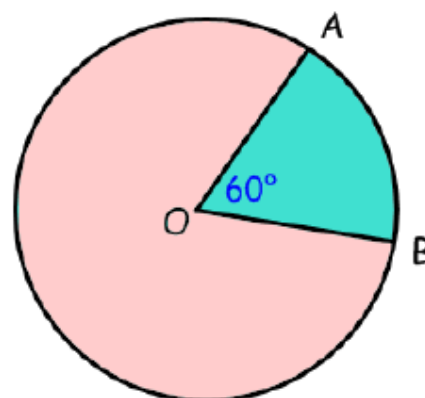


Arc Length and Sector Area

An **arc** in a circle is a fraction of its circumference. A **sector** of a circle is a fraction of its area.

If you divide a circle into two bits, you get two sectors - a bigger one (major) and a smaller one (minor). In the diagram on the right:

- The smaller blue sector OAB is the minor sector, with the minor arc AB.
- The larger pink sector OAB is called the major sector, with the longer major arc AB.



The key idea in these questions is to identify the fraction of the circle that is in the question. This depends on the **angle** at the centre of the circle. This fraction is always $\frac{\text{Angle}}{360}$.

Formula. These formula are not given on the National 5 Mathematics exam paper.

Arc length in a circle:

$$\text{Arc length} = \frac{\text{Angle}}{360} \pi d$$

Sector area of a circle:

$$\text{Sector Area} = \frac{\text{Angle}}{360} \pi r^2$$

You are always allowed to use 3.14 instead of π in calculations.

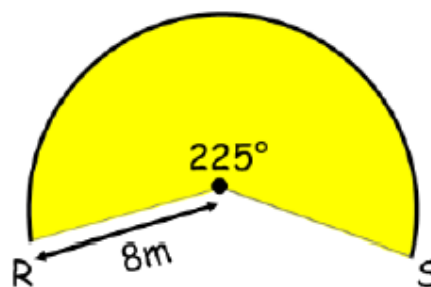
Example 1 – Arc Length

Find the length of the (major) arc RS in this sector of a circle

Solution

Radius is 8m so diameter is 16m.

$$\begin{aligned} \text{Arc length} &= \frac{225}{360} \pi d \\ &= \pi \times 16 \div 360 \times 225 \\ &= 31.41592... \\ &= \underline{31.4\text{m (1 d.p.)}} \end{aligned}$$



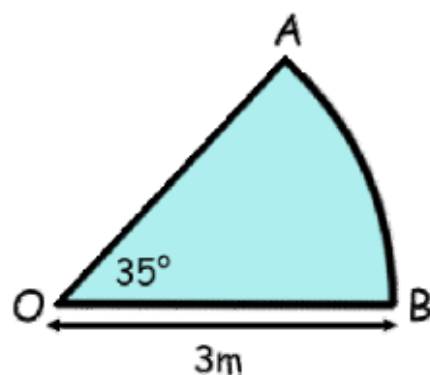
Note: units for arc length are just ‘normal’ units (i.e. not squared or cubic units).

Example 2 – Sector area

Calculate the area of (minor) sector AOB in this diagram.

Solution

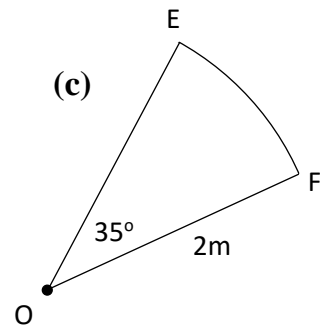
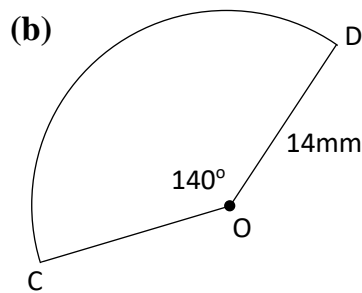
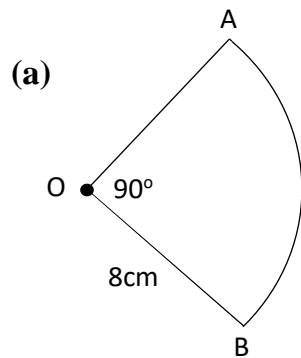
$$\begin{aligned}A &= \frac{35}{360} \pi r^2 \\&= \pi \times 3^2 \div 360 \times 35 \\&= 2.74889357... \\&= \underline{2.75\text{m}^2} \text{ (2 d.p.)}\end{aligned}$$



Note: units for sector area must always be squared units.

