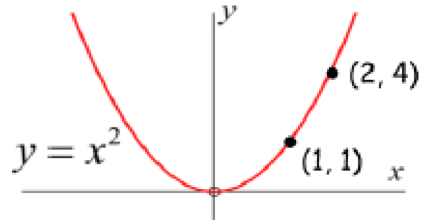
N5 Graphs of Quadratics

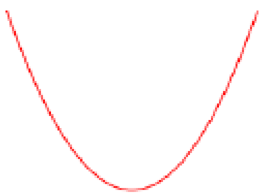
Relationships

SPTA Mathematics – Topic Questions with Notes

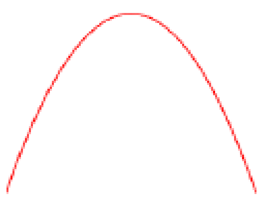
Definition: the graph of a quadratic function is called a **parabola**.

The graph on the right is the basic graph of $y = x^2$. You should know its shape.





If x^2 is positive, the graph is "happy" (it has a **minimum** turning point)



If x^2 is negative, then the graph is "unhappy" (it has a **maximum** turning point)

The coordinates of any point on a graph tells you a value for x and y.

e.g. for the coordinate point (3 , 7), $x = 3$ and $y = 7$

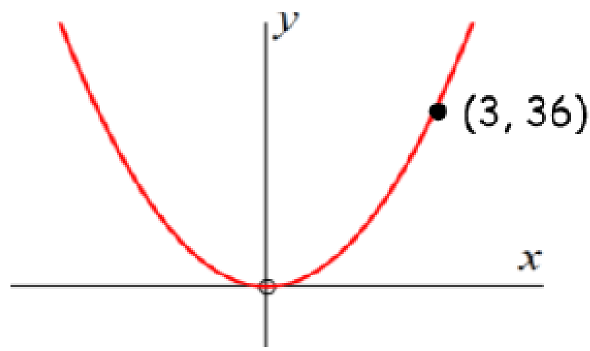
e.g. for the coordinate point (0 , 5), $x = 0$ and $y = 5$

e.g. for the coordinate point (–4 , 1), $x = –4$ and $y = 1$

These values can be put back into the equation of the graph. If you don't know the full equation of a graph, they can give you an equation to solve to complete it.

Example

The graph on the right has the equation $y = kx^2$. The graph passes through the point (3, 36). Find the value of k.



Solution

A point on the graph is (3 , 36). This means that $x = 3$ and $y = 36$.

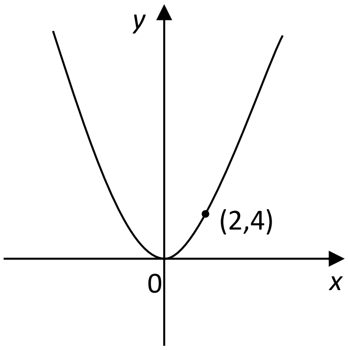
Substituting these values into the equation gives:

$$y = kx^2$$
$$36 = k \times 3^2$$
$$36 = k \times 9$$
$$k = 4$$

__Exercise 1__

1. Write down the equation of the graphs shown below, which have the form $y = kx^2$.

a)



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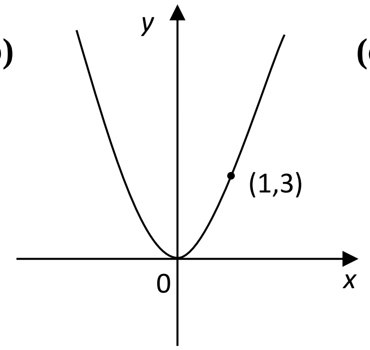

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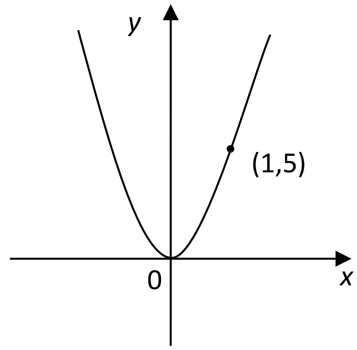

