# N5 Volume of a Solid

Expressions & Formulae

SPTA Mathematics – Topic Questions with Notes

## Volumes of Solids

You should know from National 4 how to calculate the volume of a **prism**. At National 5 level, you also need to be able to calculate the volume of a **pyramid**. Throughout this topic remember that:

- all volume questions must have answered in cubic units (e.g. $m^3$, $cm^3$, $inches^3$)
- you should always state your unrounded answer before rounding (see page 6)

**Formula.** This formula <u>is **not** given</u> on the National 5 Mathematics exam paper.

**Volume of a Prism:**

$$V = Ah$$

$$\text{Volume} = \text{Area of cross section} \times \text{Height}$$

**Formula.** This formula is <u>given</u> on the National 5 Mathematics exam paper.

**Volume of a Pyramid:**

$$V = \frac{1}{3}Ah$$

$$\text{Volume} = \frac{1}{3}\text{Area of Base} \times \text{Height}$$

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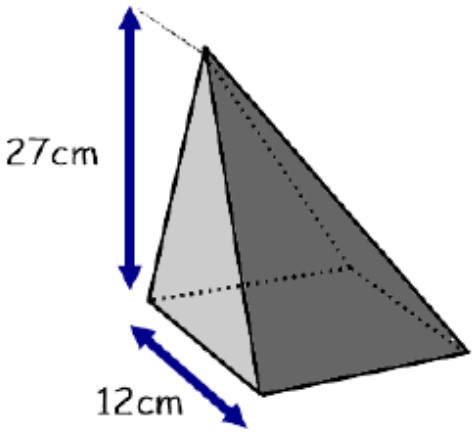
<u>Example 1 – Pyramid</u>

**The diagram shows a pyramid with height 27cm and a square base with sides of length 12cm. Calculate the volume of the pyramid.**




**Solution**

The area of a square is given by the formula $A = L^2$, so the area of the base of this pyramid is $12^2 = 144cm^2$

Therefore the volume of the whole pyramid is

$$V = \frac{1}{3}Ah$$

$$= 144 \times 27 \div 3$$

$$= \underline{\underline{1296cm^3}}$$

Special cases of prisms and pyramids are when the cross-sectional area of the prism is a circle (in which case you have a cylinder) or when the base of a pyramid is a circle (giving a cone). In these cases, we can adapt the earlier formulae to give us a quicker formula:

**Formula.** This formula is **not** given on the National 5 Mathematics exam paper.

**Volume of a Cylinder:** $V = \pi r^2 h$

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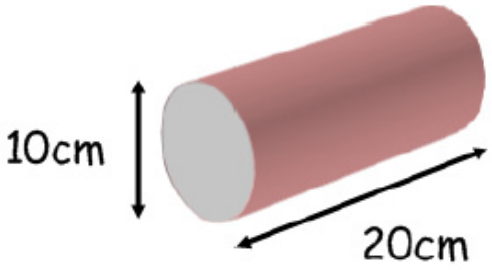
Example 2 – cylinder

**Work out the volume of this cylinder. Round your answer to 2 significant figures.**




**Solution**

Diameter is 10cm so radius is 5cm

$V = \pi r^2 h$

$= \pi \times 5^2 \times 20 \quad (\text{or } \pi \times 5 \times 5 \times 20)$

$= 1570 \cdot 796327....$

$= \underline{1600\text{cm}^3 \text{ (2 s.f.)}}$

**Formula.** This formula is given on the National 5 Mathematics exam paper.

**Volume of a Cone:** $V = \frac{1}{3}\pi r^2 h$

In the formula for the volume of a cone, the 'height' $h$ refers to the **perpendicular** height (the one that goes straight up) and not any sloping heights.

