Simplifying Surds

Definition: a surd is a square root (or cube root etc.) which does not have an exact answer. e.g. $\sqrt{2} = 1.414213562...$, so $\sqrt{2}$ is a surd. However $\sqrt{9} = 3$ and $\sqrt[3]{64} = 4$, so $\sqrt{9}$ and $\sqrt[3]{64}$ are not surds because they have an exact answer.

We can multiply and divide surds.

Facts

$$\sqrt{a} \times \sqrt{b} = \sqrt{ab} \qquad \frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}} \qquad \sqrt{x} \times \sqrt{x} = \left(\sqrt{x}\right)^2 = x$$

Example 1

Simplify $3\sqrt{2} \times 5\sqrt{2}$

Solution

$$\begin{aligned} 3\sqrt{2} \times 5\sqrt{2} &= 3 \times 5 \times \sqrt{2} \times \sqrt{2} \\ &= 15 \times 2 \qquad \text{(because } \sqrt{2} \times \sqrt{2} = 2\text{)} \\ &= \underline{\underline{30}} \end{aligned}$$

To simplify a surd, you need to look for square numbers that are factors of the original number.

Examples 2

Express $\sqrt{48}$ and $\sqrt{98}$ in their simplest form

Solution

$$\begin{aligned} \sqrt{48} &= \sqrt{16 \times 3} \\ &= \sqrt{16} \times \sqrt{3} \\ &= 4 \times \sqrt{3} \\ &= \underline{4\sqrt{3}} \end{aligned}$$

$$\begin{aligned} \sqrt{98} &= \sqrt{2 \times 49} \\ &= \sqrt{2} \times \sqrt{49} \\ &= \sqrt{2} \times 7 \\ &= \underline{7\sqrt{2}} \end{aligned}$$

You can only add or take away surds when the number underneath the surd sign is the same. e.g. Simplify $\sqrt{5}+\sqrt{3}$ is NOT $\sqrt{8}$. Instead the simplest answer is $\sqrt{5}+\sqrt{3}$ (i.e. no change), because no simplifying is possible.

Examples 3 – simplifying a surd followed by collecting like terms

Write as a single surd in its simplest form: $\sqrt{63}+\sqrt{7}-\sqrt{28}$

Solution

Step One – simplify all three surds

$$\begin{aligned} & \sqrt{63} + \sqrt{7} - \sqrt{28} \\ =\ & \sqrt{9\times 7} + \sqrt{7} - \sqrt{4\times 7} \\ =\ & \sqrt{9}\times\sqrt{7} + \sqrt{7} - \sqrt{4}\times\sqrt{7} \\ =\ & 3\sqrt{7} + \sqrt{7} - 2\sqrt{7} \end{aligned}$$

Step Two – add and take away: $3\sqrt{7}+\sqrt{7}-2\sqrt{7}=\underline{2\sqrt{7}}$

Exercise 1

1. Express each of the following in its simplest form:

a) $\sqrt{8}$	**(b)** $\sqrt{12}$	**(c)** $\sqrt{50}$	**(d)** $\sqrt{20}$	**(e)** $\sqrt{24}$	**(f)** $\sqrt{108}$
g) $\sqrt{60}$	**(h)** $\sqrt{72}$	**(i)** $\sqrt{300}$	**(j)** $\sqrt{27}$	**(k)** $\sqrt{96}$	**(l)** $\sqrt{48}$
m) $\sqrt{45}$	**(n)** $\sqrt{98}$	**(o)** $\sqrt{90}$	**(p)** $\sqrt{18}$	**(q)** $\sqrt{28}$	**(r)** $\sqrt{80}$
s) $\sqrt{32}$	**(t)** $\sqrt{160}$	**(u)** $\sqrt{150}$	**(v)** $\sqrt{44}$	**(w)** $\sqrt{63}$	**(x)** $\sqrt{175}$

2. Simplify:

a) $5\sqrt{8}$	**(b)** $3\sqrt{32}$	**(c)** $5\sqrt{40}$	**(d)** $2\sqrt{12}$	**(e)** $4\sqrt{18}$	**(f)** $3\sqrt{24}$
g) $3\sqrt{27}$	**(h)** $10\sqrt{48}$	**(i)** $2\sqrt{108}$	**(j)** $3\sqrt{45}$	**(k)** $2\sqrt{63}$	**(l)** $4\sqrt{20}$