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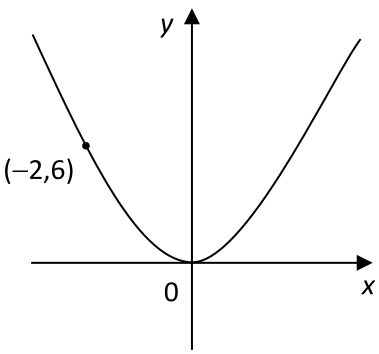

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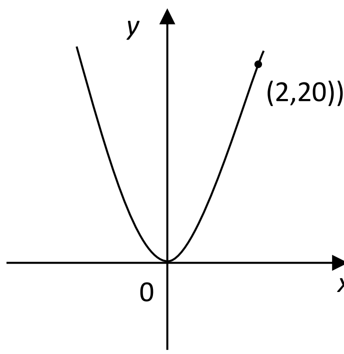

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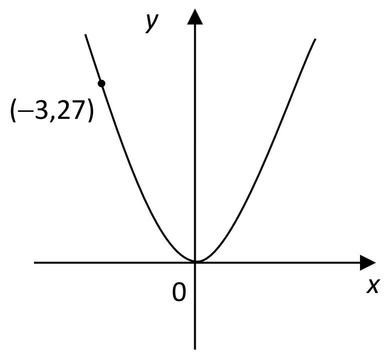

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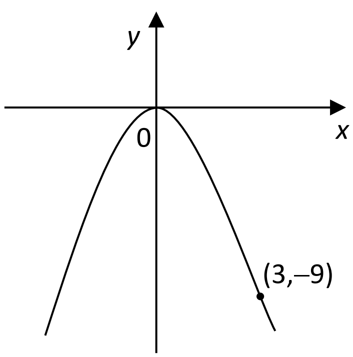

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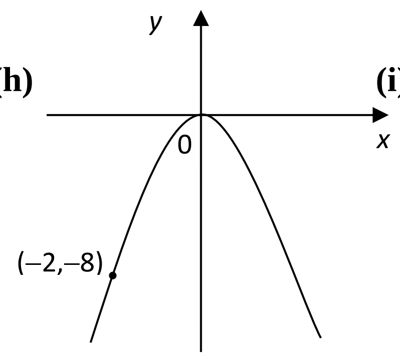

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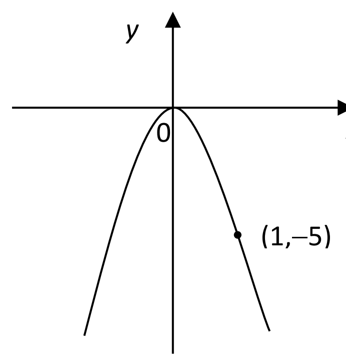

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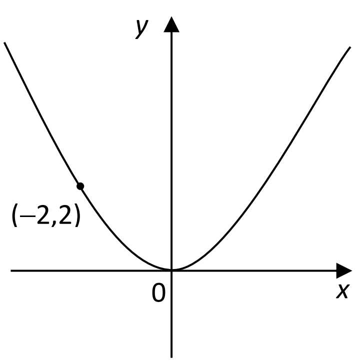

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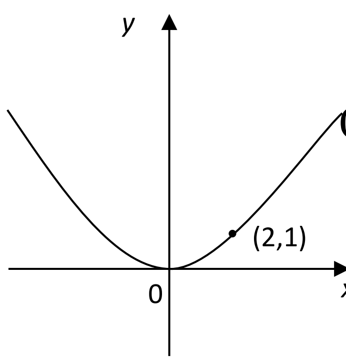

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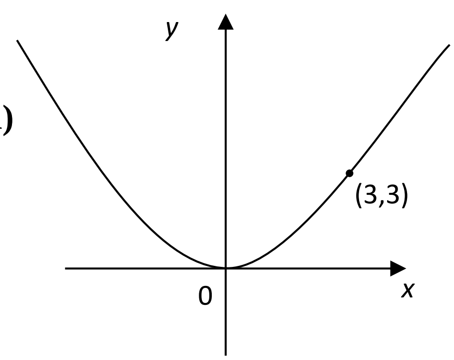

