N5 Equations/Inequations

Relationships

SPTA Mathematics – Topic Questions with Notes

Exercise 1 – National 4 Revision

1. Solve :

a)	$x + 3 = 5$	**(b)**	$x + 5 = 9$	**(c)**	$x + 9 = 12$	**(d)**	$x + 2 = 7$
e)	$a + 2 = 4$	**(f)**	$y + 3 = 8$	**(g)**	$p + 7 = 11$	**(h)**	$c + 4 = 5$
i)	$b - 7 = 9$	**(j)**	$q - 8 = 8$	**(k)**	$d - 5 = 10$	**(l)**	$x - 1 = 6$
m)	$c - 4 = 6$	**(n)**	$p - 6 = 14$	**(o)**	$a - 2 = 15$	**(p)**	$y - 5 = 14$

2. Solve:

a)	$2x = 6$	**(b)**	$5x = 20$	**(c)**	$8x = 16$	**(d)**	$3x = 27$
e)	$4a = 16$	**(f)**	$7y = 28$	**(g)**	$6p = 18$	**(h)**	$5c = 25$
i)	$9b = 36$	**(j)**	$2q = 18$	**(k)**	$7d = 70$	**(l)**	$4x = 32$
m)	$8c = 56$	**(n)**	$3p = 15$	**(o)**	$5a = 35$	**(p)**	$6y = 42$

3. Solve:

a)	$2a = -36$	**(b)**	$-5m = -55$	**(c)**	$-8q = 64$	**(d)**	$-3y = -48$
e)	$4x = -52$	**(f)**	$-7c = -63$	**(g)**	$-6d = 72$	**(h)**	$-5a = -125$
i)	$9p = -81$	**(j)**	$-2q = -17$	**(k)**	$-4x = 22$	**(l)**	$-6q = -33$
m)	$8c = -28$	**(n)**	$-5x = -90$	**(o)**	$-10a = 42$	**(p)**	$-4y = -42$

4. Solve :

a)	$2x - 3 = 5$	**(b)**	$4x + 5 = 9$	**(c)**	$3x + 3 = -12$	**(d)**	$5x + 2 = 7$
e)	$2a - 2 = -14$	**(f)**	$5y + 3 = 18$	**(g)**	$2p + 7 = 21$	**(h)**	$3c - 4 = 17$
i)	$6b + 7 = 49$	**(j)**	$8q - 8 = -8$	**(k)**	$2d - 5 = 35$	**(l)**	$3x + 5 = -25$
m)	$8c + 4 = 36$	**(n)**	$7p + 6 = 55$	**(o)**	$12a + 2 = 26$	**(p)**	$9y + 5 = 50$

5. Solve :

a)	$3x - 2 = 7$	**(b)**	$4x - 5 = 11$	**(c)**	$2x - 9 = 3$	**(d)**	$3x - 7 = 5$
e)	$7a - 2 = 12$	**(f)**	$5y - 3 = 22$	**(g)**	$6p - 7 = 29$	**(h)**	$4c - 3 = 29$
i)	$8b - 7 = 57$	**(j)**	$10q - 8 = 72$	**(k)**	$3d - 5 = 31$	**(l)**	$9x - 1 = 80$
m)	$4c - 9 = 15$	**(n)**	$6p - 2 = 40$	**(o)**	$5a - 2 = 73$	**(p)**	$3y - 14 = 40$

At National 5 level, you need to be able solve more complex equations and inequations. The method used in these notes is to *change side and do the opposite*. You have to use a method to get the answer – ***if you just write the answer down (even if you think it is obvious) you will get no marks.***

You should always check your final answer so that you know it is correct.

At National 4, you learnt to solve an equation that has letters on both sides.

At National 5 you need to be able to use this method and to extend it to inequations and more difficult equations.

The basic method is as follows:

- **Step one** – move everything with a letter in to the left-hand side, and all the numbers to the right-hand side, remembering to "change side and do the opposite".
- **Step two** – simplify each side
- **Step three** – solve the resulting equation
- **Final step** – double check your answer works by substituting it back in to both sides of the original equation and checking both sides give the same answer.