3. Express each of the following in its simplest form:

a) $5\sqrt{2} + 3\sqrt{2}$	**(b)** $3\sqrt{7} - \sqrt{7}$	**(c)** $4\sqrt{3} + 2\sqrt{3} - 3\sqrt{3}$
d) $5\sqrt{6} - 2\sqrt{6} + \sqrt{6}$	**(e)** $4\sqrt{3} + 5\sqrt{3}$	**(f)** $8\sqrt{6} - 2\sqrt{6}$
g) $\sqrt{2} + 2\sqrt{2}$	**(h)** $3\sqrt{7} - 9\sqrt{7}$	**(i)** $5\sqrt{10} - 5\sqrt{10}$
j) $\sqrt{5} + 5\sqrt{5} - 3\sqrt{5}$	**(k)** $2\sqrt{3} + \sqrt{3} - 5\sqrt{3}$	**(l)** $5\sqrt{11} + 7\sqrt{11} - \sqrt{11}$

4. Express each of the following in its simplest form:

a) $\sqrt{12} + \sqrt{27}$	**(b)** $\sqrt{32} - \sqrt{8}$	**(c)** $\sqrt{72} - \sqrt{50}$
d) $\sqrt{2} + \sqrt{98}$	**(e)** $\sqrt{80} + \sqrt{20}$	**(f)** $\sqrt{24} + \sqrt{54}$
g) $\sqrt{180} - \sqrt{45}$	**(h)** $\sqrt{1000} - \sqrt{90}$	**(i)** $\sqrt{50} - \sqrt{8}$
j) $\sqrt{3} - \sqrt{12}$	**(k)** $\sqrt{75} + \sqrt{108} - \sqrt{3}$	**(l)** $\sqrt{5} + \sqrt{20} + \sqrt{80}$
m) $\sqrt{108} + \sqrt{12}$	**(n)** $\sqrt{32} - \sqrt{8}$	**(o)** $\sqrt{72} - \sqrt{50}$
p) $\sqrt{2} + \sqrt{98}$	**(q)** $\sqrt{80} + \sqrt{20}$	**(r)** $\sqrt{24} + \sqrt{54}$
s) $\sqrt{8} + 5\sqrt{2}$	**(t)** $3\sqrt{12} + \sqrt{27}$	**(u)** $3\sqrt{2} + 2\sqrt{8} - \sqrt{18}$

5. Simplify:

a) $\sqrt{5} \times \sqrt{5}$	**(b)** $\sqrt{2} \times \sqrt{2}$	**(c)** $\sqrt{11} \times \sqrt{11}$
d) $\sqrt{a} \times \sqrt{a}$	**(e)** $\sqrt{6} \times \sqrt{6}$	**(f)** $\sqrt{c} \times \sqrt{c}$
g) $\sqrt{k} \times \sqrt{k}$	**(h)** $\sqrt{3} \times \sqrt{6}$	**(i)** $\sqrt{8} \times \sqrt{2}$
j) $\sqrt{6} \times \sqrt{2}$	**(k)** $\sqrt{3} \times \sqrt{5}$	**(l)** $\sqrt{x} \times \sqrt{y}$
m) $\sqrt{2} \times \sqrt{8}$	**(n)** $\sqrt{12} \times \sqrt{3}$	**(o)** $\sqrt{5} \times \sqrt{20}$
p) $\sqrt{2} \times \sqrt{32}$	**(q)** $\sqrt{a} \times \sqrt{b}$	**(r)** $\sqrt{10} \times \sqrt{x}$
s) $\sqrt{p} \times \sqrt{q}$	**(t)** $\sqrt{k} \times \sqrt{6}$	**(u)** $\sqrt{2} \times \sqrt{10}$
v) $\sqrt{24} \times \sqrt{3}$	**(w)** $\sqrt{5} \times \sqrt{10}$	**(x)** $\sqrt{6} \times \sqrt{12}$
y) $\sqrt{20} \times \sqrt{3}$	**(z)** $\sqrt{4} \times \sqrt{8}$	

