N5 Discriminant

Relationships

SPTA Mathematics – Topic Questions with Notes

The Discriminant

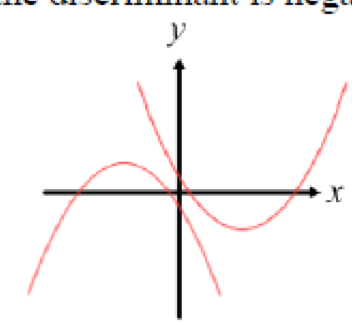
To solve any quadratic equation, we have to use the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Since this formula contains $\sqrt{b^2 - 4ac}$, and we can only take the square root of a positive number, the value of $b^2 - 4ac$ is important, and has a special name, the **discriminant**. The symbol Δ can be used for the discriminant.

Formula. This formula is **not** given on the National 5 Mathematics exam paper, though the quadratic formula that contains it is given.
The **discriminant** of a quadratic equation $ax^2 + bx + c = 0$ is given by $\Delta = b^2 - 4ac$

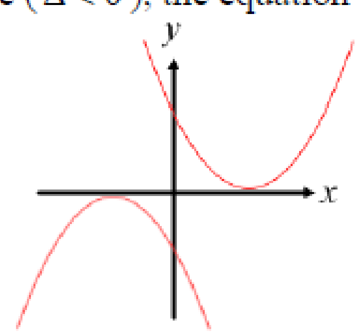
The **discriminant** of the quadratic equation $ax^2 + bx + c = 0$ is the number that has to be square rooted to complete the quadratic formula.

A quadratic equation may have two roots (known as **real and distinct** roots), one root (known as a **real and equal** root) or **no real roots**.
Simply put, the discriminant tells us how many real roots there are to a quadratic equation

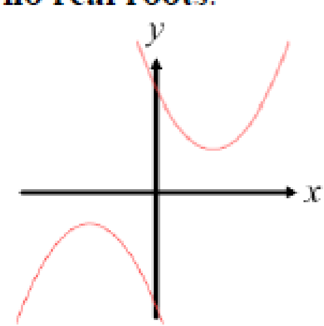
- If the discriminant is positive ($\Delta > 0$), the equation has two **distinct real** roots (also known as **unequal real** roots).
- If the discriminant is zero ($\Delta = 0$), the equation has one **real and equal** (or **real and repeated**) root
- If the discriminant is negative ($\Delta < 0$), the equation has **no real roots**.



Two real and distinct roots



One real and equal root



No real roots

Example 1

State the nature of the roots of the quadratic equation $2x^2 - x + 5 = 0$

Solution

For this equation $a = 2$, $b = -1$ and $c = 5$

$$\begin{aligned}\Delta &= (-1)^2 - 4 \times 2 \times 5 \\ &= 1 - 40 \\ &= -39\end{aligned}$$

$\Delta < 0$, so this equation has **no real roots.**

You may also be told what the nature of the roots are when the equation contains an unknown coefficient – such as $2x^2 + px + 1$ or $ax^2 - x - 3$, and you would then be expected to solve an equation to work out what p or a could be.

Example 2 – range of values

Find the range of values of p such that the equation $3x^2 - 6x + p = 0$ has real and distinct roots.

Solution

For real and distinct roots, the discriminant must be positive, so we solve $\Delta > 0$.

For this equation, $a = 3$, $b = (-6)$ and $c = p$. We can now work out Δ .

$$\begin{aligned}\Delta &= b^2 - 4ac \\ &= (-6)^2 - 4 \times 3 \times p \\ &= 36 - 12p\end{aligned}$$

Since we know the discriminant is $36 - 12p$ and that the discriminant must be positive, we can solve the equation:

$$36 - 12p > 0$$

$$-12p > -36$$

$$p < \tfrac{-36}{-12} \qquad \text{(dividing by a negative means > changes to <)}$$

$$p < 3$$

Exercise 1

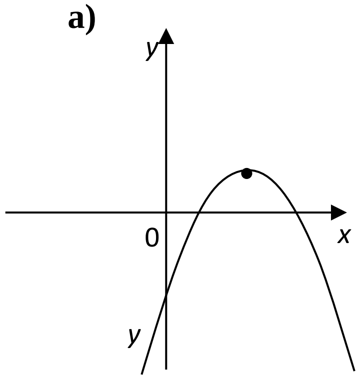
1. Find the discriminant for each of these quadratic equations

a)	$x^2 + 4x + 3 = 0$	**(b)**	$x^2 + 6x + 9 = 0$	**(c)**	$x^2 + 8x + 7 = 0$
d)	$3 - 5w - 2w^2 = 0$	**(e)**	$2x^2 + 7x + 5 = 0$	**(f)**	$x^2 - 12x + 36 = 0$
g)	$x^2 - 7x + 12 = 0$	**(h)**	$2x^2 + 7x + 9 = 0$	**(i)**	$5x^2 - 16x + 3 = 0$
j)	$6y^2 - 11y - 2 = 0$	**(k)**	$x^2 - 8x + 9 = 0$	**(l)**	$3x^2 + 2x + 7 = 0$
m)	$2x^2 - 7x + 4 = 0$	**(n)**	$4x^2 - 3x + 4 = 0$	**(o)**	$3x^2 - 2x - 1 = 0$
p)	$x^2 + 10x + 25 = 0$	**(q)**	$3x^2 - 7x + 5 = 0$	**(r)**	$x^2 - 8x + 16 = 0$