Example 1 – equation

Solve algebraically the equation $5y - 5 = 3y + 9$

Solution

Optional first step *– write in "invisible plus signs" in front of anything that does not already have a sign*

Step one – move the '+ 3y' over to the left-hand side where it becomes '–3y'. Move '–5' to the right-hand side where it becomes '+5'.

Step two – simplify both sides
Step three – divide to get the final answer

$$\begin{aligned} 5y - 5 &= 3y + 9 \\ +5y - 5 &= +3y + 9 \\ +5y - 3y &= +9 + 5 \\ 2y &= 14 \\ y &= \tfrac{14}{2} \\ y &= 7 \end{aligned}$$

Final step: check, by substituting $y = 7$ into the original equation

- The left-hand side is $5y - 5$. If we replace y with 7, we get $5 \times 7 - 5$, which makes 30.
- The left-hand side is $3y + 9$. If we replace y with 7, we get $3 \times 7 + 9$, which also makes 30. These are the same, so our answer has to be correct.

Inequalities (also known as Inequations) are solved in exactly the same way as for equations, except that there is not a '=' sign in the middle.

Example 2 – inequation

Solve algebraically the inequation $2a + 5 < 15 - 2a$

Solution

Optional first step *– write in "invisible plus signs" in front of anything that does not already have a sign*

Step one – move the '–2a' over to the left-hand side where it becomes '+2a'. Move '+5' to the right-hand side where it becomes '–5'.

Step two – simplify both sides
Step three – divide to get the final answer. It is fine to leave the final answer as a fraction, so long as it is in its simplest form. (**Note**, we could also have changed $\frac{5}{2}$ to a decimal to give the final answer as $a < 2.5$).

$$\begin{aligned} 2a + 5 &< 15 - 2a \\ +2a + 5 &< +15 - 2a \\ +2a + 2a &< +15 - 5 \\ 4a &< 10 \\ a &< \tfrac{10}{4} \\ a &< \tfrac{5}{2} \quad \text{(simplifying)} \end{aligned}$$

If an equation or an inequation contains a bracket, the bracket can be multiplied out before proceeding with the usual method.

Example 3 – inequation containing brackets

Solve algebraically the inequation $7(x+5) \geq 3x-2$

Solution

$$7(x+5) \geq 3x-2$$
$$7x+35 \geq 3x-2 \quad \text{(multiplying out brackets)}$$
$$+7x+35 \geq +3x-2 \quad \text{(optional: writing in invisible + signs)}$$
$$+7x-3x \geq -2-35 \quad \text{(collecting like terms)}$$
$$4x \geq -37 \quad \text{(simplifying)}$$
$$x \geq -\frac{37}{4}$$

Equations and Inequations containing fractions

If an equation or an inequation contains a fraction, we can use a technique called **cross multiplication** to remove the fractions.

Cross multiplication involves multiplying the numerator (top) of each fraction by the denominator (bottom) of the other. For example, $\frac{a}{b}=\frac{c}{d}$ rearranges to become $ad=bc$.

Example 1

Solve the equation $\frac{x}{4}=\frac{1}{x}$

Solution

$$\frac{x}{4} \times \frac{1}{x} \quad \text{(cross multiplying)}$$
$$x \times x = 1 \times 4$$
$$x^2 = 4$$
$$x = \pm 2$$

If either the numerator and/or the denominator of a fraction contains more than just a single letter or number then we must introduce brackets to the expression.

Example 2

Solve the equation $\dfrac{2}{x+3}=5$

Solution

$$\frac{2}{x+3}=5$$

$$\frac{2}{(x+3)}=5 \qquad \text{(introducing brackets)}$$

$$2=5(x+3) \qquad \text{(cross multiplying)}$$

$$5(x+3)=2 \qquad \text{(optional: switching sides so that } x \text{ is on the left hand side)}$$

$$5x+15=2 \qquad \text{(multiplying out the bracket)}$$

$$5x=2-15 \qquad \text{(moving } +15 \text{ to the right-hand side)}$$

$$5x=-13$$

$$x=-\frac{13}{5}$$

This is not the only method. For example, some people prefer to jump straight to the $5x+15$ stage without requiring the brackets.