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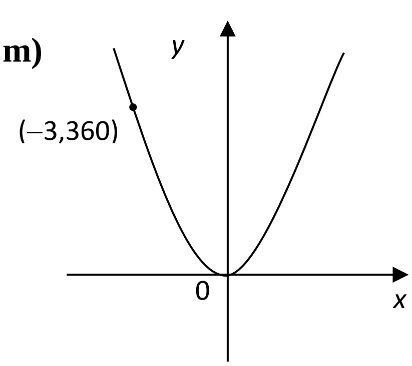

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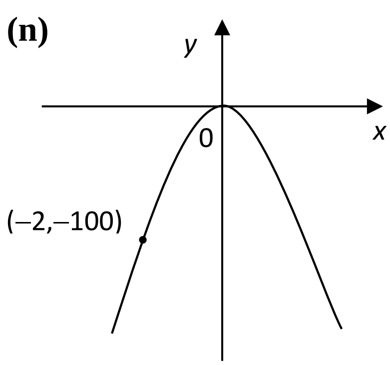

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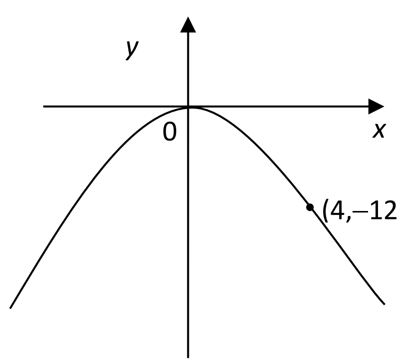
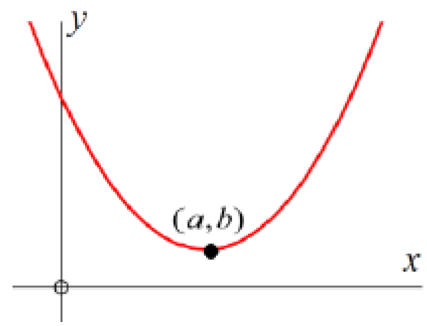


The graph of $y=(x-a)^2+b$ is still a parabola, but it has been moved so that its minimum point is no longer at (0, 0):

- The number inside the bracket (a) tells us how far the graph has been moved **left or right**. If a is positive, the graph moved to the left. If it is negative, it moved to the right.
- The number outside the bracket (b) tells us how far the graph has been moved **up or down**. If b is positive, the graph moved upwards. If it is negative, it moved downwards.



Example A: The graph of $y=(x+3)^2-2$ has been moved 3 to the left and 2 down. Its minimum point is (–3, –2)

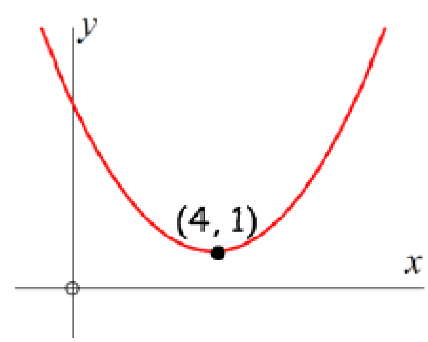
Example B: The graph of $y=(x-5)^2+4$ is a 'happy' parabola that has been moved 5 to the right and 4 up. Its minimum point is (5, 4)

Key facts:

- $y=(x-a)^2+b$ is a "happy" parabola. Its minimum turning point is (a, b)
- $y=-(x-a)^2+b$ is an "unhappy" parabola. Its maximum turning point is (a,b)
- The **axis of symmetry** of $y=(x-a)^2+b$ or $y=-(x-a)^2+b$ has the equation $x=a$

Example 1

This graph has an equation of the form $y=(x-a)^2+b$. What is its equation?



