# N5 Changing the Subject

Relationships

SPTA Mathematics – Topic Questions with Notes

Changing the subject of a formula was introduced at National 4 and is just like rearranging an equation: you move things from one side to the other and 'do the opposite'.

A useful (but optional) tip for changing the subject questions is to switch the left-hand side and the right-hand side before you begin moving things, so that the letter that will be the subject is already on the left-hand side. However this does not work on every occasion (see example 4).

It is important to move things over in the correct order. Because we are rearranging a formula (effectively going backwards), the order that we deal with terms in is the exact opposite of BODMAS – i.e. the first thing we deal with is terms that are added or subtracted; then we deal with multiplying and division; then squares and square roots, and then lastly split up any brackets.

Example 1

**Change the subject of the formula $y = ab + d$ to '$b$'**

**Solution**

Step One – flip the left and right hand sides: $ab + d = y$

Step Two – rearrange; dealing with the adding/subtracting first, then multiplying dividing

$$ab + d = y$$

$$ab = y - d \qquad (+d \text{ moves over and becomes } -d)$$

$$b = \frac{y - d}{a} \qquad (\times a \text{ moves over and becomes } \div a)$$

Example 2

**Change the subject of the formula $A = \pi d^2$ to '$d$'**

**Solution**

Step One – flip the left and right hand sides: $\pi d^2 = A$

Step Two – rearrange; dealing with the multiplying/dividing first, then the square/square root

$$\pi d^2 = A$$

$$d^2 = \frac{A}{\pi} \qquad (\times \pi \text{ moves over and becomes } \div \pi)$$

$$d = \pm\sqrt{\frac{A}{\pi}} \qquad \text{(squaring moves over and becomes square root)}$$

Just because a formula has a square (or square root) in it does not mean that the final answer will have to have a square root (or square) in it. It depends on which letter we are making the subject. If that letter is being squared, then we have to use a square root. If another letter is being squared, then we do not.

Example 3

**Change the subject of the formula $P = qt^2 + d$ to '$q$'**

**Solution**

Step One – flip the left and right hand sides: $qt^2 + d = P$

Step Two – rearrange; dealing with the adding/taking away, then the multiplying/dividing. There is no need to 'deal with' the square, as $q$ is not being squared

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$$qt^2 + d = P$$

$$qt^2 = P - d \qquad (+d \text{ moves over and becomes } -d)$$

$$q = \frac{P - d}{t^2} \qquad (\times t^2 \text{ moves over and becomes } \div t^2)$$

There are some occasions in which it does not help to flip the left and right-hand sides. This usually involves formulae where the letter that is to become the subject is on the denominator of the right-hand side.

Example 4

**Change the subject of the formula $D = \frac{V}{T}$ to '$T$'**

**Solution**

On this occasion, step one is not required. This is because $T$ will move naturally over to the left-hand side when we begin rearranging.

$$D = \frac{V}{T}$$

$$DT = V \qquad (\div T \text{ moves over and becomes } \times T)$$

$$T = \frac{V}{D} \qquad (\times D \text{ moves over and becomes } \div D)$$

***Exercise 1***

**1.** Change the subject of each formula to $x$.

| | | | | | |
|---|---|---|---|---|---|
| **a)** | $y = x + 3$ | **(b)** | $y = x - 5$ | **(c)** | $y = x + a$ |
| **d)** | $y = x - b$ | **(e)** | $y = 3x$ | **(f)** | $y = 10x$ |
| **g)** | $y = kx$ | **(h)** | $y = ax$ | **(i)** | $y = 3p + x$ |
| **j)** | $y = x - 5t$ | **(k)** | $y = 2x + 1$ | **(l)** | $y = 3x - 7$ |
| **m)** | $y = 7x + 4a$ | **(n)** | $y = 3b + 4x$ | **(o)** | $y = 8 + 10x$ |