With inequations only, there is one additional rule to bear in mind:

Rule: In an inequation, if you multiply or divide by a *negative* number, the sign reverses (e.g. $\geq$ becomes $\leq$; $<$ becomes $>$ etc.)

Example

Solve algebraically the inequation $x-8>4x+7$

Solution

$$+x-8>+4x+7$$

$$x-4x>7+8$$

$$-3x>15$$

$$x<\tfrac{15}{-3} \qquad \text{(changing } > \text{ to } < \text{ because we are dividing by a negative)}$$

$$x<-5$$

Exercise 2

1. Multiply out the brackets and solve :

a)	$2(x + 5) = 12$	**(b)**	$5(y + 7) = 45$	**(c)**	$3(a + 6) = 36$
d)	$6(x + 4) = 54$	**(e)**	$4(x + 9) = 48$	**(f)**	$3(c + 8) = 30$
g)	$7(d + 3) = 56$	**(h)**	$5(m + 5) = 55$	**(i)**	$2(y + 14) = 50$
j)	$8(d - 6) = 24$	**(k)**	$3(s - 8) = 9$	**(l)**	$4(x - 15) = 20$
m)	$10(w - 2) = 50$	**(n)**	$5(c - 5) = 35$	**(o)**	$3(a - 10) = 33$

2. Solve :

a)	$6y + 3 = y + 18$	**(b)**	$5a + 7 = a + 15$	**(c)**	$9c + 5 = c + 21$
d)	$10x + 1 = 4x + 19$	**(e)**	$5b + 3 = 2b + 9$	**(f)**	$7n + 6 = 3n + 18$
g)	$3x + 2 = x + 14$	**(h)**	$9c + 58 = 6c + 73$	**(i)**	$16 + 7y = 2y + 31$
j)	$15a + 4 = 3a + 76$	**(k)**	$16 + 25x = 5x + 96$	**(l)**	$6n + 3 \cdot 5 = 3n + 5$
m)	$19b + 8 = 10b + 80$	**(n)**	$14x + 4 = 3x + 125$	**(o)**	$250 + 3x = 295$
p)	$20y + 4 = 3y + 55$	**(q)**	$13a + 6 = a + 150$	**(r)**	$50x + 40 = 10x + 200$
s)	$19y + 3 = 8y + 80$	**(t)**	$5b + 2 = 2b + 50$	**(u)**	$2 + 14x = 2x + 110$
v)	$20x + 11 = 13x + 60$	**(w)**	$19x + 10 = 4x + 70$	**(x)**	$205a + 13 = 10a + 403$

3. Solve :

a)	$6y - 3 = 3y + 15$	**(b)**	$5a - 9 = a + 15$	**(c)**	$9c - 8 = 4c + 12$
d)	$10x - 1 = 4x + 5$	**(e)**	$5b - 3 = 2b + 9$	**(f)**	$3n - 10 = n + 2$
g)	$7x - 14 = 3x + 2$	**(h)**	$6c - 13 = 3c + 59$	**(i)**	$7y - 16 = 2y + 34$
j)	$15a - 8 = 3a + 76$	**(k)**	$25x - 16 = 5x + 84$	**(l)**	$6n - 3 \cdot 5 = 3n + 4$
m)	$b + 13 = 9b - 7$	**(n)**	$3x + 12 = 4x - 4$	**(o)**	$x + 25 = 3x - 5$
p)	$5y + 4 = 20y - 26$	**(q)**	$a + 6 = 13a - 18$	**(r)**	$10x + 40 = 50x - 120$
s)	$8y + 3 = 19y - 74$	**(t)**	$2b + 2 = 5b - 16$	**(u)**	$2 + 2x = 10x - 14$
v)	$13x + 11 = 20x - 38$	**(w)**	$4x + 10 = 9x - 50$	**(x)**	$10a + 13 = 20a - 387$

4. Solve :

a)	$x + 4 > 5$	**(b)**	$x + 6 > 9$	**(c)**	$x + 8 > 12$	**(d)**	$x + 3 > 7$
e)	$a + 1 > 4$	**(f)**	$y + 5 > 8$	**(g)**	$p + 2 > 11$	**(h)**	$c + 4 > 5$
i)	$b + 3 > 9$	**(j)**	$q + 8 > 8$	**(k)**	$d + 7 > 10$	**(l)**	$x + 2 > 6$
m)	$c + 1 > 6$	**(n)**	$p + 4 > 13$	**(o)**	$a + 3 > 15$	**(p)**	$y + 2 > 14$

