = 0$ into the equation.
- the **turning point**. We can find this by symmetry (using the fact that the turning point is exactly in the middle of the roots). We could also do it by completing the square (see page 20).

Example 1 – factorised

Sketch the graph with equation $y = (x - 1)(x + 3)$. Mark clearly where the graph crosses the axes and the coordinates of the turning point.

Solution

Shape – the x^2 term is positive, so the graph is a ‘happy’ parabola (if it was negative, it would be ‘unhappy’).

Roots – to find the roots, solve the equation $(x - 1)(x + 3) = 0$:

The bracket $(x - 1)$ tells us that one root is $x = 1$.

The bracket $(x + 3)$ tells us that the other root is $x = -3$.

Answer: the roots are $x = 1$ and $x = -3$.

y-intercept – to find the y-intercept, substitute $x = 0$ into the equation.

$$y = (x - 1)(x + 3)$$

$$= (0 - 1)(0 + 3)$$

$$= (-1) \times (3)$$

$$= -3$$

Answer: the y-intercept is at $y = -3$.

Turning Point – the simplest way to find the turning point is to use symmetry.

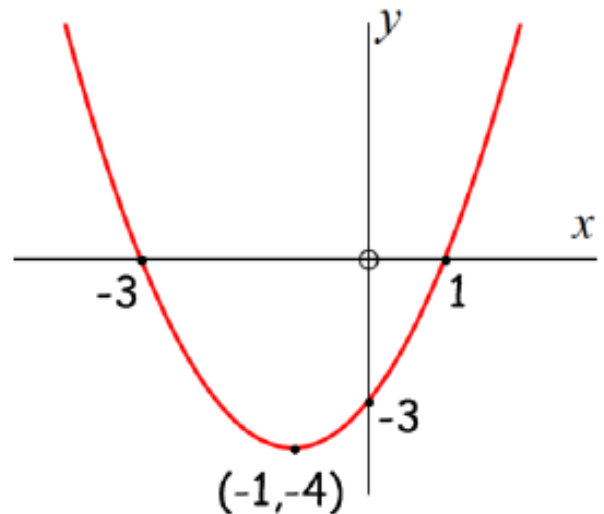
Because the parabola is symmetrical, the turning point is exactly midway between the roots. Since the roots are $x = 1$ and $x = -3$, the turning point must be at $x = -1$.

We now substitute $x = -1$ into the original equation.

$$\begin{aligned}y &= (x - 1)(x + 3) \\&= (-1 - 1)(-1 + 3) \\&= (-2) \times (2) \\&= -4\end{aligned}$$

Answer: the turning point is $(-1, -4)$.

Sketch – sketch the graph, including all the key points found above. A possible sketch is shown on the right.



Exercise 1

1. Sketch the graphs with the following equations

- | | | |
|--------------------------------|---------------------------------|---------------------------------|
| a) $y = (x - 1)(x - 5)$ | (b) $y = (x - 4)(x - 2)$ | (c) $y = (x - 3)(x - 7)$ |
| d) $y = (x - 6)(x - 8)$ | (e) $y = (x - 5)(x - 2)$ | (f) $y = (x - 8)(x - 5)$ |
| g) $y = (x + 2)(x + 3)$ | (h) $y = (x + 5)(x + 2)$ | (i) $y = (x + 4)(x + 6)$ |
| j) $y = (x + 3)(x + 4)$ | (k) $y = (x + 9)(x + 5)$ | (l) $y = (x + 3)(x + 8)$ |

2. Sketch the graphs with the following equations

- | | | |
|---------------------------------|----------------------------------|-----------------------------------|
| a) $y = (x - 1)(x + 5)$ | (b) $y = (3 + x)(7 - x)$ | (c) $y = -(3 + x)(5 - x)$ |
| d) $y = -(x + 8)(x - 4)$ | (e) $y = (x + 1)(x - 7)$ | (f) $y = (1 + x)(7 - x)$ |
| g) $y = -(x - 3)(x + 9)$ | (h) $y = (x - 10)(x + 2)$ | (i) $y = -(x - 9)(x + 7)$ |
| j) $y = -(x + 4)(x - 6)$ | (k) $y = (1 + x)(1 - x)$ | (l) $y = (x + 2)(x - 6)$ |
| m) $y = (x - 3)(x + 3)$ | (n) $y = -(x - 7)(x + 1)$ | (o) $y = -(x + 10)(x - 6)$ |

Example 2 – completed square

Sketch the graph with equation $y = (x - 2)^2 - 9$, showing the coordinates of the turning point and the point of intersection with the y -axis.

Solution

Shape – the x^2 term is positive, so the graph is a ‘happy’ parabola (if it was negative, it would be ‘unhappy’).

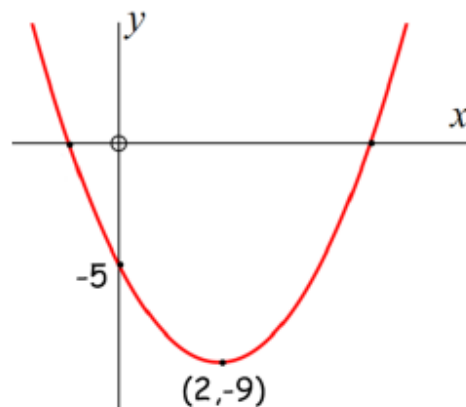
Turning Point – this can be found directly from the equation using the technique on page 43. This tells us that the turning point of $y = (x - 2)^2 - 9$ is $(2, -9)$.

y -intercept – to find the y intercept, substitute $x = 0$ into the equation.

$$\begin{aligned}y &= (x - 2)^2 - 9 \\&= (0 - 2)^2 - 9 \\&= (-2)^2 - 9 \\&= 4 - 9 \\&= -5\end{aligned}$$

Answer: the y intercept is at $y = -5$.