6. **a)** $3\sqrt{2} \times \sqrt{2}$ **(b)** $2\sqrt{5} \times 3\sqrt{5}$ **(c)** $3\sqrt{2} \times 2\sqrt{7}$ **(d)** $4\sqrt{3} \times 2\sqrt{3}$

e) $\sqrt{5} \times 3\sqrt{2}$ **(f)** $2\sqrt{6} \times 3\sqrt{3}$ **(g)** $8\sqrt{2} \times \sqrt{12}$ **(h)** $5\sqrt{3} \times 3\sqrt{5}$

7. Simplify:

a) $\frac{\sqrt{8}}{\sqrt{2}}$ **(b)** $\frac{\sqrt{27}}{\sqrt{12}}$ **(c)** $\frac{\sqrt{2}}{\sqrt{32}}$ **(d)** $\frac{\sqrt{3}}{\sqrt{27}}$

e) $\frac{\sqrt{20}}{\sqrt{5}}$ **(f)** $\frac{\sqrt{12}}{\sqrt{48}}$ **(g)** $\frac{\sqrt{54}}{\sqrt{24}}$ **(h)** $\frac{\sqrt{175}}{\sqrt{63}}$

i) $\frac{\sqrt{18}}{\sqrt{72}}$ **(j)** $\frac{\sqrt{6}}{\sqrt{54}}$ **(k)** $\frac{\sqrt{288}}{\sqrt{8}}$ **(l)** $\frac{\sqrt{1000}}{\sqrt{90}}$

m) $\frac{\sqrt{48}}{\sqrt{6}}$ **(n)** $\frac{\sqrt{3}}{\sqrt{24}}$ **(o)** $\frac{\sqrt{98}}{\sqrt{7}}$ **(p)** $\frac{\sqrt{50}}{\sqrt{250}}$

8. Expand and simplify:

a) $\sqrt{2}(1 - \sqrt{2})$ **(b)** $\sqrt{3}(\sqrt{3} + 1)$ **(c)** $\sqrt{5}(\sqrt{5} - 1)$

d) $\sqrt{2}(5 + \sqrt{2})$ **(e)** $\sqrt{2}(3 + \sqrt{6})$ **(f)** $2\sqrt{3}(\sqrt{8} + 1)$

g) $\sqrt{3}(\sqrt{6} - 2\sqrt{8})$ **(h)** $\sqrt{5}(\sqrt{5} + 2)$ **(i)** $4\sqrt{6}(2\sqrt{6} - \sqrt{8})$

j) $\sqrt{8}(\sqrt{2} + 4)$ **(k)** $2\sqrt{12}(\sqrt{3} + \sqrt{6})$ **(l)** $\sqrt{5}(\sqrt{200} + \sqrt{50})$

m) $\sqrt{3}(\sqrt{2} + 1)$ **(n)** $\sqrt{2}(\sqrt{8} + \sqrt{2})$ **(o)** $\sqrt{3}(\sqrt{2} + \sqrt{6})$

(p) $\sqrt{5}(3 - \sqrt{5})$

9. Expand and simplify where possible:

a) $(\sqrt{2} + 3)(\sqrt{2} - 1)$ **(b)** $(\sqrt{5} + 1)(2\sqrt{5} - 4)$ **(c)** $(2\sqrt{2} + 3)(\sqrt{2} + 4)$

d) $(\sqrt{3} + 1)(\sqrt{3} - 1)$ **(e)** $(2 + \sqrt{5})(2 - \sqrt{5})$ **(f)** $(\sqrt{3} + \sqrt{2})(\sqrt{3} - \sqrt{2})$

g) $(\sqrt{2} - 4)(3\sqrt{2} - 1)$ **(h)** $(\sqrt{8} + 2)(\sqrt{8} + 1)$ **(i)** $(2\sqrt{3} + \sqrt{2})(\sqrt{3} + 3\sqrt{2})$

j) $(\sqrt{2} + 3)^2$ **(k)** $(\sqrt{2} + \sqrt{3})^2$ **(l)** $(2\sqrt{3} - 1)^2$

m) $(2\sqrt{7} - \sqrt{2})^2$ **(n)** $(5 - 2\sqrt{3})^2$ **(o)** $(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})$

p) $(\sqrt{7} + 1)^2$ **(q)** $(\sqrt{6} + \sqrt{2})^2$ **(r)** $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$

Rationalising the Denominator

For various mathematical reasons, it is not good to have a surd on a bottom of a fraction.