5. Solve :

a)	$x + 5 < 7$	**(b)**	$x + 1 < 8$	**(c)**	$x + 3 < 13$	**(d)**	$x + 5 < 9$
e)	$a + 3 < 6$	**(f)**	$y + 5 < 11$	**(g)**	$p + 2 < 10$	**(h)**	$c + 1 < 5$
i)	$b + 8 < 13$	**(j)**	$q + 3 < 20$	**(k)**	$d + 7 < 7$	**(l)**	$x + 10 < 15$
m)	$c + 3 < 9$	**(n)**	$p + 2 < 16$	**(o)**	$a + 4 < 15$	**(p)**	$y + 9 < 10$

6. Solve:

a)	$2x > 6$	**(b)**	$5x > 20$	**(c)**	$8x > 16$	**(d)**	$3x > 27$
e)	$4a > 16$	**(f)**	$7y > 28$	**(g)**	$6p > 18$	**(h)**	$5c > 25$
i)	$9b < 36$	**(j)**	$2q < 18$	**(k)**	$7d < 70$	**(l)**	$4x < 32$
m)	$8c < 56$	**(n)**	$3p < 15$	**(o)**	$5a < 35$	**(p)**	$6y < 42$

7. Solve :

a)	$x - 3 < 4$	**(b)**	$x - 5 > 1$	**(c)**	$x - 9 > 2$	**(d)**	$x - 2 < 7$
e)	$a - 2 < 4$	**(f)**	$y - 3 > 8$	**(g)**	$p - 7 < 11$	**(h)**	$c - 4 > 5$
i)	$b - 7 > 9$	**(j)**	$q - 8 < 8$	**(k)**	$d - 5 > 10$	**(l)**	$x - 1 > 6$
m)	$c - 4 > 6$	**(n)**	$p - 6 < 14$	**(o)**	$a - 2 < 15$	**(p)**	$y - 5 < 14$

8. Solve :

a)	$2x + 1 < 5$	**(b)**	$4x + 1 > 9$	**(c)**	$3x + 3 > 12$	**(d)**	$5x + 2 > 12$
e)	$2a + 2 < 8$	**(f)**	$5y + 3 < 13$	**(g)**	$2p + 5 > 21$	**(h)**	$3c + 1 < 16$
i)	$6b + 13 > 49$	**(j)**	$8q + 8 < 8$	**(k)**	$3d + 5 < 35$	**(l)**	$4x + 5 > 21$
m)	$8c + 12 < 36$	**(n)**	$7p + 6 < 55$	**(o)**	$12a + 2 > 26$	**(p)**	$9y + 23 < 50$

9. Solve :

a)	$3x - 1 > 8$	**(b)**	$4x - 3 > 13$	**(c)**	$2x - 7 < 5$	**(d)**	$3x - 5 > 4$
e)	$7a - 1 < 13$	**(f)**	$5y - 2 < 23$	**(g)**	$6p - 5 > 31$	**(h)**	$4c - 7 > 25$
i)	$8b - 3 > 61$	**(j)**	$10q - 7 < 73$	**(k)**	$3d - 2 < 34$	**(l)**	$9x - 8 > 73$
m)	$4c - 5 < 19$	**(n)**	$6p - 1 < 41$	**(o)**	$5a - 4 < 71$	**(p)**	$3y - 24 < 30$

10. Solve each of the following inequations where x can only take values from the set of numbers $\{-2, -1, 0, 1, 2, 3, 4, 5\}$.

a)	$6x+2 \le 3x+5$	**(b)**	$7x \ge 13x+3$
c)	$3(2x+1) \ge 5x+8$	**(d)**	$2(6+5x) < 8x+12$
e)	$14-2(3-x) \le 8$	**(f)**	$5+3(2-x) \ge 14-6x$
g)	$2x-(4-x) < x+2$	**(h)**	$3-4(2+x) > 6(2-x)-17$