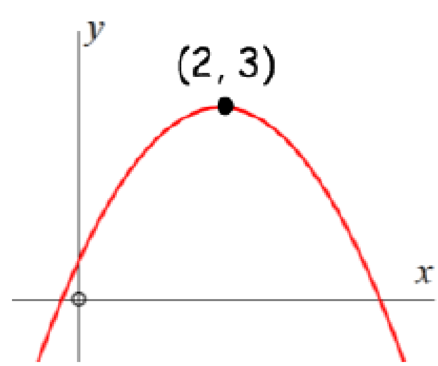
Solution

The minimum point is (4, 1). This tells us that the graph has been moved 4 to the right (so a = 4) and 1 up (so b = 1)

Therefore its equation is $\underline{y=(x-4)^2+1}$

Example 2

What is the equation of this parabola?



Solution

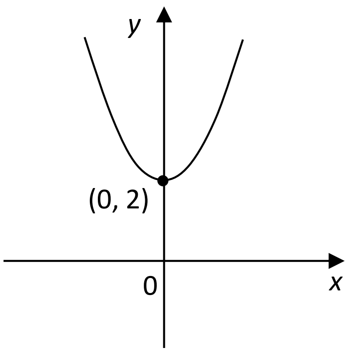
The graph is unhappy, so it has an equation of the form
$y=-(x-a)^2+b$ (as opposed to $y=(x-a)^2+b$)

The maximum point is (2, 3). The graph has moved 2 to the right and 3 up. Therefore its equation is $\underline{y=-(x-2)^2+3}$.

Exercise 2

1. Write down the equation of the graphs shown below, which have the form $y = ax^2 + b$.

a)



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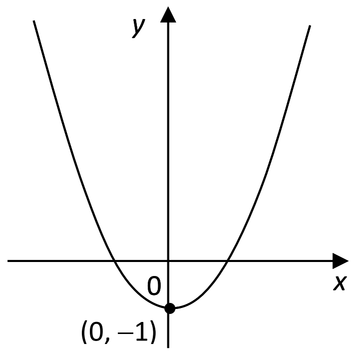

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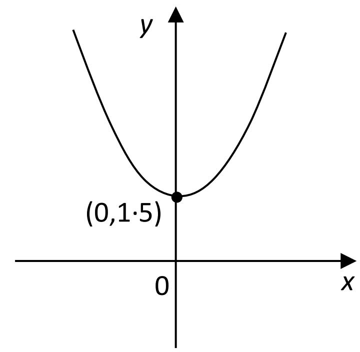

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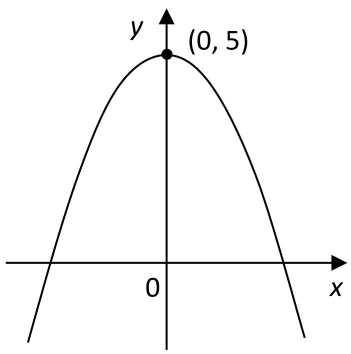

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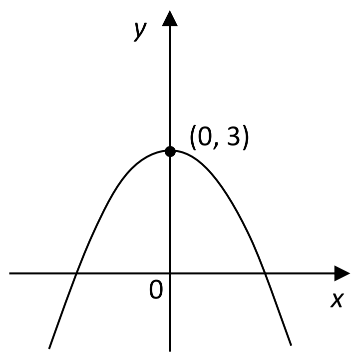

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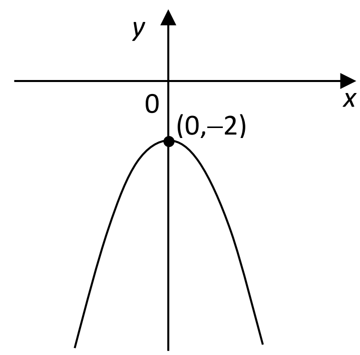

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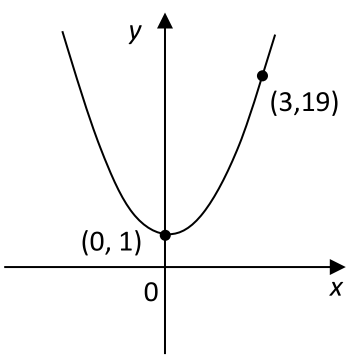