2. Use the discriminants from Q1 to state the nature of the roots of each of the quadratic equations.

3. Here are some graphs of quadratic functions. What can you say about the discriminant for each one?

a)



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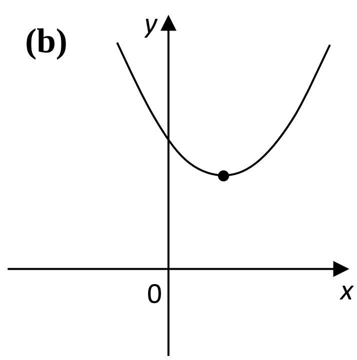

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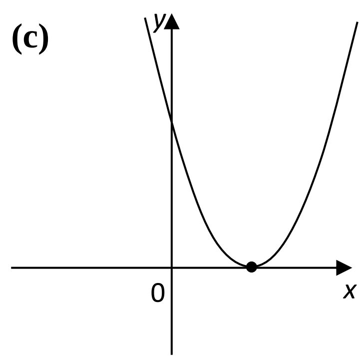

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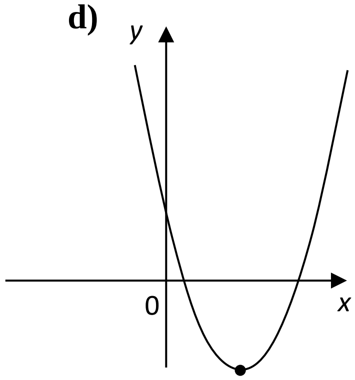

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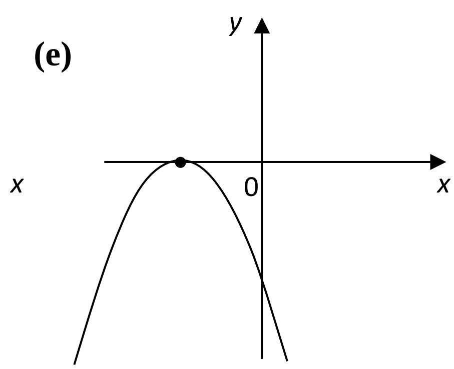

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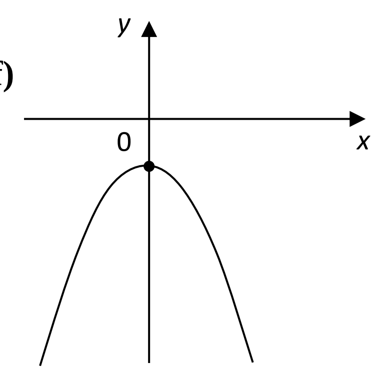

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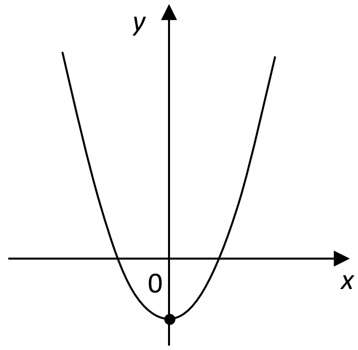

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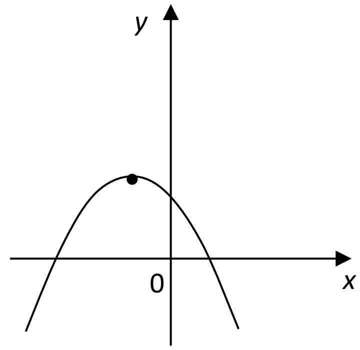

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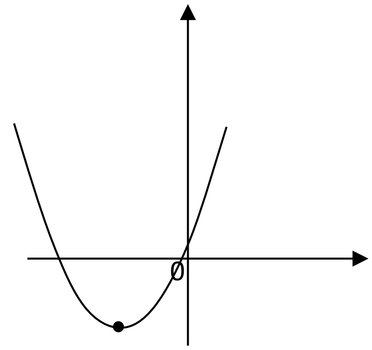

