

Angles

You will be expected to work out angles in diagrams using the properties of angles. :

- Two angles on a straight line add up to make 180° .
- Opposite angles in **X-shapes** are equal.
- The three angles in a **triangle** add up to make 180° .
 - And in particular, **Isosceles triangles** are symmetrical, meaning that the two angles in the base are the same size.
- The four angles in a **quadrilateral** add up to make 360° .
- Angles in **Z shapes** (made by parallel lines) are the same.
- A **tangent line** to a circle meets a radius at right-angles.
- The **angle in a semicircle** is a right angle.

Questions about angles in the exam require you to identify the right-angles and isosceles triangles, and then using the rules of angles to find any remaining angles.

You can only write a right angle in a diagram if you *know* it is a right angle. There are some occasions when you can *know* an angle is a right-angle:

1. If you are told that a shape is a **square** or **rectangle**, you know the angles in its corners are right angles.
2. A **tangent** always makes a right angle with the radius.
3. Triangles in **semi-circles** are always right angled.
4. If a line in a circle diagram is a line of symmetry, it will cross any other line at right-angles.

Essential Exam Tips

- Always copy (or trace) the diagram onto your answer sheet and mark in all angles clearly.
- Make sure you make it clear which angle is your final answer – writing it on the diagram isn't enough unless you indicate clearly which angle is the one we are looking for.

Two examples are given below – however every question is different. The only way to get used to them is to practice them from past exam papers and textbooks.

Example 1 – angles and circles

The diagram shows a circle centre O . DE is a tangent to the circle at point C . Angle OAB is 35° .

Calculate the size of angle BCE

Solution

Using the diagram on your exam paper, mark each angle in turn.

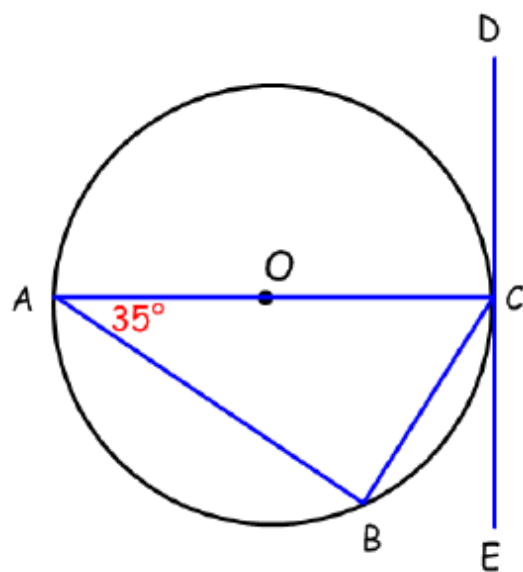
A tangent and a radius meet at right angles, so ACD and ACE are 90° .

The angle in a semi-circle is a right angle, so ABC is also 90° .

We know two of the angles in triangle ABC . Since we know the angles in a triangle must add to make 180° , angle ACB must be 55° .

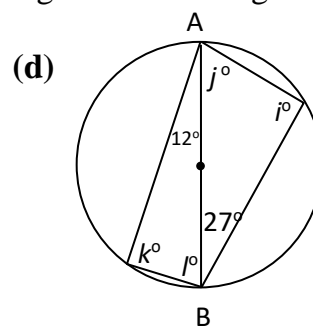
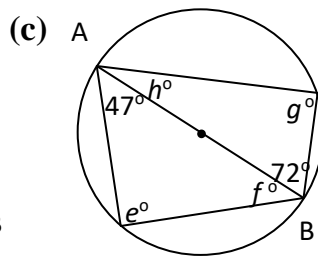
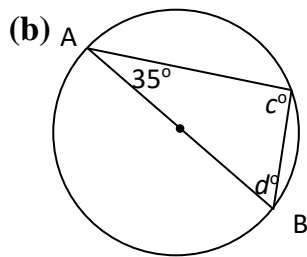
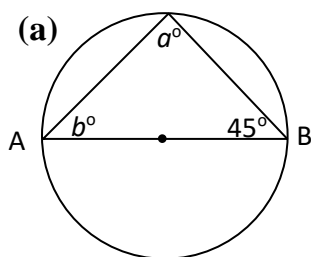
Finally since we already knew ACE is 90° , this tells us that ACB and BCE must add to make 90° . Therefore angle BCE is 35° .