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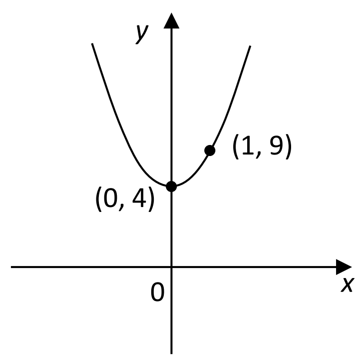

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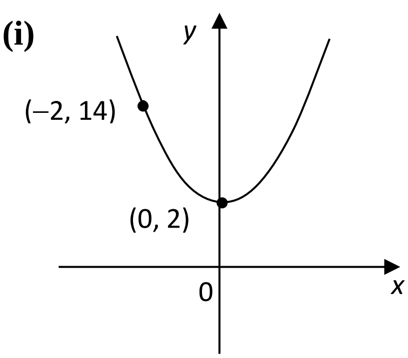

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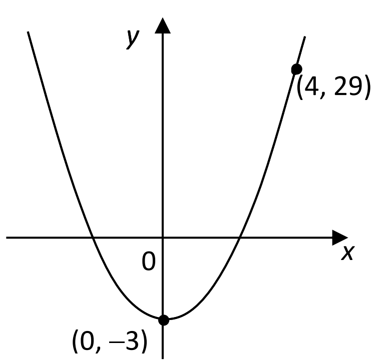

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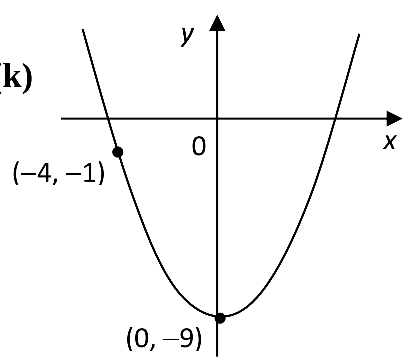

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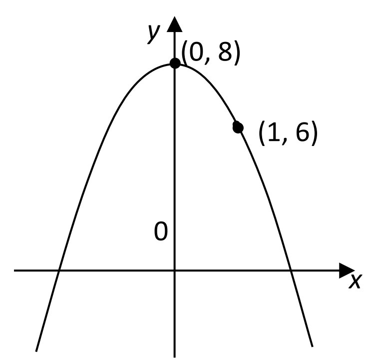

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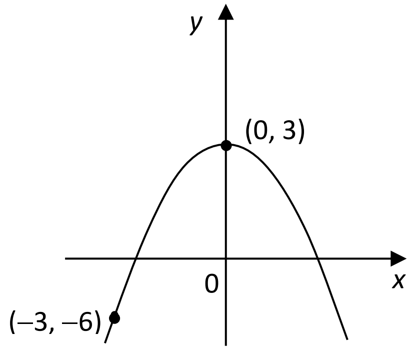

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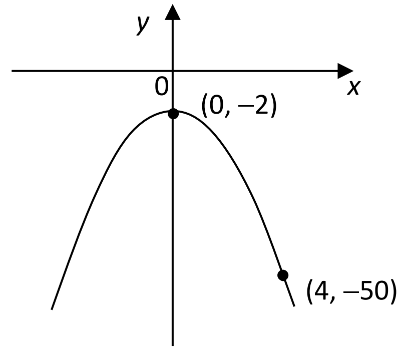
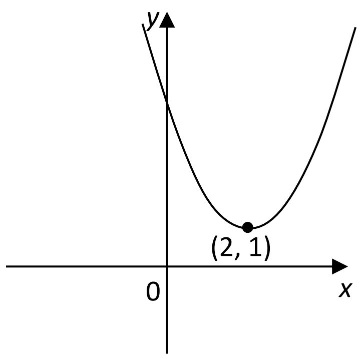


2. Write down the equation of the graphs shown below, which have the form $y = (x + a)^2 + b$.

a)