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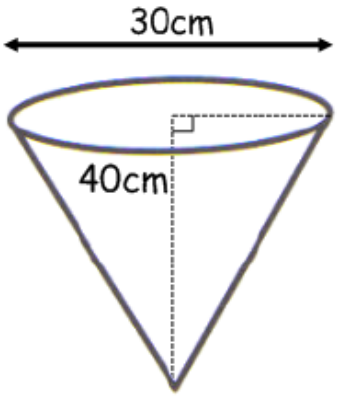
Example 3 – cone

**Calculate the volume of this cone. Round your answer to 3 significant figures.**




**Solution**

Diameter is 30cm so radius is 15cm

$V = \frac{1}{3}\pi r^2 h$

$= \pi \times 15^2 \times 40 \div 3 \quad (\text{or } 1 \div 3 \times \pi \times 15^2 \times 40)$

$= 9424 \cdot 777961....$

$= \underline{9420\text{cm}^3} \text{ (3 s.f.)}$

You are also expected to know how to calculate the volume of a sphere

**Formula.** This formula is given on the National 5 Mathematics exam paper.

**Volume of a Sphere:** $V = \frac{4}{3}\pi r^3$

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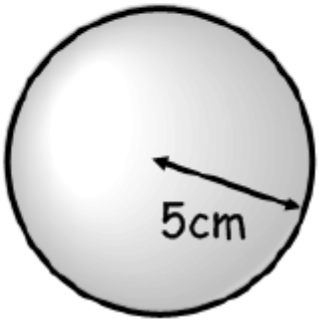
Example 4 – sphere

**Calculate the volume of this sphere. Round your answer to 1 significant figure.**




**Solution**

Radius is 5cm

$$V = \frac{4}{3}\pi r^3$$
$$= \pi \times 5^3 \div 3 \times 4 \quad (\text{or } 4 \div 3 \times \pi \times 5 \times 5 \times 5)$$
$$= 523 \cdot 5987756....$$
$$= \underline{500\text{cm}^3 \text{ (1 s.f.)}}$$

**Definition:** A **hemisphere** is half of a sphere.

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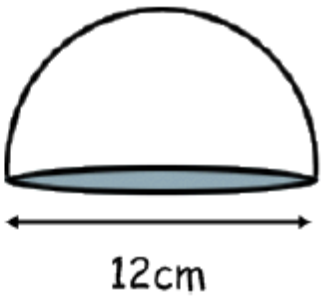
Example 5 – hemisphere

**Calculate the volume of this hemisphere. Round your answer to 4 significant figures.**




**Solution**

Diameter is 12cm so radius is 6cm

$$V = \frac{4}{3}\pi r^3 \div 2$$
$$V = \pi \times 6^3 \div 3 \times 4 \div 2$$
$$V = 452 \cdot 3893421....$$
$$V = \underline{452 \cdot 4\text{cm}^3 \text{ (4 s.f.)}}$$

# Composite Shapes

In the exam, you may be expected to deal with a shape formed from more than one other shape joined together. If the diagram is confusing you and you are not sure what the shape in the question is, then read the question carefully.

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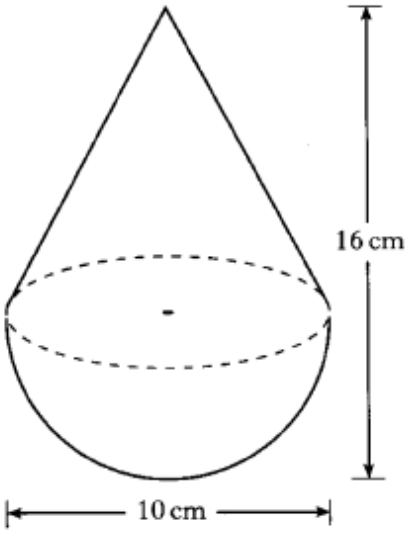
Example (2006 Intermediate 2 Paper 2)

**A child's toy is in the shape of a hemisphere with a cone on top, as shown in the diagram. The toy is 10 centimetres wide and 16 centimetres high. Calculate the volume of the toy. Give your answer correct to 2 significant figures.**




**Solution**

The cone and the hemisphere have the same radius, 5cm.

The 16cm line in the picture is made of the height of the cone plus the radius of the sphere.
Height of cone + 5cm = 16cm
i.e. height of cone is 16 – 5 = 11cm.