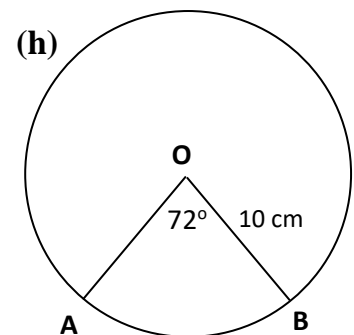
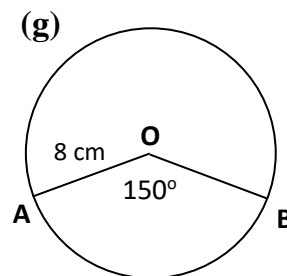
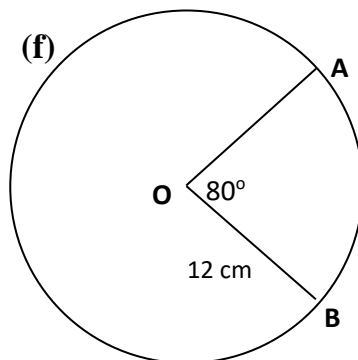
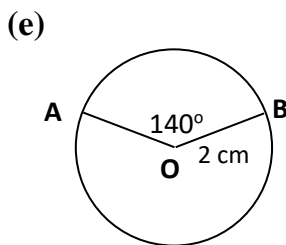
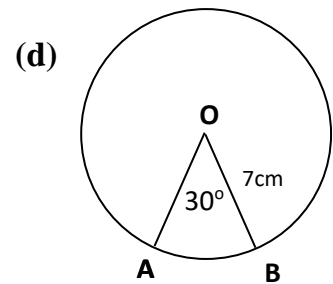
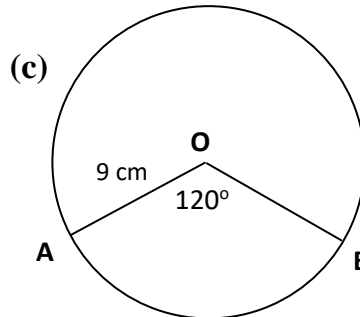
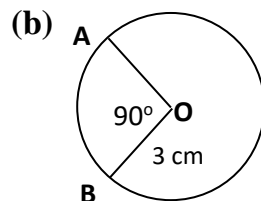
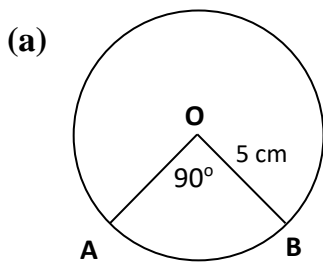
Exercise 1

1. Calculate the length of the arc in each diagram below, giving your answer correct to 2d.p.



2. Calculate the perimeter of each sector in Question 1. Giving your answers correct to 2 d.p.

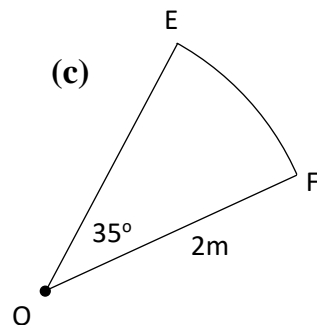
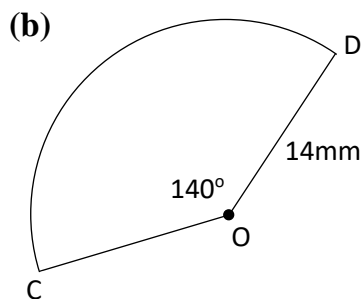
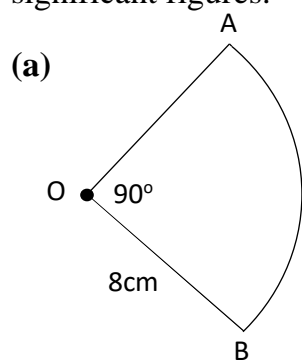
3. Find the length of the minor arc AB in each of the following circles, giving your answers correct to 2 d.p.