Definition: Rationalising the denominator means turning the surd at the bottom of the fraction into a whole number, whilst keeping the fraction the same.

The method is very simple: **multiply top and bottom by the surd.**

Example 1

Express with a rational denominator: $\frac{4}{\sqrt{5}}$

Solution

Multiply top and bottom by $\sqrt{5}$: $\qquad \frac{4}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \underline{\frac{4\sqrt{5}}{5}}$

Example 2

Express with a rational denominator: $\frac{1}{3\sqrt{2}}$

Solution

Multiply top and bottom by $\sqrt{2}$ (not $3\sqrt{2}$): $\qquad \frac{1}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{3\sqrt{2} \times \sqrt{2}} = \underline{\frac{\sqrt{2}}{6}}$

You may be able to simplify your answer after rationalising:

Example 3

Express with a rational denominator: $\frac{6}{\sqrt{8}}$

Solution

Multiply top and bottom by $\sqrt{8}$: $\qquad \frac{6}{\sqrt{8}} \times \frac{\sqrt{8}}{\sqrt{8}} = \frac{{}^{3}\cancel{6}\sqrt{8}}{{}_{4}\cancel{8}} = \underline{\frac{3\sqrt{8}}{4}}$

Exercise 2

1. Express each of the following with a *rational denominator* and simplify where possible:

a) $\frac{1}{\sqrt{2}}$ **(b)** $\frac{1}{\sqrt{3}}$ **(c)** $\frac{1}{\sqrt{5}}$ **(d)** $\frac{6}{\sqrt{3}}$

e) $\frac{10}{\sqrt{5}}$ **(f)** $\frac{2}{\sqrt{3}}$ **(g)** $\frac{3}{\sqrt{5}}$ **(h)** $\frac{20}{\sqrt{2}}$

i) $\frac{2}{\sqrt{2}}$ **(j)** $\frac{12}{\sqrt{3}}$ **(k)** $\frac{3}{\sqrt{6}}$ **(l)** $\frac{4}{\sqrt{5}}$

m) $\frac{10}{\sqrt{2}}$ **(n)** $\frac{35}{\sqrt{7}}$

2. Express each of the following with a ***rational denominator*** and simplify where possible:

a) $\frac{1}{2\sqrt{5}}$ **(b)** $\frac{4}{5\sqrt{2}}$ **(c)** $\frac{3}{3\sqrt{2}}$ **(d)** $\frac{12}{5\sqrt{6}}$

e) $\frac{8}{3\sqrt{2}}$ **(f)** $\frac{20}{7\sqrt{5}}$ **(g)** $\frac{50}{3\sqrt{10}}$ **(h)** $\frac{10}{3\sqrt{2}}$

3. Express each of the following in its simplest form with a rational denominator.

a) $\frac{\sqrt{3}}{\sqrt{2}}$ **(b)** $\frac{\sqrt{2}}{\sqrt{5}}$ **(c)** $\frac{\sqrt{8}}{\sqrt{2}}$ **(d)** $\frac{\sqrt{18}}{\sqrt{3}}$

e) $\frac{\sqrt{5}}{\sqrt{20}}$ **(f)** $\frac{\sqrt{2}}{\sqrt{12}}$ **(g)** $\frac{\sqrt{15}}{\sqrt{5}}$ **(h)** $\frac{\sqrt{8}}{\sqrt{6}}$

i) $\frac{\sqrt{5}}{\sqrt{2}}$ **(j)** $\frac{\sqrt{11}}{\sqrt{2}}$ **(k)** $\frac{\sqrt{7}}{\sqrt{3}}$ **(l)** $\frac{\sqrt{13}}{\sqrt{5}}$

m) $\frac{\sqrt{8}}{3\sqrt{2}}$ **(n)** $\frac{2\sqrt{3}}{3\sqrt{2}}$ **(o)** $\frac{5\sqrt{3}}{3\sqrt{5}}$ **(p)** $\frac{4\sqrt{5}}{5\sqrt{3}}$

q) $\frac{\sqrt{6}}{\sqrt{18}}$ **(r)** $\frac{\sqrt{50}}{\sqrt{10}}$ **(s)** $\sqrt{\frac{3}{12}}$ **(l)** $\sqrt{\frac{5}{2}}$