Volume of cone:

$$V = \frac{1}{3}\pi r^2 h$$
$$= \pi \times 5^2 \times 11 \div 3$$
$$= 287 \cdot 98...$$

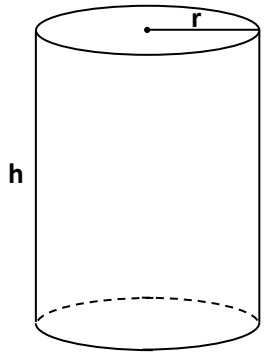
Volume of hemisphere

$$V = \frac{4}{3}\pi r^3 \div 2$$
$$= \pi \times 5^3 \div 3 \times 4 \div 2$$
$$= 261 \cdot 8...$$

Total volume = 287·98 + 261·8 = 549·7… = 550cm³ (2 s.f.)

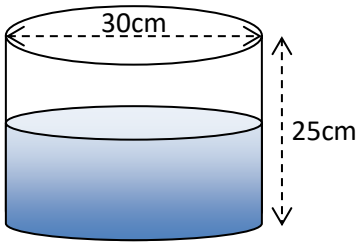
***Exercise 1***

**1**. Find the volume of a circular-based prism for the values of ***r*** and ***h*** given.




| | | |
|---|---|---|
| **(a)** | $r$ = 6 cm | $h$ = 15 cm |
| **(b)** | $r$ = 8 cm | $h$ = 24 cm |
| **(c)** | $r$ = 4 cm | $h$ = 12 cm |
| **(d)** | $r$ = 10 cm | $h$ = 8 cm |
| **(e)** | $r$ = 20 cm | $h$ = 60 cm |
| **(f)** | $r$ = 7 cm | $h$ = 20 cm |
| **(g)** | $r$ = 15 cm | $h$ = 40 cm |
| **(h)** | $r$ = 11 cm | $h$ = 35 cm |
| **(i)** | $r$ = 44 cm | $h$ = 125 cm |
| **(j)** | $r$ = 8.8 cm | $h$ = 30 cm |

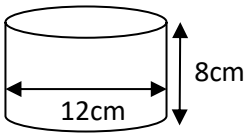
**2.** A milk dispenser is cylindrical in shape with diameter 30cm.




**a)** If 14 litres of milk are poured into it, calculate the depth of the milk in the cylinder.

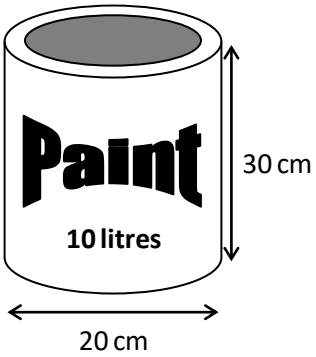
**b)** The height of the cylinder is 25cm.
How many **more** litres of milk are needed to completely fill it?

**3.**




Calculate the volume of a cylinder with diameter 12cm and height 8cm.

**4.** This paint tin has diameter 20 cm and height 30 cm as shown in the diagram.

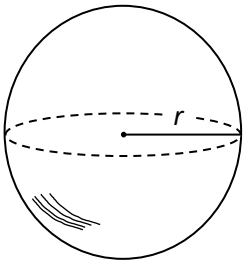



It is claimed that it can hold 10 litres of paint.
Is this claim correct?

**You must show all working and give a reason for your answer.**

***Exercise 2***

**1**. Calculate the volume of each sphere described below, rounding your answer to 1 decimal place.




**(a)** $r$ = 6cm

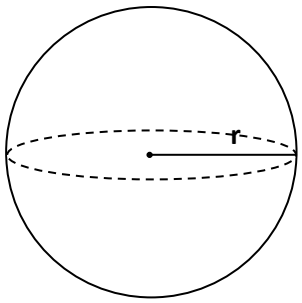
**(b)** $r$ = 2m

**(c)** $r$ = 9mm

**(d)** $r$ = 3cm

**2**. Find the volume of a sphere for the following values of ***r*** and ***d***.

(give your answers correct to 3 significant figures)