**2.** Make $a$ the subject of each formula.

| | | | | | |
|---|---|---|---|---|---|
| **a)** | $b = 4 - a$ | **(b)** | $d = 12 - a$ | **(c)** | $y = 5x - a$ |
| **d)** | $m = 2 - 2a$ | **(e)** | $q = 7 - 5a$ | **(f)** | $c = 20 - 3a$ |
| **g)** | $r = s - 2a$ | **(h)** | $t = d - 4a$ | **(i)** | $z = 4b - 5a$ |
| **j)** | $k = 2h - 7a$ | **(k)** | $p = 6q - 11a$ | **(l)** | $g = 2x - 9a$ |

**3.** Make $x$ the subject of each formula below.

| | | | | | |
|---|---|---|---|---|---|
| **a)** | $y = ax + b$ | **(b)** | $y = mx + c$ | **(c)** | $t = sx - r$ |
| **d)** | $p = qx + 2r$ | **(e)** | $m = fx - 3n$ | **(f)** | $a = b + cx$ |
| **g)** | $k = h - mx$ | **(h)** | $d = 3b + cx$ | **(i)** | $g = kc - hx$ |

**4.** Change the subject of each formula to the letter shown in brackets.

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **a)** | $P = 4l$ | $(l)$ | **(b)** | $V = IR$ | $(I)$ | **(c)** | $S = DT$ | $(T)$ |
| **d)** | $A = lb$ | $(b)$ | **(e)** | $C = \pi d$ | $(d)$ | **(f)** | $G = UT$ | $(U)$ |
| **g)** | $v = u + at$ | $(t)$ | **(h)** | $P = 2l + 2b$ | $(l)$ | **(i)** | $H = xy + 5m$ | $(y)$ |

**5**. Change the subject of each formula to $c$.

| | | | | | |
|---|---|---|---|---|---|
| **a)** | $b = {}^1\!/_2\, c$ | **(b)** | $x = {}^1\!/_5\, c$ | **(c)** | $y = {}^1\!/_4\, c$ |
| **d)** | $m = {}^1\!/_6\, c$ | **(e)** | $k = {}^1\!/_9\, c$ | **(f)** | $d = {}^1\!/_{10}\, c$ |
| **g)** | $a = {}^1\!/_2\, c + 2$ | **(h)** | $h = {}^1\!/_3\, c - 5$ | **(i)** | $p = {}^1\!/_4\, c + q$ |
| **j)** | $y = {}^1\!/_{10}\, c - x$ | **(k)** | $t = {}^1\!/_8\, c + 2s$ | **(l)** | $r = {}^1\!/_5\, c - 3q$ |

**6.** Change the subject of each formula to $x$.

| | | | | | |
|---|---|---|---|---|---|
| **a)** | $y = \frac{3}{x}$ | **(b)** | $d = \frac{c}{x}$ | **(c)** | $m = \frac{y}{x}$ |
| **d)** | $s = \frac{a+2}{x}$ | **(e)** | $w = \frac{z-1}{x}$ | **(f)** | $a = \frac{b+c}{x}$ |
| **g)** | $a = \frac{x+8}{9}$ | **(h)** | $k = \frac{x-5}{2}$ | **(i)** | $p = \frac{3-x}{4}$ |
| **j)** | $y = \frac{2}{x} + 1$ | **(k)** | $z = \frac{6}{x} - 7$ | **(l)** | $h = \frac{m}{x} + k$ |