4. Express each of the following with a ***rational denominator*** and simplify where possible:

a) $\dfrac{1}{\sqrt{50}}$ **(b)** $\dfrac{18}{\sqrt{27}}$ **(c)** $\dfrac{5}{\sqrt{50}}$ **(d)** $\dfrac{3}{\sqrt{20}}$

e) $\dfrac{6}{\sqrt{18}}$ **(f)** $\dfrac{2}{\sqrt{8}}$ **(g)** $\dfrac{10}{\sqrt{12}}$ **(h)** $\dfrac{3}{\sqrt{50}}$

i) $\dfrac{4}{\sqrt{32}}$ **(j)** $\dfrac{2\sqrt{3}}{\sqrt{54}}$ **(k)** $\dfrac{3\sqrt{2}}{\sqrt{24}}$ **(l)** $\dfrac{2\sqrt{5}}{\sqrt{45}}$

5. Rationalise the denominator, in each fraction, using the appropriate conjugate surd.

a) $\dfrac{1}{\sqrt{2}-1}$ **(b)** $\dfrac{1}{\sqrt{5}+1}$ **(c)** $\dfrac{12}{2-\sqrt{3}}$ **(d)** $\dfrac{1}{1-\sqrt{2}}$

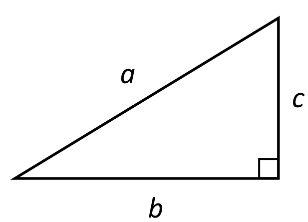
e) $\dfrac{1}{1+\sqrt{3}}$ **(f)** $\dfrac{3}{\sqrt{5}-1}$ **(g)** $\dfrac{2}{\sqrt{2}+2}$ **(h)** $\dfrac{3}{2-\sqrt{6}}$

i) $\dfrac{5}{3+\sqrt{2}}$ **(j)** $\dfrac{4}{1+\sqrt{3}}$ **(k)** $\dfrac{1}{\sqrt{7}-2}$ **(l)** $\dfrac{1}{\sqrt{3}-\sqrt{2}}$

m) $\dfrac{6}{\sqrt{3}+\sqrt{2}}$ **(n)** $\dfrac{12}{\sqrt{10}-\sqrt{2}}$ **(o)** $\dfrac{3}{\sqrt{5}+\sqrt{6}}$ **(p)** $\dfrac{14}{9-\sqrt{2}}$

Mixed Exercise

1. A right angled triangle has sides a, b and c as shown.
For each case below calculate the length of the third side, expressing your answer as a surd in its simplest form.



a) Find a if $b = 6$ and c = 3. **(b)** Find c if $a = 2$ and $b = 1$.

c) Find c if $a = 18$ and $b = 12$ **(d)** Find b if $a = 2\sqrt{8}$ and $c = 2\sqrt{6}$.

2. Given that $x = 1 + \sqrt{2}$ and $y = 1 - \sqrt{2}$, simplify:

a) $5x + 5y$ **(b)** $2xy$ **(c)** $x^2 + y^2$ **(d)** $(x + y)(x - y)$

3. Given that $p = \sqrt{5} + \sqrt{3}$ and $q = \sqrt{5} - \sqrt{3}$, simplify:

a) $2p - 2q$ **(b)** $4pq$ **(c)** $p^2 - q^2$

4. A rectangle has sides measuring $(2 + \sqrt{2})$ cm and $(2 - \sqrt{2})$ cm.

Calculate the exact value of: **a)** its area

b) the length of a diagonal.

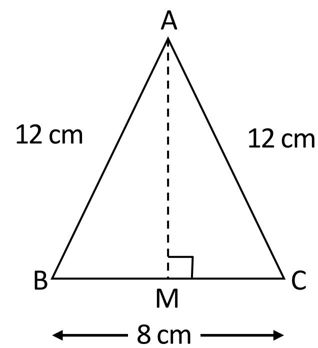
5. A curve has as its equation $y = 2 + \frac{1}{2}x^2$.

a) If the point P($\sqrt{2}$, k) lies on this curve find the exact value of k.

b) Find the exact length of OP where O is the origin.