**(a)** $r$ = 10cm

**(b)** $r$ = 25cm

**(c)** $d$ = 2m

**(d)** $r$ = 200mm

**(e)** $d$ = 11cm

**(f)** $d$ = 18cm

**(g)** $r$ = 80mm

**(h)** $d$ = 55cm

**(i)** $r$ = 3·5m

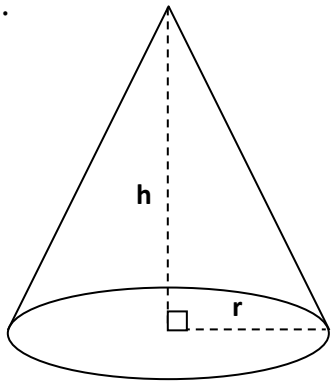
**(j)** $d$ = 48cm

**3.** A sphere has a diameter of 8cm.

Calculate its volume giving your answer correct to 3 significant figures.

**4**. Find the volume of a cone for the following values of ***r*** and ***h***.

(give your answers correct to 3 significant figures)




**(a)** $r$ = 5cm $h$ = 14cm

**(b)** $r$ = 7cm $h$ = 25cm

**(c)** $r$ = 3cm $h$ = 22cm

**(d)** $r$ = 12cm $h$ = 7cm

**5.** Find the volume of a cone for the following values of ***d*** and ***h***.
(give your answers correct to 3 significant figures)

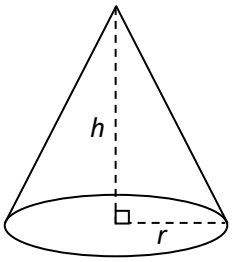
**(a)** $d$ = 15cm $h$ = 40cm

**(b)** $d$ = 11cm $h$ = 37cm

**(c)** $d$ = 22cm $h$ = 125cm

**(d)** $d$ = 8·8cm $h$ = 30cm

**6.** Calculate the volume of each cone described below, rounding your answers to 1 decimal place.




**(a)** $r$ = 3cm and $h$ = 6cm

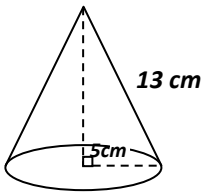
**(b)** $r$ = 8mm and $h$ = 12mm

**(c)** $r$ = 3cm and $h$ = 5cm

**(d)** $r$ = 2m and $h$ = 6m

**7.** A cone has a base diameter of 8cm and a height of 5cm. Calculate the volume of this cone.

**8.** A cone has a base diameter of 10cm and a **slant height** of 13cm.
Calculate the volume of the cone.




**9.** A cone has a base radius of 9cm and a **slant height** of 15cm.
Calculate the volume of the cone.

**10.** A pyramid has a square base of side 4cm and a vertical height of 7cm.
Calculate the volume of the pyramid correct to 2 significant figures.

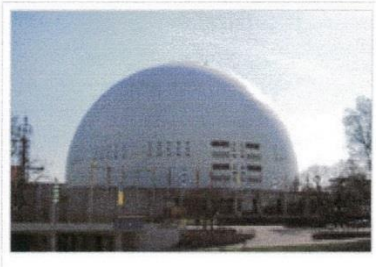
**11.** A pyramid has a rectangular base measuring 16mm by 12mm and a vertical height of 10mm.
Calculate the volume of the pyramid.

***Exercise 3***

**1.** The Stockholm Globe Arena is the largest hemispherical building in the world.

The radius of the building is 110 m.

Calculate the volume of the building in cubic metres, giving your answer in scientific notation correct to 3 significant figures.



**2.** A metal bottle stopper is made up from a cone topped with a sphere.

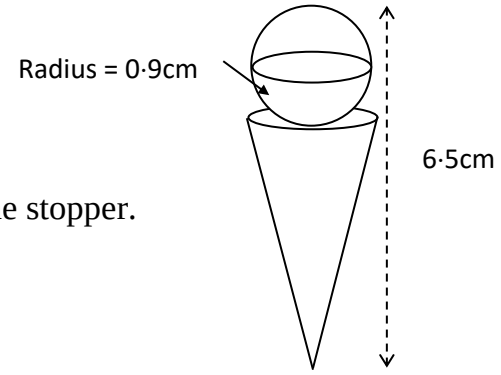
The sphere has diameter 1·5cm.

The cone has radius 0·9cm.

The overall length of the stopper is 6·5cm.