Final answer: clearly state that angle BCE is 35° (just marking it on the diagram isn't enough as it doesn't make it clear that you know which angle is angle BCE).

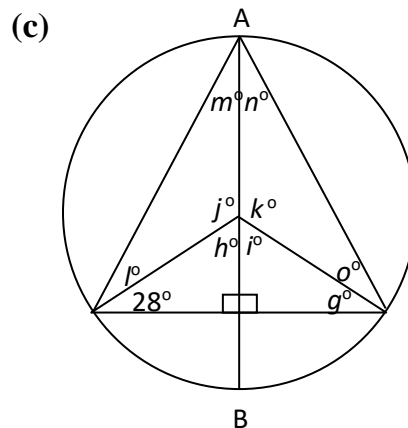
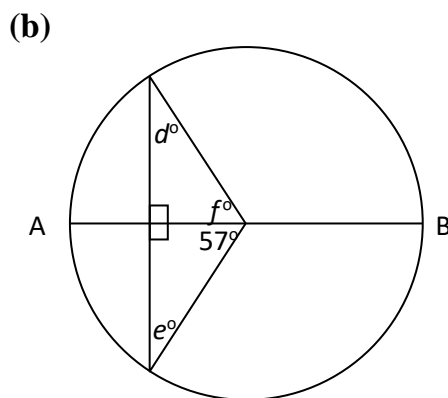
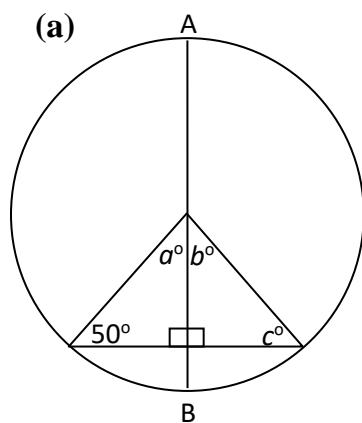


Exercise 1

1. In each of the diagrams below AB is a diameter. Find the missing angles in each diagram.

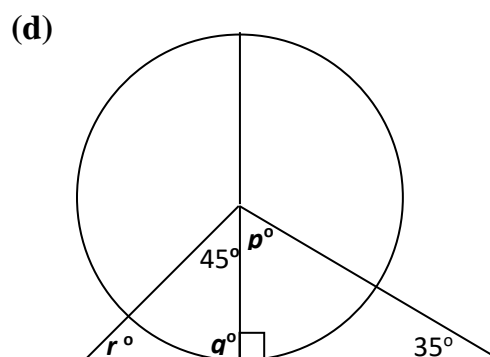
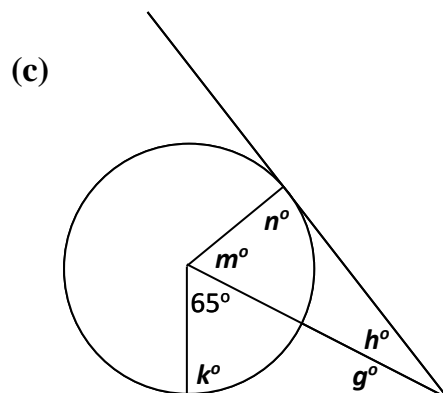
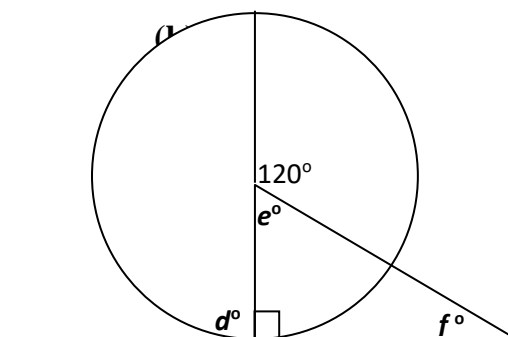
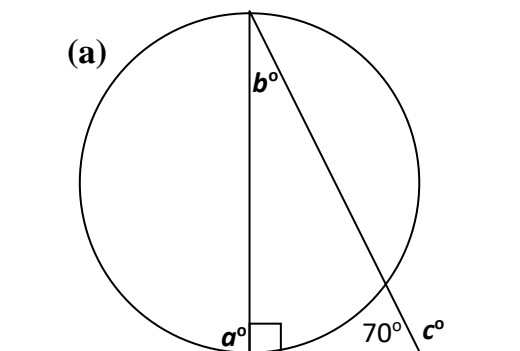


2. Use the symmetry properties of the circle to find the missing angles in the diagrams below. In each diagram AB is a diameter.