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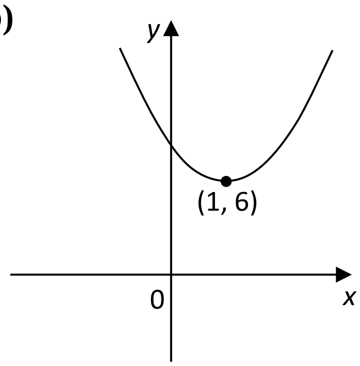

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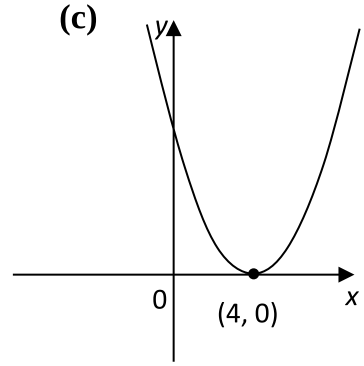

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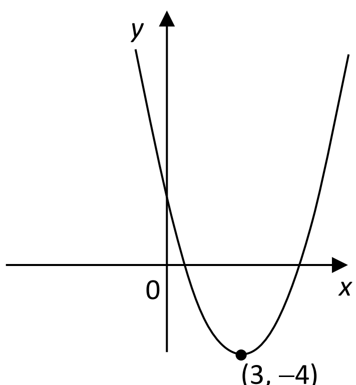

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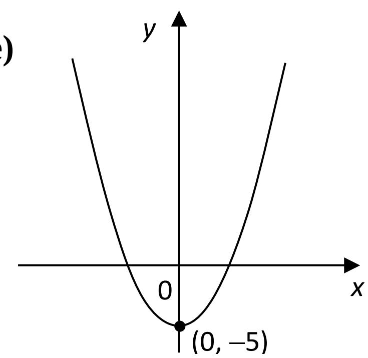

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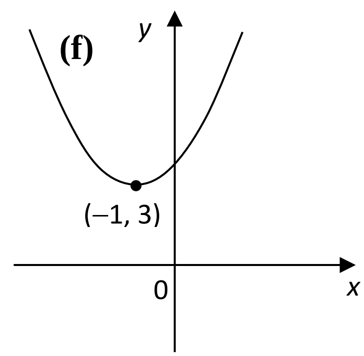

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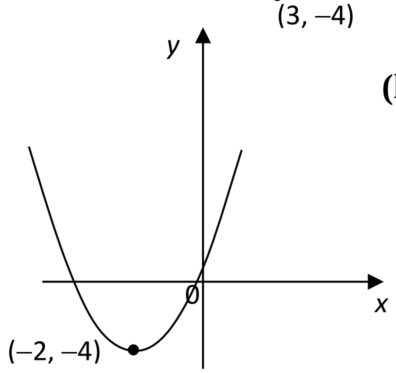

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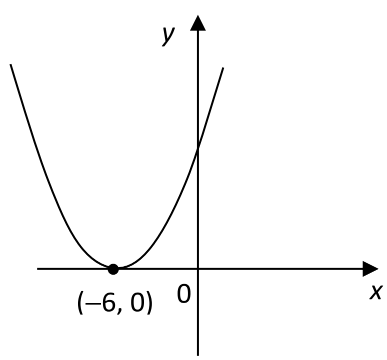

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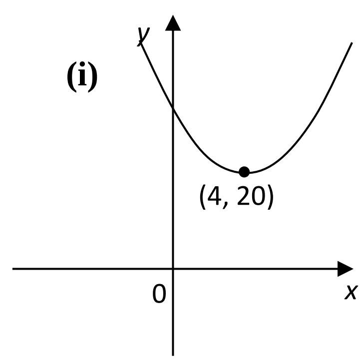

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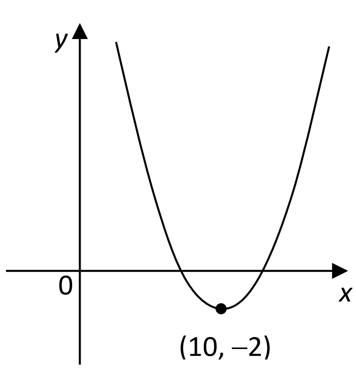