6. In ΔABC, AB = AC = 12cm and BC = 8cm.

Express the length of the altitude from A to BC as a surd in its simplest form. [The line AM in the diagram]



7. An equilateral triangle has each of its sides measuring $2a$ metres.

a) Find the exact length of an altitude of the triangle in terms of a.

b) Hence find the exact area of the triangle in terms of a.
[Draw a diagram to help you with this question]

8. The exact **area** of a rectangle is $2(\sqrt{6} + \sqrt{3})$ square centimetres.

Given that the breadth of the rectangle is $\sqrt{6}$ cm,

show that the length is equal to $(2 + \sqrt{2})$ cm.

9. (Challenging!!) Given that $\tan 75^\circ = \dfrac{\sqrt{3} + 1}{\sqrt{3} - 1}$, show that $\tan 75^\circ = 2 + \sqrt{3}$

Answers

Exercise 1

1.	**a)**	$2\sqrt{2}$	**(b)**	$2\sqrt{3}$	**(c)**	$5\sqrt{2}$	**(d)**	$2\sqrt{5}$	**(e)**	$2\sqrt{6}$	**(f)**	$6\sqrt{3}$
	g)	$2\sqrt{15}$	**(h)**	$6\sqrt{2}$	**(i)**	$10\sqrt{3}$	**(j)**	$3\sqrt{3}$	**(k)**	$4\sqrt{6}$	**(l**	$4\sqrt{3}$
	m)	$3\sqrt{5}$	**(n)**	$7\sqrt{2}$	**(o)**	$3\sqrt{10}$	**(p)**	$3\sqrt{2}$	**(q)**	$2\sqrt{7}$	**(r)**	$4\sqrt{5}$
	s)	$4\sqrt{2}$	**(t)**	$4\sqrt{10}$	**(u)**	$5\sqrt{6}$	**(v)**	$2\sqrt{11}$	**(w)**	$3\sqrt{7}$	**(x)**	$5\sqrt{7}$

2.	**a)**	$10\sqrt{2}$	**(b)**	$12\sqrt{2}$	**(c)**	$10\sqrt{10}$	**(d)**	$4\sqrt{3}$	**(e)**	$12\sqrt{2}$	**(f)**	$6\sqrt{6}$
	g)	$9\sqrt{3}$	**(h)**	$40\sqrt{3}$	**(i)**	$12\sqrt{3}$	**(j)**	$9\sqrt{5}$	**(k)**	$6\sqrt{7}$	**(l)**	$8\sqrt{5}$

3.	**a)**	$8\sqrt{2}$	**(b)**	$2\sqrt{7}$	**(c)**	$3\sqrt{3}$	**(d)**	$4\sqrt{6}$	**(e)**	$9\sqrt{3}$	**(f)**	$6\sqrt{6}$
	g)	$3\sqrt{2}$	**(h)**	$-6\sqrt{7}$	**(i)**	0	**(j)**	$3\sqrt{5}$	**(k)**	$-2\sqrt{3}$	**(l)**	$11\sqrt{11}$

4.	**a)**	$5\sqrt{3}$	**(b)**	$2\sqrt{2}$	**(c)**	$\sqrt{2}$	**(d)**	$8\sqrt{2}$	**(e)**	$6\sqrt{5}$	**(f)**	$5\sqrt{6}$
	g)	$3\sqrt{5}$	**(h)**	$7\sqrt{10}$	**(i)**	$3\sqrt{2}$	**(j)**	$-\sqrt{3}$	**(k)**	$10\sqrt{3}$	**(l)**	$7\sqrt{5}$
	m)	$8\sqrt{3}$	**(n)**	$2\sqrt{2}$	**(o)**	$\sqrt{2}$	**(p)**	$8\sqrt{2}$	**(q)**	$6\sqrt{5}$	**(r)**	$5\sqrt{6}$
	s)	$7\sqrt{2}$	**(t)**	$8\sqrt{3}$	**(u)**	$4\sqrt{2}$						