11. Solve each of the following inequations.

a)	$3a+2 \le 17-2a$	**(b)**	$7(2x+3) > 8x+27$
c)	$2(5p-12) \ge 7p-18$	**(d)**	$40+3k < 28-k$
e)	$3-5(2-m) \le 2(m+7)$	**(f)**	$3(2y-4)-1 > 4(4-y)$
g)	$2(3-4h) < 13-15h$	**(h)**	$2-3(2-x) > 2(1-x)-5$

12. Solve each of the following inequations.

a) $2a+18 \leq 12+4a$ **(b)** $14-3x > x+6$

c) $3(p-2) \geq 5p-10$ **(d)** $16-3k < 20-k$

e) $7(2-d) \leq 2(d-12)$ **(f)** $2(2y-1)-8 > 10(1+y)$

g) $4(3-4h) < 12+h$ **(h)** $3(2-y) > 2(1+3y)-7$

13. I think of a <u>whole</u> number, treble it and subtract 3. The answer must be less than or equal to 12.

Form an inequation and solve it to find the possible starting whole numbers.

14. I subtract a whole number from 8 and double the answer. The result must be greater than 10.

Form an inequation and solve it to find the possible starting whole numbers.

15. Fred and Jane are brother and sister. Fred is 3 years older than twice Jane's age.

The sum of their ages is less than 36 years.

Taking Jane's age to be x years form an inequation. What can you say about Jane's age?

Answers

Exercise 1

1.	**a)**	2	**(b)**	4	**(c)**	3	**(d)**	5	**(e)**	2	**(f)** 5
	g)	4	**(h)**	1	**(i)**	16	**(j)**	16	**(k)**	15	**(l)** 7
	m)	10	**(n)**	20	**(o)**	17	**(p)**	19			
2.	**a)**	3	**(b)**	4	**(c)**	2	**(d)**	9	**(e)**	4	**(f)** 4
	g)	2	**(h)**	5	**(i)**	4	**(j)**	9	**(k)**	10	**(l)** 8
	m)	7	**(n)**	5	**(o)**	7	**(p)**	7			
3.	**a)**	–18	**(b)**	11	**(c)**	–8	**(d)**	16	**(e)**	–13	**(f)** 9
	g)	–12	**(h)**	25	**(i)**	–9	**(j)**	8·5	**(k)**	–5·5	**(l)** 5·5
	m)	–3·5	**(n)**	18	**(o)**	–4·2	**(p)**	10·5			
4.	**a)**	4	**(b)**	1	**(c)**	–5	**(d)**	1	**(e)**	–6	**(f)** 3
	g)	7	**(h)**	7	**(i)**	7	**(j)**	0	**(k)**	20	**(l)** –10
	m)	4	**(n)**	7	**(o)**	2	**(p)**	5			
5.	**a)**	3	**(b)**	4	**(c)**	6	**(d)**	4	**(e)**	2	**(f)** 5
	g)	6	**(h)**	8	**(i)**	8	**(j)**	8	**(k)**	12	**(l)** 9
	m)	6	**(n)**	7	**(o)**	15	**(p)**	18			

Exercise 2

1. **a)** 1 **(b)** 2 **(c)** 6 **(d)** 5 **(e)** 3 **(f)** 2

g) 5 **(h)** 6 **(i)** 11 **(j)** 9 **(k)** 11 **(l)** 20

m) 7 **(n)** 12 **(o)** 21

2. **a)** 3 **(b)** 2 **(c)** 2 **(d)** 3 **(e)** 2 **(f)** 3

g) 6 **(h)** 5 **(i)** 3 **(j)** 6 **(k)** 4 **(l)** 0·5

m) 8 **(n)** 11 **(o)** 15 **(p)** 3 **(q)** 12 **(r)** 4

s) 7 **(t)** 16 **(u)** 9 **(v)** 7 **(w)** 4 **(x)** 2

3. **a)** 6 **(b)** 6 **(c)** 4 **(d)** 1 **(e)** 4 **(f)** 6

g) 4 **(h)** 24 **(i)** 8 **(j)** 7 **(k)** 5 **(l)** 2·5

m) 2·5 **(n)** 16 **(o)** 15 **(p)** 2 **(q)** 2 **(r)** 4

s) 7 **(t)** 6 **(u)** 2 **(v)** 7 **(w)** 12 **(x)** 40

4. **a)** $x > 1$ **(b)** $x > 3$ **(c)** $x > 4$ **(d)** $x > 4$ **(e)** $a > 3$ **(f)** $y > 3$