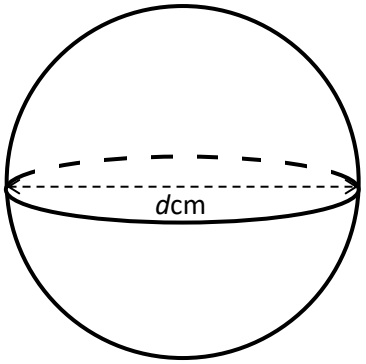
Calculate the volume of metal required to make the stopper.

Give your answer correct to 3 significant figures.




**3.** The volume of this sphere is $524\text{cm}^3$.

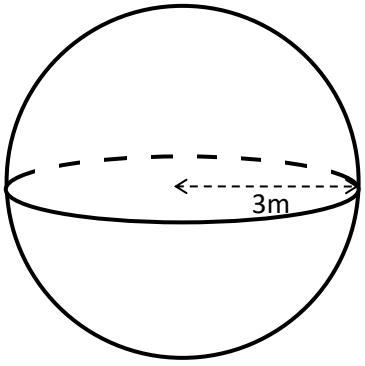
Calculate the diameter, $d$ cm.




**4.** **Non Calculator!**

Calculate the volume of this sphere which has radius 3m.

[Take $\pi = 3{\cdot}14$]



**5.** Sherbet in a sweet shop is stored in a cylindrical container like the one shown in *diagram 1*.

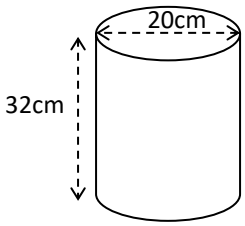



*Diagram 1*

The volume of the cylinder, correct to the nearest $1000cm^3$, is $10\,000\ cm^3$.

The sherbet is sold in conical containers with diameter 5 cm as shown in *diagram 2*.

250 of these cones can be filled from the contents of the cylinder.

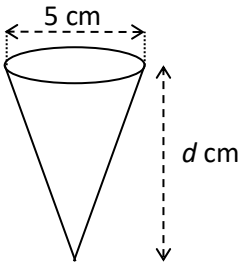


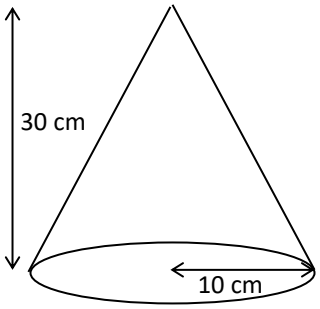

*Diagram 2*

Calculate the depth, *d* cm, of a sherbet cone.

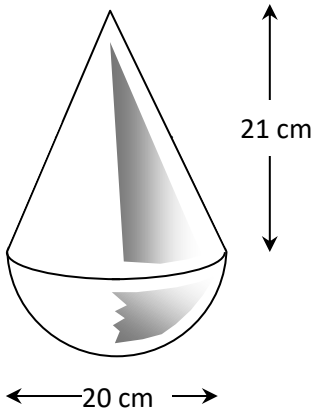
**6.** **Non Calculator!**

The diagram shows a cone with radius 10 centimetres and height 30 centimetres.

Taking $\pi = 3{\cdot}14$, calculate the volume of the cone.




**7**.

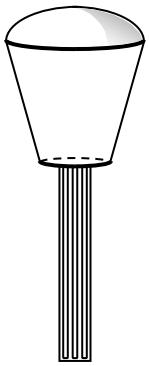



A children's wobbly toy is made from a cone, 21 cm high, on top of a hemispherical base of diameter 20 cm.

The toy has to be filled with liquid foam.

Calculate the volume of foam which will be required.

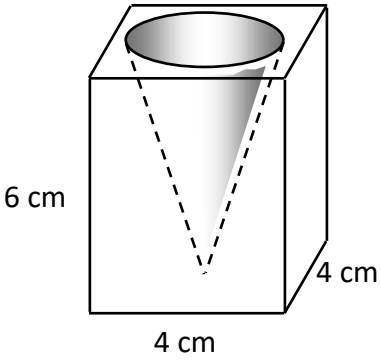
**8**.