**7**. Change the subject of each formula to $k$.

**a)** $y = \sqrt{k}$ **(b)** $x = \sqrt{k}$ **(c)** $m = \sqrt{k}$

**d)** $a = \sqrt{\frac{k}{b}}$ **(e)** $c = \sqrt{\frac{k}{d}}$ **(f)** $h = \sqrt{\frac{k}{g}}$

**g)** $s = \sqrt{\frac{t}{k}}$ **(h)** $q = \sqrt{\frac{p}{k}}$ **(i)** $w = \sqrt{\frac{z}{k}}$

**j)** $r = k^2$ **(k)** $ab = k^2$ **(l)** $\frac{p}{q} = k^2$

**m)** $y = x + k^2$ **(n)** $c = k^2 - d$ **(o)** $x = 3k^2 - 1$

**8.** Change the subject of each formula to the letter shown in brackets.

**a)** $v^2 = u^2 + 2as$ $(s)$ **(b)** $v^2 = u^2 + 2as$ $(u)$

**c)** $V = \pi r^2 h$ $(h)$ **(d)** $V = \pi r^2 h$ $(r)$

**e)** $r = \sqrt{\frac{A}{\pi}}$ $(A)$ **(f)** $L = 3 + \sqrt{6a}$ $(a)$

**g)** $2k = \sqrt{(p+4)}$ $(p)$ **(h)** $x^2 = \frac{4yz}{t}$ $(y)$

**i)** $ar = \frac{1}{2}\sqrt{\frac{x}{b}}$ $(b)$ **(j)** $st = A^2(x - 3y)$ $(A)$

**k)** $R = A^2(x - 3y)$ $(x)$ **(l)** $na = \sqrt{(1 - n^2)}$ $(n)$

**m)** $d = \frac{t(n-1)}{n}$ $(n)$ **(n)** $\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2}$ $(R)$

**o**) $d = \frac{a^2(x+b)}{4}$ $(a)$

***Exercise 2***

**1.** Change the subject of the formula to $c$.

$$ab = \frac{1}{2}\sqrt{\frac{x}{c^2}}$$

**2.** The formula for the velocity that a body must have to escape the gravitational pull of Earth is

$$V = \sqrt{2gR}$$

Change the subject of the formula to $g$.

**3.** For the formula given below, change the subject to $x$

$$A^2 = \sqrt{x} + 5$$

**4.** The formula for kinetic energy is

$$E = \tfrac{1}{2}mv^2$$

Change the subject of the formula to $v$.

**5.** Change the subject of the formula to $a$:

$$V = 3a^2b$$

**6.** Change the subject of the formula to $k$.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

**7.** A formula to convert temperature from degrees Celsius to degrees Farenheit is

$$F = \frac{9}{5}C + 32$$

Change the subject of the formula to C.

**8.** The formula for finding the volume of a cone is given by

$$V = \frac{1}{3}\pi r^2 h$$

Change the subject of the formula to $r$.

**9.** Change the subject of the formula $m = \frac{k^2 + 2}{ab}$ to $k$.

**10.** Change the subject of the formula $a = b^2 c + 7$ to $b$.

**11.** Change the subject of this formula to $m$ $k = \frac{mn}{3}$

**12.** The formula for finding the volume of a sphere is given by

$$V = \frac{4}{3}\pi r^3$$

Change the subject of the formula to $r$.

