N5 Gradient

Expressions & Formulae

SPTA Mathematics – Topic Questions with Notes

The Meaning of Gradient

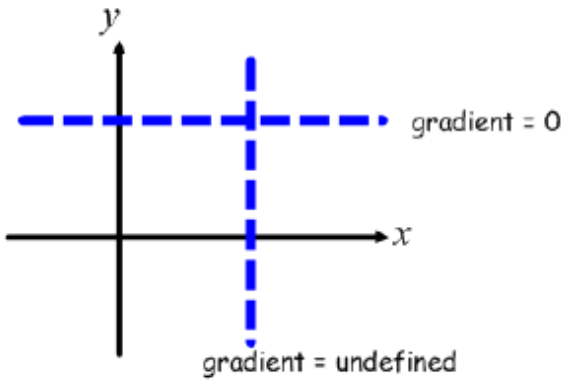
The **gradient** of a line is its steepness. It measures how much the line goes up (or down) for every one unit that you move along.

The basic definition of gradient is: $\text{gradient} = \frac{\text{up}}{\text{along}} = \frac{\text{vertical}}{\text{horizontal}}$.

A **positive** gradient (e.g. 2, $\frac{1}{2}$, $\frac{5}{7}$) means the line slopes **upwards**. A **negative** gradient (e.g. -2, $-\frac{1}{2}$, $-\frac{5}{7}$) means the line slopes **downwards**.

- A gradient of 2 means 'along 1, up 2'.
- A gradient of -3 means 'along 1, down 3'
- A gradient of $\frac{3}{4}$ means 'along 1, up $\frac{3}{4}$', [more easily thought of as 'along 4, up 3']

If a line is horizontal, it has a gradient of **zero**. If a line is vertical, we say its gradient is **undefined**.

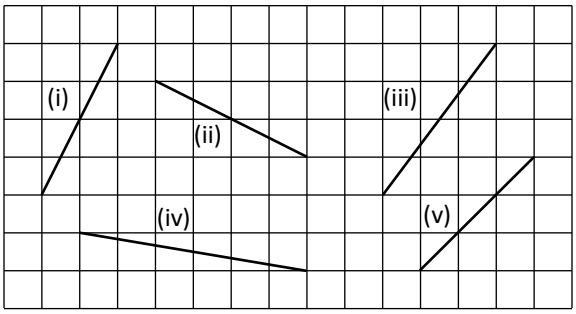


We usually use the letter m to mean gradient.

Two lines are **parallel** if they have the same gradient.

Exercise 1

1. **a)** Calculate the gradient of each line in the diagram opposite.



b) **Copy** and complete each statement below:

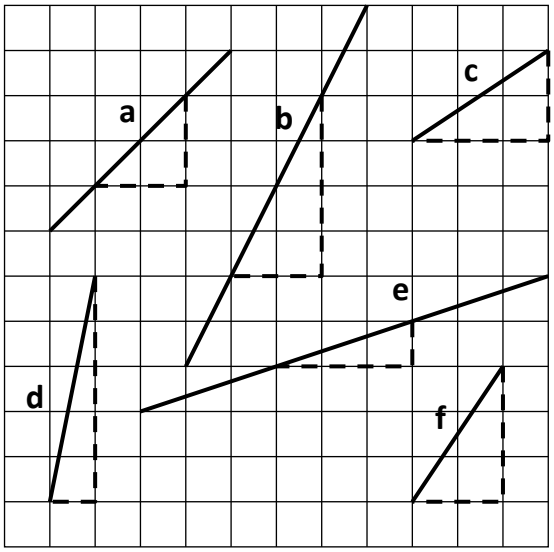
The gradient of any horizontal line is _____ .

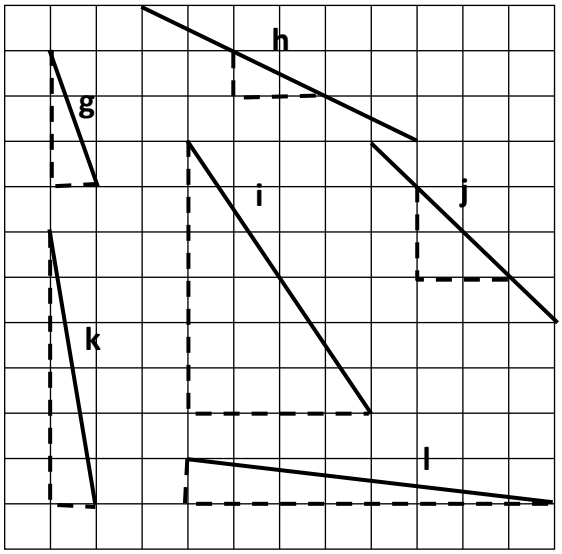
The gradient of any vertical line is _____.

A line sloping upwards from left to right has a _____ gradient.

A line sloping upwards from right to left has a _____ gradient.