The lamp cover in a street lamp is in the shape of a cone with the bottom cut off. The height of the cone is 50cm and its radius is 25cm. The height of the lamp is 30cm and the base of the lamp has a radius of 18cm

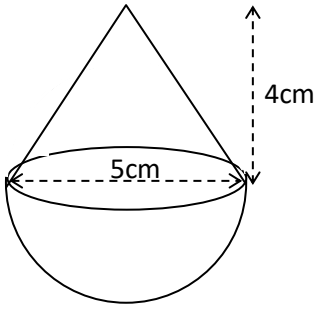
Calculate the volume of the lamp cover. [Answer to 3 significant figures.]

**9**.




A glass candle holder is in the shape of a cuboid with a cone removed. The cuboid measures 4cm by 4cm by 6cm. The cone has a diameter of 3cm and a height of 5cm. Calculate the volume of glass in the candle holder.

**10.** For the Christmas market a confectioner has created a chocolate Santa. It consists of a solid hemisphere topped by a solid cone. Both have diameter 5cm and the height of the cone is 4cm as shown in the diagram.




Calculate the volume of chocolate required to make one chocolate Santa, giving your answer correct to 3 significant figures.

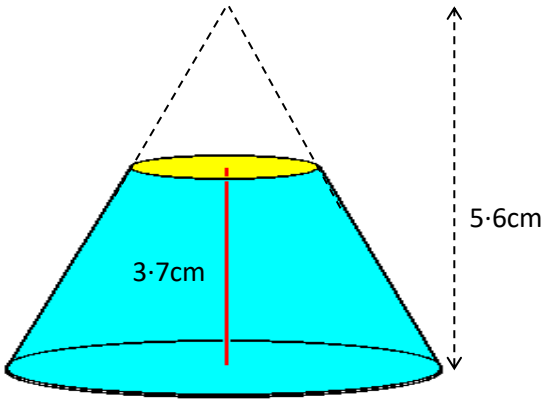
**11.** The diameter of an ordinary snooker ball is 5·25cm.

Calculate the volume of a snooker ball giving your answer correct to 3 significant figures.

**12.** A dessert is in the shape of a truncated cone [a cone with a 'slice' taken from the top].

The radius of the base is 4·1cm and is 1·6 cm at the top.

The other dimensions are shown in the diagram.




Calculate the volume of the dessert.

**13.** A young child was given a slab of moulding clay. It was a cuboid and measured 15·2cm by 4·8cm by 3·4cm.

**(a)** Calculate the volume of the cuboid rounding your answer to 2 significant figures.

The clay was made into 25 identical spheres.

**(b)** Using your answer from part (a), calculate the radius of one of the spheres.

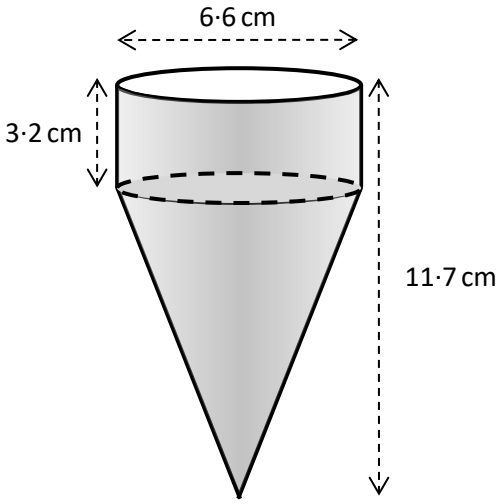
**14.** An ice cream is shaped like the one in the diagram.

The overall height of is 11·7 cm.

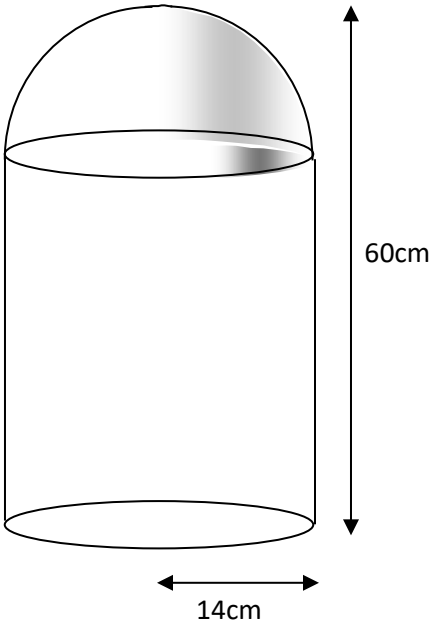
The height of the cylinder is 3·2 cm.