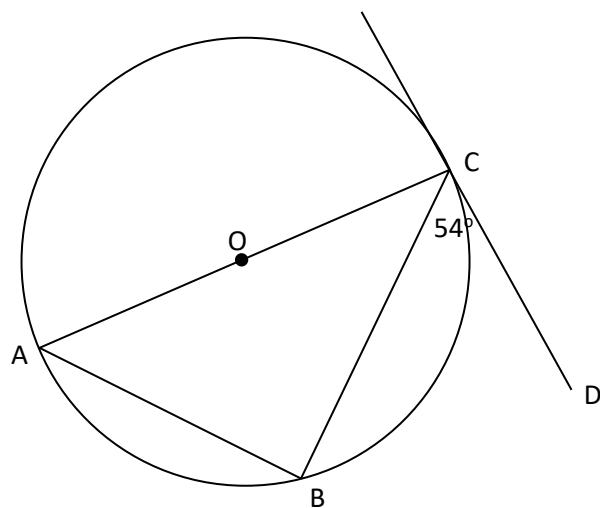
Exercise 2

1. Calculate the sizes of the angles marked a , b , \dots , r , in the diagrams below.



Exercise 3

- 1.

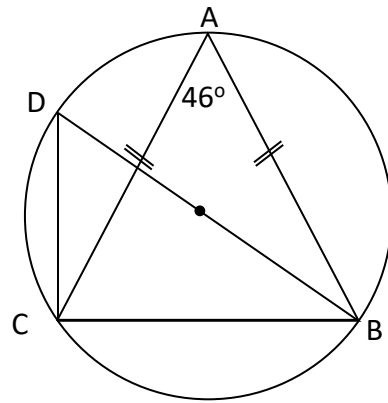


AC is a diameter and O is the centre of the circle shown opposite. CD is a tangent to the circle with C the point of contact.

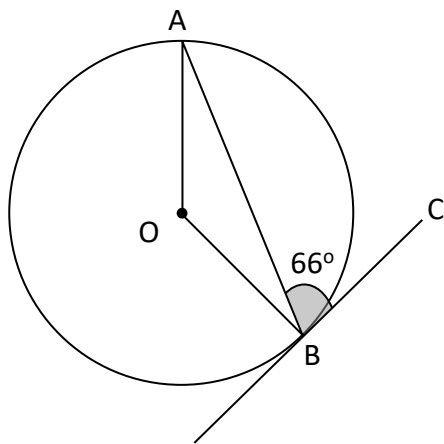
If $\angle BCD = 54^\circ$, find the size of $\angle CAB$.

2. In the diagram triangle ABC is isosceles and BD is a diameter of the circle.

Calculate the size of angle ACD.



3.

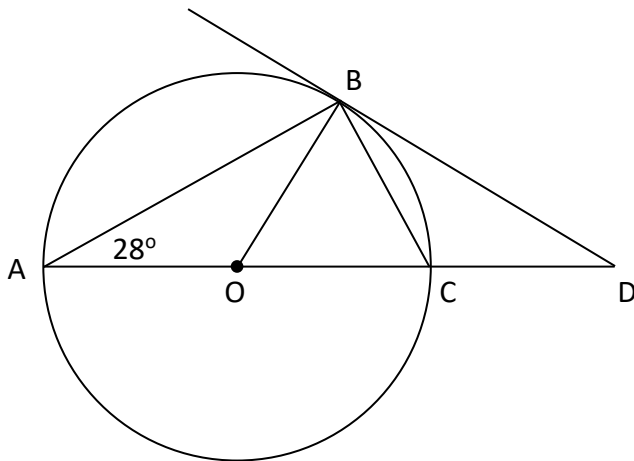


A and B are points on the circumference of a circle centre O. BC is a tangent to the circle.

Angle $ABC = 66^\circ$.