g) $p > 9$ **(h)** $c > 1$ **(i)** $b > 6$ **(j)** $q > 0$ **(k)** $d > 3$ **(l)** $x > 4$

m) $c > 5$ **(n)** $p > 9$ **(o)** $a > 12$ **(p)** $y > 12$

5. **a)** $x < 2$ **(b)** $x < 7$ **(c)** $x < 10$ **(d)** $x < 4$ **(e)** $a < 3$ **(f)** $y < 6$

g) $p < 8$ **(h)** $c < 4$ **(i)** $b < 5$ **(j)** $q < 17$ **(k)** $d < 0$ **(l)** $x < 5$

m) $c < 6$ **(n)** $p < 14$ **(o)** $a < 11$ **(p)** $y < 1$

6. **a)** $x > 3$ **(b)** $x > 4$ **(c)** $x > 4$ **(d)** $x > 9$ **(e)** $a > 4$ **(f)** $y > 4$

g) $p > 3$ **(h)** $c > 5$ **(i)** $b < 4$ **(j)** $q < 9$ **(k)** $d < 10$ **(l)** $x < 8$

m) $c < 7$ **(n)** $p < 5$ **(o)** $a < 7$ **(p)** $y < 7$

7. **a)** $x < 7$ **(b)** $x > 6$ **(c)** $x > 11$ **(d)** $x < 9$ **(e)** $a < 6$ **(f)** $y > 11$

g) $p < 18$ **(h)** $c > 9$ **(i)** $b > 16$ **(j)** $q < 16$ **(k)** $d > 15$ **(l)** $x > 7$

m) $c > 10$ **(n)** $p < 20$ **(o)** $a < 17$ **(p)** $y < 19$

8. **a)** $x < 2$ **(b)** $x > 2$ **(c)** $x > 3$ **(d)** $x > 2$ **(e)** $a < 3$ **(f)** $y < 2$

g) $p > 8$ **(h)** $c < 5$ **(i)** $b > 6$ **(j)** $q < 0$ **(k)** $d < 10$ **(l)** $x > 4$

m) $c < 3$ **(n)** $p < 7$ **(o)** $a > 2$ **(p)** $y < 3$

9. **a)** $x > 3$ **(b)** $x > 4$ **(c)** $x < 6$ **(d)** $x > 3$ **(e)** $a < 2$ **(f)** $y < 5$

g) $p > 6$ **(h)** $c > 8$ **(i)** $b > 8$ **(j)** $q < 8$ **(k)** $d < 12$ **(l)** $x > 9$

m) $c < 6$ **(n)** $p < 7$ **(o)** $a < 15$ **(p)** $y < 18$

10. **a)** $\{-2, -1, 0, 1\}$ **(b)** $\{-2, -1\}$ **(c)** $\{5\}$ **(d)** $\{-2, -1\}$

e) $\{-2, -1, 0\}$ **(f)** $\{1,2,3,4,5\}$ **(g)** $\{-2, -1, 0, 1, 2\}$ **(h)** $\{1,2,3,4,5\}$

11. **a)** $a \leq 3$ **(b)** $x > 1$ **(c)** $p \geq 2$ **(d)** $k < -3$

e) $m \leq 7$ **(f)** $y > 2 \cdot 9$ **(g)** $h < 1$ **(h)** $x > \frac{1}{5}$

12. **a)** $a \geq 3$ **(b)** $x < 2$ **(c)** $p \leq 2$ **(d)** $k > -2$

e) $d \geq \frac{38}{9}$ **(f)** $y < -\frac{10}{3}$ **(g)** $h > 0$ **(h)** $y < \frac{11}{9}$

13. {0, 1, 2, 3, 4, 5}

14. {0, 1, 2}

15. Jane must be younger than 11