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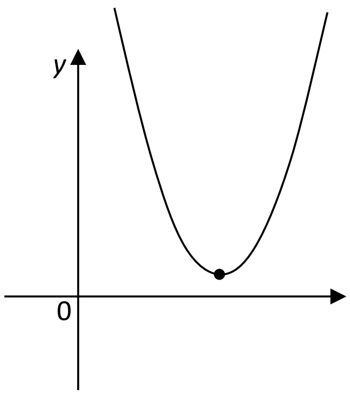
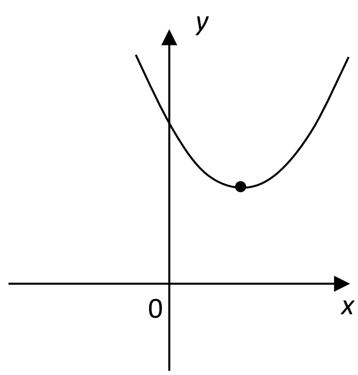


(k)



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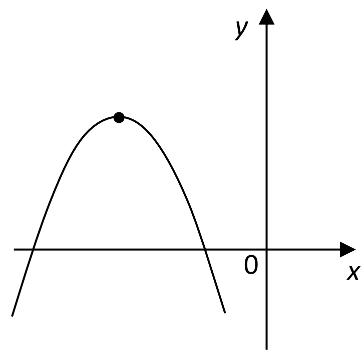

4. Find the value of a so that these quadratic equations have equal roots.

a)	$x^2 - 4x - a = 0$	**(b)**	$2x^2 + 10x + a = 0$	**(c)**	$ax^2 - 2x + 5 = 0$
d)	$ax^2 + (4a - 3)x + a = 0$	**(e)**	$3x^2 + 8x + a = 0$	**(f)**	$ax^2 - 7x - 5 = 0$

5. Find the value of k so that these quadratic equations have equal roots.

a)	$kx^2 - 8x + 4 = 0$	**(b)**	$kx^2 + 6x + 18 = 0$	**(c)**	$x^2 - 2kx + 5 = 0$
d)	$x^2 - 6x + k = 0$	**(e)**	$kx^2 + 5x + 10 = 0$	**(f)**	$x^2 - 3kx + 36 = 0$

Exercise 2

1. The following words can be used to describe the roots of a quadratic.

I Real II Equal III Distinct IV Non-real

Which of the above words can be used to describe the roots of the equation

$$2x^2 + 3x - 4 = 0?$$

2. **a)** Find the value of the discriminant for the quadratic equation

$$x^2 - 5x + 3 = 0$$

b) Use the discriminant to state the nature of the roots in part (a).

3. For what values of p does the equation $x^2 - 2x + p = 0$ have equal roots?

4. The roots of a quadratic equation can be described as:

I Real II Equal III Distinct IV Non-real

Which of the above can be used to describe the roots of the equation $3x^2 - 4x + 5 = 0$?

5. **(a)** For the quadratic equation $x^2 - 5x + 3 = 0$, find the value of the discriminant.

(b) Use the words from question 4 to describe the nature of the roots of the equation.

Answers

Exercise 1

1.	**a)**	4	**(b)**	0	**(c)**	36	**(d)**	49	**(e)**	9	**(f)**	0
	g)	1	**(h)**	–23	**(i)**	196	**(j)**	169	**(k)**	28	**(l)**	–80
	m)	17	**(n)**	–55	**(o)**	16	**(p)**	0	**(q)**	–11	**(r)**	0

2.	**a)**	real, distinct	**(b)**	equal	**(c)**	real, distinct
	d)	real, distinct	**(e)**	real, distinct	**(f)**	equal
	g)	real, distinct	**(h)**	non real	**(i)**	real, distinct
	j)	real, distinct	**(k)**	real, distinct	**(l)**	non real
	m)	real, distinct	**(n)**	non real	**(o)**	real, distinct
	p)	equal	**(q)**	non real	**(r)**	equal

3.	**a)**	$b^2 - 4ac > 0$	**(b)**	$b^2 - 4ac < 0$	**(c)**	$b^2 - 4ac = 0$	**(d)**	$b^2 - 4ac > 0$
	e)	$b^2 - 4ac = 0$	**(f)**	$b^2 - 4ac < 0$	**(g)**	$b^2 - 4ac > 0$	**(h)**	$b^2 - 4ac > 0$
	i)	$b^2 - 4ac > 0$	**(j)**	$b^2 - 4ac < 0$	**(k)**	$b^2 - 4ac < 0$	**(l)**	$b^2 - 4ac > 0$

4.	**a)**	–4	**(b)**	12·5	**(c)**	0·2
	d)	0·5 or 1·5	**(e)**	$5\frac{1}{3}$	**(f)**	$-2\frac{9}{20}$
5.	**a)**	4	**(b)**	$\frac{1}{2}$	**(c)**	$\pm\sqrt{5}$
	d)	9	**(e)**	$\frac{5}{8}$	**(f)**	± 4

Exercise 2

1. 41, real and distinct

2. **a)** 13 **(b)** real and distinct

3. 1

4. Non real

5. 13, real and distinct