Calculate the size of angle AOB.

4.



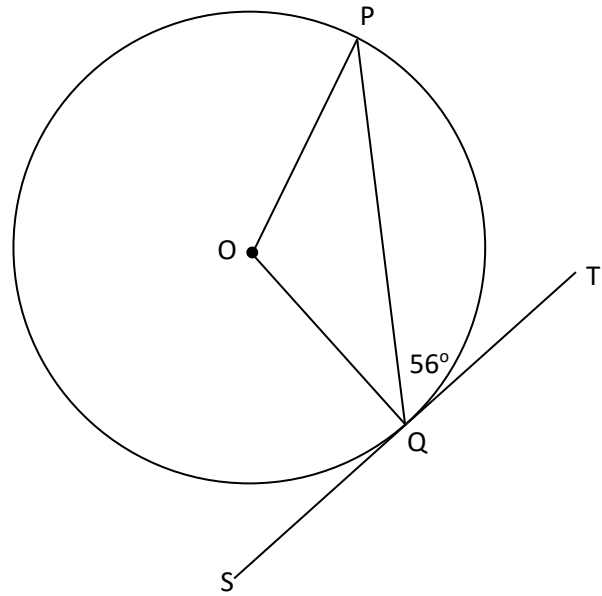
In the diagram shown, BD is a tangent to the circle centre O.

Angle $BAC = 28^\circ$.

Calculate the size of angle CBD.

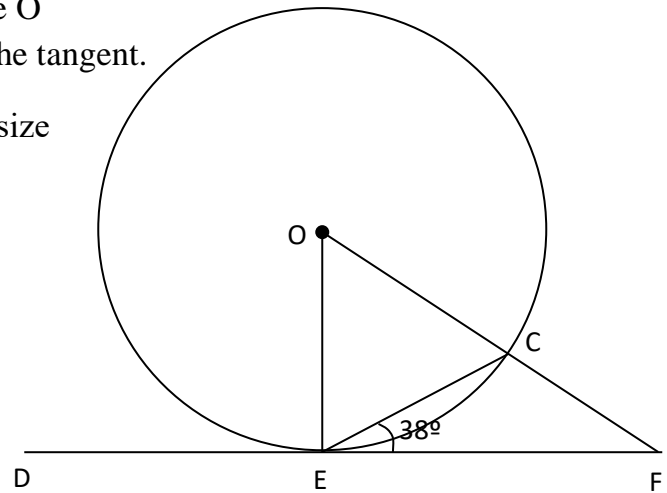
5. The diagram shows a circle with centre O. ST is a tangent to the circle with point of contact Q. $\angle PQT = 56^\circ$.

Calculate the size of $\angle POQ$.



6. The line DF is a tangent to the circle centre O shown below. E is the point of contact of the tangent.

Given that angle CEF is 38° , calculate the size of angle EOC.



Answers

Exercise 1

- | | | | | | | | | | | |
|-----------|------------|------|------------|-----|------------|-----|------------|-----|------------|------|
| 1. | (a) | 90° | (b) | 45° | (c) | 90° | (d) | 55° | (e) | 90° |
| | (f) | 43° | (g) | 90° | (h) | 18° | (i) | 90° | (j) | 63° |
| | (k) | 90° | (l) | 78° | | | | | | |
| 2. | (a) | 40° | (b) | 40° | (c) | 50° | (d) | 33° | (e) | 33° |
| | (f) | 57° | (g) | 28° | (h) | 62° | (i) | 62° | (j) | 118° |
| | (k) | 118° | (l) | 31° | (m) | 31° | (n) | 31° | (o) | 31° |

Exercise 2

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|-----------|------------|-----|------------|-----|------------|------|------------|-----|------------|-----|
| 1. | (a) | 90° | (b) | 20° | (c) | 110° | (d) | 90° | (e) | 60° |
| | (f) | 30° | (g) | 35° | (h) | 35° | (k) | 90° | (m) | 65° |
| | (n) | 90° | (p) | 55° | (q) | 90° | (r) | 45° | | |

Exercise 3

- | | | | | | |
|-----------|-----|-----------|------|-----------|------|
| 1. | 54° | 2. | 23° | 3. | 132° |
| 4. | 28° | 5. | 112° | 6. | 76° |