5.	**a)**	5	**(b)**	2	**(c)**	11	**(d)**	a	**(e)**	6	**(f)**	c
	g)	k	**(h)**	$3\sqrt{2}$	**(i)**	4	**(j)**	$2\sqrt{3}$	**(k)**	$\sqrt{15}$	**(l)**	$\sqrt{xy}$
	m)	4	**(n)**	6	**(o)**	10	**(p)**	8	**(q)**	$\sqrt{ab}$	**(r)**	$\sqrt{10x}$
	s)	$\sqrt{pq}$	**(t)**	$\sqrt{6k}$	**(u)**	$2\sqrt{5}$	**(v)**	$6\sqrt{2}$	**(w)**	$5\sqrt{2}$	(**x)**	$6\sqrt{2}$
	y)	$2\sqrt{15}$	**(z)**	$4\sqrt{2}$								

6.	**a)**	6	**(b)**	30	(c)	$6\sqrt{14}$	(d)	24	**(e)**	$3\sqrt{10}$	**(f)**	$18\sqrt{2}$
	g)	$16\sqrt{6}$	**(h)**	$15\sqrt{15}$								

7.	**a)**	2	**(b)**	$\frac{3}{2}$	**(c)**	$\frac{1}{4}$	**(d)**	$\frac{1}{3}$	**(e)**	2	**(f)**	$\frac{1}{2}$
	g)	$\frac{3}{2}$	**(h)**	$\frac{5}{3}$	**(i)**	$\frac{1}{2}$	**(j)**	$\frac{1}{3}$	**(k)**	6	**(l)**	$\frac{10}{3}$
	m)	$2\sqrt{2}$	**(n)**	$\frac{1}{2\sqrt{2}}$	**(o)**	$\sqrt{14}$	**(p)**	$\frac{1}{\sqrt{5}}$				

8.	**a)**	$\sqrt{2} - 2$	**(b)**	$3 + \sqrt{3}$	**(c)**	$5 - \sqrt{5}$	**(d)**	$5\sqrt{2} + 2$
	e)	$3\sqrt{2} + 2\sqrt{3}$	**(f)**	$4\sqrt{6} + 2\sqrt{3}$	**(g)**	$3\sqrt{2} - 4\sqrt{6}$	**(h)**	$5 + 2\sqrt{5}$
	i)	$48 - 16\sqrt{3}$	**(j)**	$4 + 8\sqrt{2}$	**(k)**	$12 + 12\sqrt{2}$	**(l)**	$15\sqrt{10}$
	m)	$\sqrt{6} + \sqrt{3}$	**(n)**	6	**(o)**	$\sqrt{6} + 3\sqrt{2}$	**(p)**	$3\sqrt{5} - 5$

9.	**a)**	$2\sqrt{2} - 1$	**(b)**	$6 - 2\sqrt{5}$	**(c)**	$16 + 11\sqrt{2}$	**(d)**	2
	e)	-1	**(f)**	1	**(g)**	$10 - 13\sqrt{2}$	**(h)**	$10 + 3\sqrt{8}$
	i)	$12 + 7\sqrt{6}$	**(j)**	$11 + 6\sqrt{2}$	**(k)**	$5 + 2\sqrt{6}$	**(l)**	$13 - 4\sqrt{3}$
	m)	$30 - 4\sqrt{14}$	**(n)**	$37 - 20\sqrt{3}$	**(o)**	-2	**(p)**	$8 + 2\sqrt{7}$
	q)	$8 + 4\sqrt{3}$	**(r)**	-1				

Exercise 2

1.	**a)**	$\frac{\sqrt{2}}{2}$	**(b)**	$\frac{\sqrt{3}}{3}$	**(c)**	$\frac{\sqrt{5}}{5}$	**(d)**	$2\sqrt{3}$	**(e)**	$2\sqrt{5}$	**(f)**	$\frac{2\sqrt{3}}{3}$
	g)	$\frac{3\sqrt{5}}{5}$	**(h)**	$10\sqrt{2}$	**(i)**	$\sqrt{2}$	**(j)**	$4\sqrt{3}$	**(k)**	$\frac{\sqrt{6}}{2}$	**(l)**	$\frac{4\sqrt{5}}{5}$
	m)	$5\sqrt{2}$	**(n)**	$5\sqrt{7}$								