2. Find the **gradients** of the lines shown in each of the diagrams below:





Calculating the Gradient

Formula

Gradient between two points (x_1, y_1) and (x_2, y_2): $\quad m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Example 1 – from coordinates

Find the gradient between the points (–2, 5) and (1, 4)

Solution

Step 1 – label the coordinates: $\underset{x_1}{(-2,} \ \underset{y_1}{5)} \quad \underset{x_2}{(1,} \ \underset{y_2}{4)}$

Step 2 – put into the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4-5}{1--2}$$

$$= \frac{-1}{3}$$

Answer: $m = -\dfrac{1}{3}$

Example 2 – from a diagram

Calculate the gradient of this straight line

Solution

Step 1 – identify any two 'nice' coordinates on the line: $(0, 3) \quad (2, 4)$

Step 2 – label the coordinates:

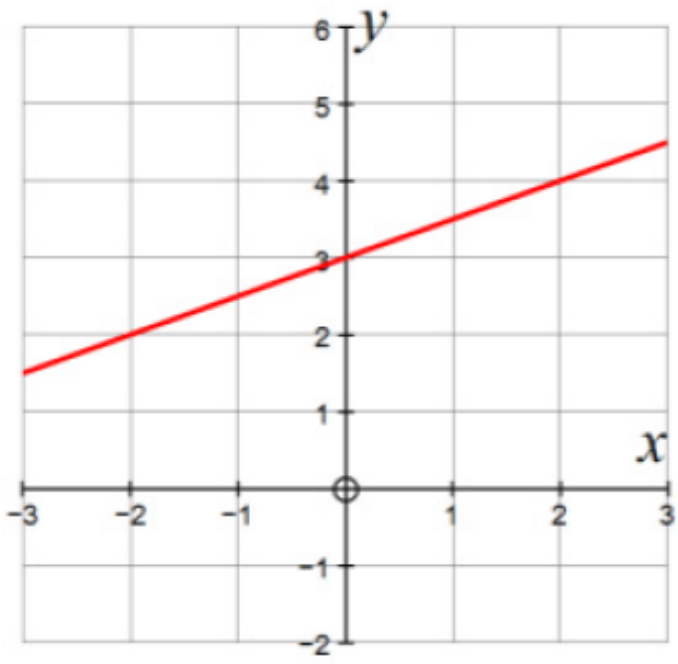
$\underset{x_1}{(0,} \ \underset{y_1}{3)} \quad \underset{x_2}{(2,} \ \underset{y_2}{4)}$

Step 3 – put into the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{4-3}{2-0}$$

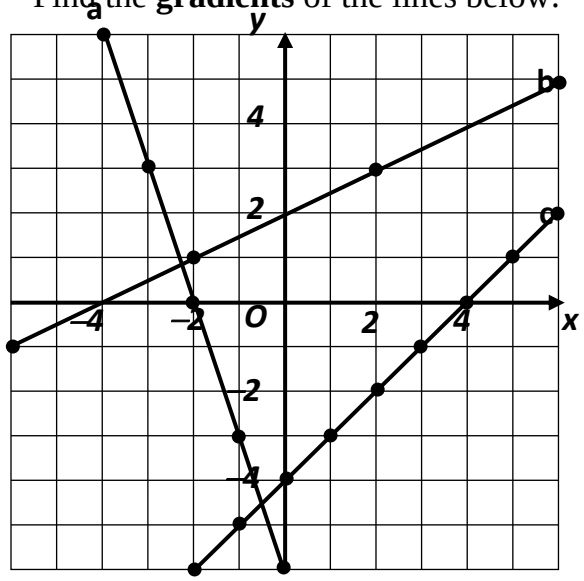
$$= \frac{1}{2}$$

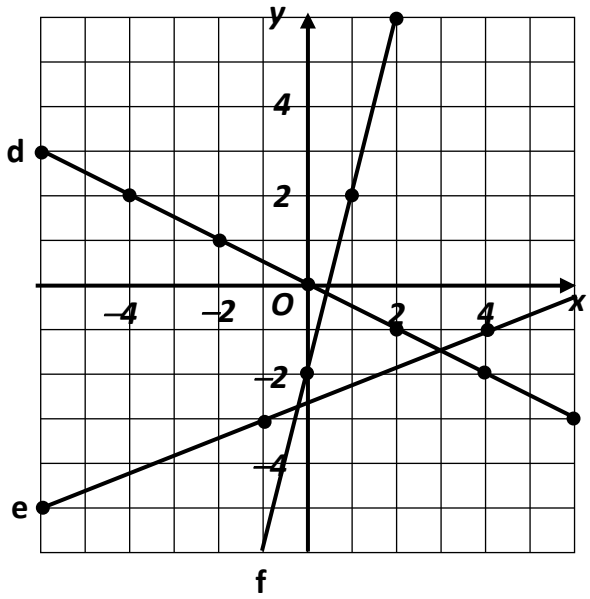


Answer: $m = \dfrac{1}{2}$

Exercise 1

1. Find the **gradients** of the lines below:





2. Calculate the gradient of the line joining each pair of points below:

a) (2, 1) and (6, 3) **(b)** (1, 5) and (3, 1) **(c)** (2, 0) and (4, 6)

d) (4, 3) and (8, 11) **(e)** (1, 9) and (3, 1) **(f)** (7, 3) and (5, 2)

g) (–2, –3) and (2, 3) **(h)** (–1, 2) and (5, –1) **(i)** (–4, 2) and (4, –4)

j) (–6, –2) and (–5, 3) **(k)** (4, –3) and (6, 5) **(l)** (–2, 3) and (0, –2)