The diameter of the cone and cylinder is 6·6 cm.

Calculate the volume of ice cream.




**15.** A company that produces bins uses the design of a cylindrical base with a hemispherical lid.

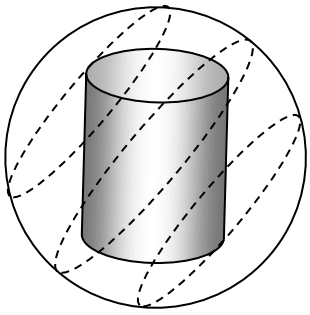



If the total height of the bin is 60cm and the radius of the bin is 14cm, calculate the total volume of the bin in litres correct to 3 significant figures.

(Volume of cylinder = $\pi r^2 h$;

Volume of sphere = $^4/_3\pi r^3$)

**16**.



A Christmas bauble is made from a sphere of perspex with a coloured cylinder in the middle. The volume round the cylinder is filled with a thick liquid.

The sphere has a diameter of 8 cm. The cylinder has a radius of 2·6 cm with a height of 6 cm.

Calculate the volume of liquid needed to fill the sphere, giving your answer correct to 2 sig fig

## *Answers*

### *Exercise 1*

**1.** **(a)** 1696·5 $cm^3$ **(b)** 4825·5 $cm^3$ **(c)** 603·2 $cm^3$ **(d)** 2513·3 $cm^3$ **(e)** 75398·2 $cm^3$

**(f)** 3078·8 $cm^3$ **(g)** 28274·3 $cm^3$ **(h)** 13304·6 $cm^3$ **(i)** 760265 $cm^3$ **(j)** 7298·5 $cm^3$

**2.** **(a)** 19·8cm **(b)** 3·7 litres **3.** 904$cm^3$

**4.** No; volume is 9·72 litres

### *Exercise 2*

**1.** **(a)** 904·3$cm^3$ **(b)** 33·5$m^3$ **(c)** 3052·1$mm^3$ **(d)** 113·0$cm^3$

**2.** **(a)** 4190$cm^3$ **(b)** 65400$cm^3$ **(c)** 4·19$m^3$ **(d)** 33500000$mm^3$

**(e)** 697$cm^3$ **(f)** 3050$cm^3$ **(g)** 2140000$mm^3$ **(h)** 87100$cm^3$

**(i)** 180$m^3$ **(j)** 57900$cm^3$

**3.** 268$cm^3$

**4.** **(a)** 366$cm^3$ **(b)** 1280$cm^3$ **(c)** 207$cm^3$ **(d)** 1060$cm^3$

**5.** **(a)** 2369$cm^3$ **(b)** 1170$cm^3$ **(c)** 15800$cm^3$ **(d)** 608$cm^3$

**6.** **(a)** 56·5$cm^3$ **(b)** 803·8$mm^3$ **(c)** 47·1$cm^3$ **(d)** 25·1$cm^3$

**7.** 83·7$cm^3$ **8.** 314$cm^3$ **9.** 1020$cm^3$ **10.** 37$cm^3$

**11.** 640$mm^3$

### *Exercise 3*

**1.** $2{\cdot}79 \times 10^6$ $m^3$ **2.** 6·01$cm^3$ **3.** 10cm **4.** 113·04$m^3$

**5.** 6·11cm **6.** 3140$cm^3$ **7.** 4291$cm^3$ **8.** 25900$cm^3$

**9.** 84·225$cm^3$ **10.** 58·9$cm^3$ **11.** 75·7$cm^3$ **12.** 93·4$cm^3$

**13.** **(a)** 250$cm^3$ **(b)** 1·3cm **14.** 206$cm^3$

**15.** 34·1 litres **16.** 140 $cm^3$