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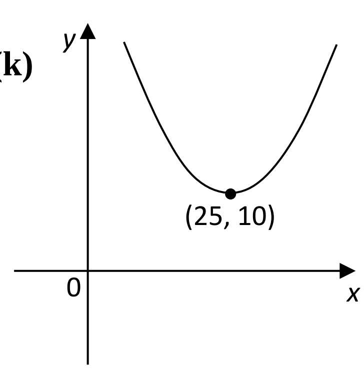

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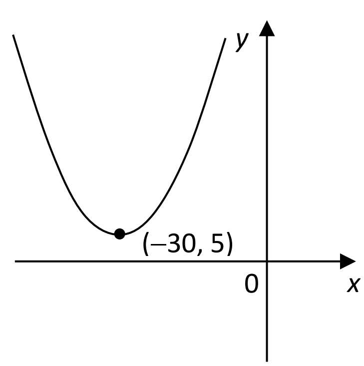

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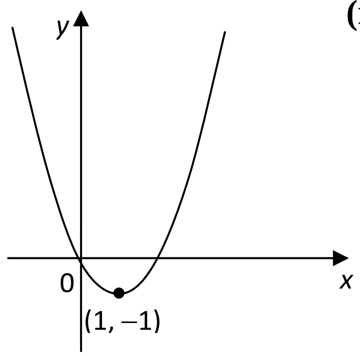

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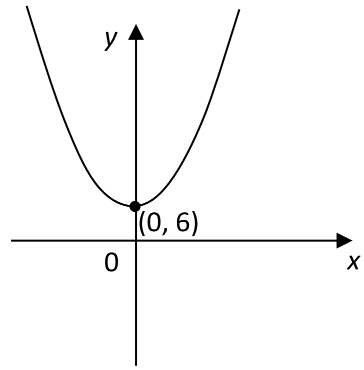

Answers

Exercise 1

1. **a)** $y = x^2$ **(b)** $y = 3x^2$ **(c)** $y = 5x^2$ **(d)** $y = 1\cdot 5x^2$

e) $y = 5x^2$ **(f)** $y = 3x^2$ **(g)** $y = -x^2$ **(h)** $y = -2x^2$

i) $y = -5x^2$ **(j)** $y = \frac{1}{2}x^2$ **(k)** $y = \frac{1}{4}x^2$ **(l)** $y = \frac{1}{3}x^2$

m) $y = 40x^2$ **(n)** $y = -25x^2$ **(o)** $y = -\frac{3}{4}x^2$

Exercise 2

1. **a)** $y = x^2 + 2$ **(b)** $y = x^2 - 1$ **(c)** $y = x^2 + 1\cdot 5$ **(d)** $y = -x^2 + 5$

(e) $y = -x^2 + 3$ **(f)** $y = -x^2 - 2$ **(g)** $y = 2x^2 + 1$ **(h)** $y = 5x^2 + 4$

(i) $y = 3x^2 + 2$ **(j)** $y = 2x^2 - 3$ **(k)** $y = \frac{1}{2}x^2 - 9$ **(l)** $y = -2x^2 + 8$

(m) $y = -x^2 + 3$ **(n)** $y = -3x^2 - 2$

2. **a)** $y = (x-2)^2 + 1$ **(b)** $y = (x-1)^2 + 6$ **(c)** $y = (x-4)^2$

d) $y = (x-3)^2 - 4$ **(e)** $y = x^2 - 5$ **(f)** $y = (x+1)^2 + 3$

g) $y = (x+2)^2 - 4$ **(h)** $y = (x+6)^2$ **(i)** $y = (x-4)^2 + 20$

j) $y = (x-10)^2 - 2$ **(k)** $y = (x-25)^2 + 10$ **(l)** $y = (x+30)^2 + 5$

m) $y = (x-1)^2 - 1$ **(n)** $y = x^2 + 6$