***Answers***

***Exercise 1***

**1.** **a)** $x = y - 3$ **(b)** $x = y + 5$ **(c)** $x = y - a$ **(d)** $x = y + b$

**e)** $x = \frac{y}{3}$ **(f)** $x = \frac{y}{10}$ **(g)** $x = \frac{y}{k}$ **(h)** $x = \frac{y}{a}$

**i)** $x = y - 3p$ **(j)** $x = y + 5t$ **(k)** $x = \frac{y-1}{2}$ **(l)** $x = \frac{y+7}{3}$

**m)** $x = \frac{y-4a}{7}$ **(n)** $x = \frac{y-38}{4}$ **(o)** $x = \frac{y-8}{10}$

**2.** **a)** $a = 4 - b$ **(b)** $a = 12 - d$ **(c)** $a = 5x - y$ **(d)** $a = \frac{2-m}{2}$

**e)** $a = \frac{7-q}{5}$ **(f)** $a = \frac{20-c}{3}$ **(g)** $a = \frac{s-r}{2}$ **(h)** $a = \frac{d-t}{4}$

**i)** $a = \frac{4b-z}{5}$ **(j)** $a = \frac{2h-k}{7}$ **(k)** $a = \frac{6p-q}{11}$ **(l)** $a = \frac{2x-g}{9}$

**3.** **a)** $x = \frac{y-b}{a}$ **(b)** $x = \frac{y-c}{m}$ **(c)** $x = \frac{t+r}{s}$ **(d)** $x = \frac{p-2r}{q}$

**e)** $x = \frac{m+3n}{f}$ **(f)** $x = \frac{a-b}{c}$ **(g)** $x = \frac{h-k}{m}$ **(h)** $x = \frac{d-3b}{c}$

**i)** $x = \frac{kc-g}{h}$

**4.** **a)** $l = \frac{P}{4}$ **(b)** $I = \frac{V}{R}$ **(c)** $T = \frac{S}{D}$ **(d)** $b = \frac{A}{l}$

**e)** $d = \frac{C}{\pi}$ **(f)** $U = \frac{G}{T}$ **(g)** $t = \frac{v-u}{a}$ **(h)** $l = \frac{P-2b}{2}$

**i)** $y = \frac{H-5m}{x}$

**5.** **a)** $c = 2b$ **(b)** $c = 5x$ **(c)** $c = 4y$ **(d)** $c = 6m$

**e)** $c = 9k$ **(f)** $c = 10d$ **(g)** $c = 2a - 4$ **(h)** $c = 3h + 15$

**i)** $c = 4p - 4q$ **(j)** $c = 10y + 10x$ **(k)** $c = 8t - 16s$ **(l)** $c = 5r + 15q$

**6.** **a)** $x = \frac{3}{y}$ **(b)** $x = \frac{c}{d}$ **(c)** $x = \frac{y}{m}$ **(d)** $x = \frac{a+2}{s}$

**e)** $x = \frac{z-1}{w}$ **(f)** $x = \frac{b+c}{a}$ **(g)** $x = 9a - 8$ **(h)** $x = 2k + 5$

**i)** $x = 3 - 4p$ **(j)** $x = \frac{2}{y-1}$ **(k)** $x = \frac{6}{z-7}$ **(l)** $x = \frac{m}{h-k}$

**7.** **a)** $k = y^2$ **(b)** $k = x^2$ **(c)** $k = m^2$ **(d)** $k = a^2 b$

**e)** $k = c^2 d$ **(f)** $k = gh^2$ **(g)** $k = \frac{t}{s^2}$ **(h)** $k = \frac{p}{q^2}$

**i)** $k = \frac{z}{w^2}$ **(j)** $k = \sqrt{r}$ **(k)** $k = \sqrt{ab}$ **(l)** $k = \sqrt{\frac{p}{q}}$

**m)** $k = \sqrt{y - x}$ **(n)** $k = \sqrt{c + d}$ **(o)** $k = \sqrt{\frac{x+1}{3}}$

**8.** **a)** $s = \frac{v^2 - u^2}{2a}$ **(b)** $u = \sqrt{v^2 - 2as}$ **(c)** $h = \frac{V}{\pi r^2}$ **(d)** $r = \sqrt{\frac{V}{\pi h}}$

**e)** $A = \pi r^2$ **(f)** $a = \frac{(L-3)^2}{6}$ **(g)** $p = 4k^2 - 4$ **(h)** $y = \frac{tx^2}{4z}$

**i)** $b = \frac{x}{4a^2 r^2}$ **(j)** $A = \sqrt{\frac{st}{x - 3y}}$ **(k)** $x = \frac{R + 3A^2 y}{A^2}$ **(l)** $n = \sqrt{\frac{1}{a^2 + 1}}$

**m)** $n = \frac{t}{t-d}$ **(n)** $R = \frac{r_1 r_2}{r_2 + r_1}$ **(o)** $a = \sqrt{\frac{4d}{x+b}}$

***Exercise 2***

**1.** $\sqrt{\dfrac{x}{4a^2b^2}}$ or $\dfrac{\sqrt{x}}{2ab}$

**2.** $g = \dfrac{V^2}{2R}$

**3.** $x = (A^2 - 5)^2$

**4.** $v = \sqrt{\dfrac{2E}{m}}$

**5.** $a = \sqrt{\dfrac{v}{3b}}$

**6.** $k = \dfrac{4\pi^2 m}{T^2}$

**7.** $C = \dfrac{5}{9}(F - 32)$

**8.** $r = \sqrt{\dfrac{3V}{\pi h}}$

**9.** $k = \sqrt{mab - 2}$

**10.** $b = \sqrt{\dfrac{a-7}{c}}$

**11.** $m = \dfrac{3k}{n}$

**12.** $r = \sqrt[3]{\dfrac{3V}{4\pi}}$