Sketch – sketch the graph, including all the key points found above. A possible sketch is shown on the right.



Exercise 2

1. Sketch the graphs with the following equations

- | | | |
|--------------------------------|--|------------------------------------|
| a) $y = (x - 4)^2 + 1$ | (b) $y = (x - 2)^2 + 5$ | (c) $y = (x - 1)^2 + 7$ |
| d) $y = (x - 2)^2 - 3$ | (e) $y = (x - 3)^2 - 4$ | (f) $y = (x - 5)^2 - 2$ |
| g) $y = (x + 4)^2 + 6$ | (h) $y = (x + 1)^2 + 5$ | (i) $y = (x + 8)^2 + 1$ |
| j) $y = (x + 3)^2 - 1$ | (k) $y = (x + \frac{1}{2})^2 - \frac{3}{4}$ | (l) $y = (x + 0.5)^2 - 2.5$ |
| m) $y = -(x - 1)^2 + 4$ | (n) $y = -(x + 6)^2 + 3$ | (o) $y = -(x + 7)^2 - 2$ |
| p) $y = (2 - x)^2 + 12$ | (q) $y = (5 - x)^2 - 1$ | (r) $y = (4 - x)^2 + 3.75$ |

Answers

Exercise 1

1. Graphs should all be minimum T.P. and show the following points:

- | | |
|--|---|
| a) (1, 0), (5, 0) and (3, -4) | (b) (2, 0), (4, 0) and (3, -1) |
| c) (3, 0), (7, 0) and (5, -4) | (d) (6, 0), (8, 0) and (7, -4) |
| e) (2, 0), (5, 0) and (3.5, -2.25) | (f) (5, 0), (8, 0) and (6.5, -2.25) |
| g) (-2, 0), (-3, 0) and (-2.5, -0.25) | (h) (-2, 0), (-5, 0) and (-3.5, -2.25) |
| i) (-4, 0), (-6, 0) and (-5, -1) | (j) (-3, 0), (-4, 0) and (-3.5, -0.25) |
| k) (-9, 0), (-5, 0) and (-7, -4) | (l) (-8, 0), (-3, 0) and (-5.5, -6.25) |

3. Graphs should show the following:

- | | |
|---|--|
| a) (1, 0), (-5, 0), (-2, -9), minimum | (b) (7, 0), (-3, 0), (2, 25), maximum |
| c) (5, 0), (-3, 0), (1, 16), minimum | (d) (4, 0), (-8, 0), (-2, 36), maximum |
| e) (7, 0), (-1, 0), (3, -16), minimum | (f) (7, 0), (-1, 0), (3, 16), maximum |
| g) (3, 0), (-9, 0), (-3, 36), maximum | (h) (10, 0), (-2, 0), (4, -36), minimum |
| i) (-7, 0), (9, 0), (1, 64), maximum | (j) (6, 0), (-4, 0), (1, 25), maximum |
| k) (1, 0), (-1, 0), (0, 1), maximum | (l) (6, 0), (-2, 0), (2, -16), minimum |
| m) (3, 0), (-3, 0), (0, -9), minimum | (n) (7, 0), (-1, 0), (3, 16), maximum |
| o) (6, 0), (-10, 0), (-2, 64), maximum | |

Exercise 2

1. Graphs should show the following:
 - a) Turning point (4, 1); minimum; y – intercept (0, 17)
 - b) Turning point (2, 5); minimum; y – intercept (0, 9)
 - c) Turning point (1, 7); minimum; y – intercept (0, 8)
 - d) Turning point (2, -3); minimum; y – intercept (0, 1)
 - e) Turning point (3, -4); minimum; y – intercept (0, 5)
 - f) Turning point (5, -2); minimum; y – intercept (0, 23)
 - g) Turning point (-4, 6); minimum; y – intercept (0, 22)
 - h) Turning point (-1, 5); minimum; y – intercept (0, 6)
 - i) Turning point (-8, 1); minimum; y – intercept (0, 65)
 - j) Turning point (-3, -1); minimum; y – intercept (0, 8)
 - k) Turning point ($-\frac{1}{2}$, $-\frac{3}{4}$); minimum; y – intercept (0, $-\frac{1}{2}$)
 - l) Turning point (-0.5, -2.5); minimum; y – intercept (0, -2.25)
 - m) Turning point (1, 4); maximum; y – intercept (0, 3)
 - n) Turning point (-6, 3); maximum; y – intercept (0, -33)
 - o) Turning point (-7, -2); maximum; y – intercept (0, -51)
 - p) Turning point (2, 12); minimum; y – intercept (0, 16)
 - q) Turning point (5, -1); minimum; y – intercept (0, 24)
 - p) Turning point (4, 3.75); minimum; y – intercept (0, 19.75)