Completing the Square

The process of writing $y = ax^2 + bx + c$ in the form $y = a(x+p)^2 + q$ is called **completing the square**. The completed square form of the equation is useful because from it we can easily determine the maximum or minimum value of a function. It also has uses when we consider the graphs of these functions (see page 42).

To rewrite $y = x^2 + bx + c$ in the form $y = (x+p)^2 + q$, **we use the fact that the number in the bracket (p) is half of the coefficient of x (b).**

Example

Express $x^2 + 8x + 3$ in the form $(x+p)^2 + q$

Solution

We can immediately see that $p = 4$ (half of 8), so $x^2 + 8x + 3 = (x+4)^2 + q$

Now we expand the bracket and compare to the original expression to work out q :

$$x^2 + 8x + 3 = (x+4)^2 + q$$
$$x^2 + 8x + 3 = (x+4)(x+4) + q$$
$$x^2 + 8x + 3 = x^2 + 8x + 16 + q$$

$$3 = 16 + q$$
$$q = -13$$

Final answer: $x^2 + 8x + 3 = (x+4)^2 - 13$

Exercise 1

1. Write the following in the form $(x + a)^2 + b$.

   (a) $x^2 + 4x$ (b) $x^2 + 10x$ (c) $x^2 + 7x$ (d) $x^2 + 9x$

   (e) $x^2 - 6x$ (f) $x^2 - 8x$ (g) $x^2 - 5x$ (h) $x^2 - 11x$

2. Write the following in the form $(x + a)^2 + b$.

   (a) $x^2 + 2x + 7$ (b) $x^2 + 6x + 2$ (c) $x^2 + 8x + 9$

   (d) $x^2 + 10x + 27$ (e) $x^2 + 4x - 8$ (f) $x^2 + 16x - 3$

   (g) $x^2 - 6x + 11$ (h) $x^2 - 2x + 5$ (i) $x^2 - 8x + 8$

   (j) $x^2 - 14x - 15$ (k) $x^2 - 12x + 21$ (l) $x^2 - 20x - 6$

3. Write the following in the form $(x + a)^2 + b$.

   (a) $4 + 2x - x^2$ (b) $7 + 4x - x^2$ (c) $3 - 6x - x^2$

   (d) $10 - 10x - x^2$ (e) $14 + 3x - x^2$ (f) $5 - 7x - x^2$

Answers

Exercise 1

1. **a)** $(x+2)^2 - 4$ **(b)** $(x+5)^2 - 25$ **(c)** $(x+3\cdot5)^2 - 12\cdot25$

     d) $(x+4\cdot5)^2 - 20\cdot25$ **(e)** $(x-3\cdot)^2 - 9$ **(f)** $(x-4)^2 - 16$

     g) $(x-2\cdot5)^2 - 6\cdot25$ **(h)** $(x-5\cdot5)^2 - 30\cdot25$

2. **a)** $(x-1)^2 + 6$ **(b)** $(x+3)^2 - 7$ **(c)** $(x+4)^2 - 7$

     d) $(x+5)^2 + 2$ **(e)** $(x+2)^2 - 12$ **(f)** $(x+8)^2 - 67$

     g) $(x-3)^2 + 2$ **(h)** $(x-1)^2 + 4$ **(i)** $(x-4)^2 - 8$

     j) $(x-7)^2 - 64$ **(k)** $(x-6)^2 - 15$ **(l)** $(x-10)^2 - 106$

3. **a)** $5 - (x-1)^2$ **(b)** $11 - (x-2)^2$ **(c)** $12 - (x+3)^2$

     d) $35 - (x+5)^2$ **(e)** $16\cdot25 - (x-1\cdot5)^2$ **(f)** $17\cdot25 - (x+3\cdot5)^2$