2.	**a)**	$\frac{\sqrt{5}}{10}$	**(b)**	$\frac{2\sqrt{2}}{5}$	**(c)**	$\frac{\sqrt{2}}{2}$	**(d)**	$\frac{2\sqrt{6}}{5}$	**(e)**	$\frac{4\sqrt{2}}{3}$	**(f)**	$\frac{4\sqrt{5}}{7}$
	g)	$\frac{5\sqrt{10}}{3}$	**(h)**	$\frac{5\sqrt{2}}{3}$								

3.	**a)**	$\frac{\sqrt{6}}{2}$	**(b)**	$\frac{\sqrt{10}}{5}$	**(c)**	2	**(d)**	$\sqrt{6}$
	e)	$\frac{1}{2}$	**(f)**	$\frac{\sqrt{6}}{6}$	**(g)**	$\sqrt{3}$	**(h)**	$\frac{2\sqrt{3}}{3}$
	i)	$\frac{\sqrt{10}}{2}$	**(j)**	$\frac{\sqrt{22}}{2}$	**(k)**	$\frac{\sqrt{21}}{3}$	**(l)**	$\frac{\sqrt{65}}{5}$
	m)	$\frac{2}{3}$	**(n)**	$\frac{\sqrt{6}}{3}$	**(o)**	$\frac{\sqrt{15}}{3}$	**(p)**	$\frac{4\sqrt{15}}{15}$
	q)	$\frac{\sqrt{3}}{3}$	**(r)**	$\sqrt{5}$	**(s)**	$\frac{1}{2}$	**(l)**	$\frac{\sqrt{10}}{2}$
4.	**a)**	$\frac{\sqrt{2}}{10}$	**(b)**	$2\sqrt{3}$	**(c)**	$\frac{\sqrt{2}}{2}$	**(d)**	$\frac{3\sqrt{5}}{10}$
	e)	$\sqrt{2}$	**(f)**	$\frac{\sqrt{2}}{2}$	**(g)**	$\frac{5\sqrt{3}}{3}$	**(h)**	$\frac{3\sqrt{2}}{10}$
	i)	$\frac{\sqrt{2}}{2}$	**(j)**	$\frac{\sqrt{2}}{3}$	**(k)**	$\frac{\sqrt{3}}{2}$	**(l)**	$\frac{2}{3}$
5.	**a)**	$\sqrt{2+1}$	**(b)**	$\frac{\sqrt{5}-1}{4}$	**(c)**	$-12(2+\sqrt{3})$	**(d)**	$-(1+\sqrt{2})$
	e)	$-\frac{1}{2}(1-\sqrt{3})$	**(f)**	$\frac{3(\sqrt{5}+1)}{4}$	**(g)**	$-(\sqrt{2}-2)$	**(h)**	$-\frac{3}{2}(2+\sqrt{6})$
	i)	$\frac{5(3-\sqrt{2})}{7}$	**(j)**	$-2(1+\sqrt{3})$	**(k)**	$\frac{\sqrt{7}+2}{3}$	**(l)**	$\sqrt{3}+\sqrt{2}$
	m)	$6(\sqrt{3}-\sqrt{2})$	**(n)**	$\frac{3}{2}(\sqrt{10}+\sqrt{2})$	**(o)**	$-3(\sqrt{5}-\sqrt{6})$	**(p)**	$\frac{14(9+\sqrt{2})}{79}$

Mixed Exercise

1.	**a)**	$3\sqrt{5}$	**(b)**	$\sqrt{3}$	**(c)**	$6\sqrt{5}$	**(d)**	$2\sqrt{2}$
2.	**a)**	10	**(b)**	-2	**(c)**	6	**(d)**	$4\sqrt{2}$
3.	**a)**	$4\sqrt{3}$	**(b)**	8	**(c)**	$4\sqrt{15}$		
4.	**a)**	2 cm^2	**(b)**	$2\sqrt{3}$ cm				
5.	**a)**	3	**(b)**	11				
6.	$8\sqrt{2}$							
7.	**a)**	$\sqrt{3}a$	**(b)**	$\sqrt{3}a^2$				
8.	Proof							
9.	Proof							