3. Calculate the gradient of the line joining each pair of points below:

a) A(–2, 6) and B(8, 8) **(b)** C(3, –3) and D(4, –1)

c) E(5, –9) and F(8, –15) **(d)** G(0, 6) and H(5, 11)

e) I(–1, –3) and J(7, –9) **(f)** K(–4, 0) and L(–1, 5)

g) M(2, 2) and N(–3, 4) **(h)** P(5, –1) and Q(–2, 10)

i) R(–3, –5) and S(8, –4) **(j)** T(4, –6) and U(7, –2)

k) V(5, –6) and W(–2, 6) **(l)** X(–1, 7) and Y(–2, 6)

m) J(6, 8) and K(–3, –5) **(n)** S(3, –5) and T(–2, 8)

o) D(6, –3) and E(0, 4) **(p)** F(6, 9) and G(–5, –5)

4. Prove that the following sets of points are collinear:

a) A(−6,−1), B(2, 3) and C(4, 4)

b) P(1, −1), Q(−3, 5) and R(7, −10)

c) E(5, −3), F(11, −2) and G(−7, −5)

d) K(5, −4), L(−1, 4) and M(9½ , −10)

5. Given that each set of points are collinear, find the value of k in each case:

a) P(−4, −2), Q(−1, −1) and R(8, k)

b) A(1, 3), B(3, k) and C(4, −6)

c) E(−4, −1), F(k, −1) and G(8, 7)

d) S(k, 2), T(9, 1) and U(−3, 4)

6. The points E and F have coordinates $(2, -5)$ and $(-4, a)$ respectively. Given that the gradient of the line EF is $\frac{2}{3}$, find the value of a .

7. If the points $(3, 2)$, $(-1, 0)$ and $(4, k)$ are collinear, find k .

8. Given that the points $(3, -2)$, $(4, 5)$ and $(-1, a)$ are collinear , find the value of a.

9. The line which passes through (1, 4) and (2, 5) is parallel to the line through (3, 7) and $(k, 5)$. Find the value of k.

10. The line which passes through (−2, 3) and (−5, −9) is parallel to the line through (4, k) and (−1, −1). Find the value of k.

Answers

Exercise 1

1. **a)** **(i)** 2 **(ii)** $-\frac{1}{2}$ **(iii)** $\frac{4}{3}$ **(iv)** $-\frac{1}{6}$ **(v)** **1**

b) 0; undefined; positive; negative

2. **a)** 1 **(b)** 2 **(c)** $\frac{2}{3}$ **(d)** **5** **(e)** $\frac{1}{3}$ **(f)** $\frac{3}{2}$

g) -3 **(h)** $-\frac{1}{2}$ **(i)** $-\frac{3}{2}$ **(j)** -1 **(k)** -6 **(l)** $-\frac{1}{8}$

3. **a)** -3 **(b)** $\frac{1}{2}$ **(c)** 1 **(d)** $-\frac{1}{2}$ **(e)** $\frac{2}{5}$ **(f)** 4

4. **a)** $\frac{1}{2}$ **(b)** -2 **(c)** 3 **(d)** 2 **(e)** -4 **(f)** $\frac{1}{2}$

g) $\frac{3}{2}$ **(h)** $-\frac{1}{2}$ **(i)** $-\frac{3}{4}$ **(j)** 5 **(k)** 4 **(l)** $-\frac{5}{2}$

5. **a)** $\frac{1}{5}$ **(b)** 2 **(c)** -2 **(d)** 1 **(e)** $-\frac{3}{4}$ **(f)** $\frac{5}{3}$

g) $-\frac{2}{5}$ **(h)** $-\frac{11}{7}$ **(i)** $\frac{1}{11}$ **(j)** $\frac{4}{3}$ **(k)** $-\frac{12}{7}$ **(l)** 1

m) $\frac{13}{9}$ **(n)** $-\frac{13}{5}$ **(o)** $-\frac{7}{6}$ **(p)** $\frac{14}{11}$

6. **a)** both gradients $\frac{1}{2}$ **(b)** both gradients $-\frac{3}{2}$

c) both gradients $\frac{1}{6}$ **(d)** both gradients $-\frac{4}{3}$

7. **a)** $k = 2$ **(b)** $k = -3$ **(c)** $k = -4$ **(d)** $k = 5$

8. $a = -9$

9. $k = 2 \cdot 5$

10. $a = -30$

11. $k = 1$

12. $k = 19$