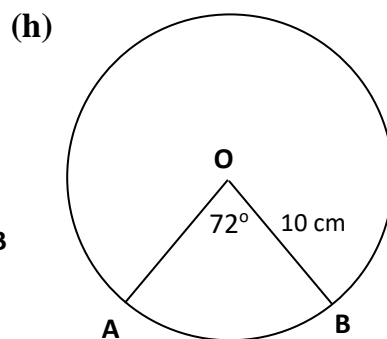
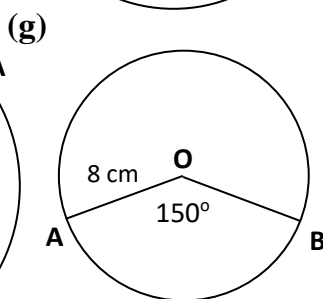
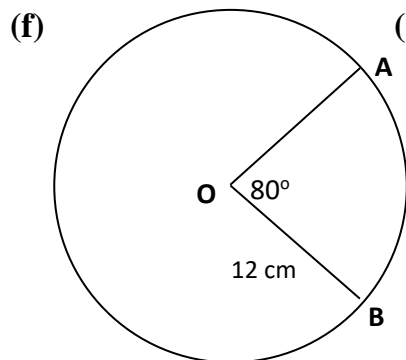
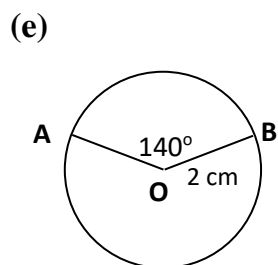
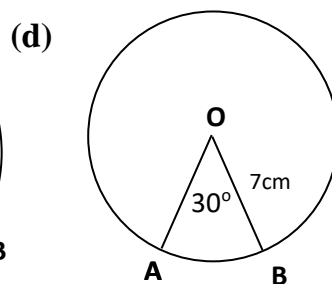
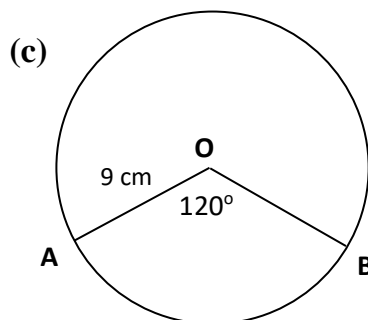
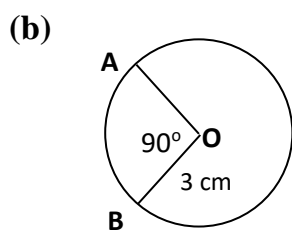
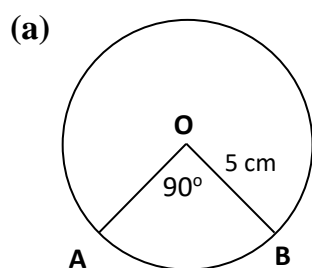
4. Calculate the length of the major arc in the circles shown in Question 3, giving your answers correct to 2 d.p.

Exercise 2

1. Calculate the area of the sector in each diagram below, giving your answer correct to 3 significant figures.



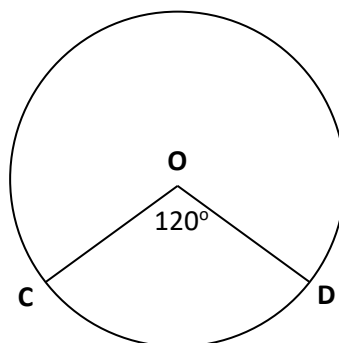
2. Calculate the area of minor sector OAB in the circles shown below, giving your answers correct to 3 significant figures.



3. Calculate the area of the major sector for the circles in Question 2, giving your answers correct to 3 significant figures.

4. The length of minor arc CD is 7.33 cm.

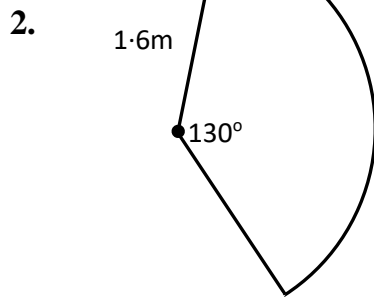
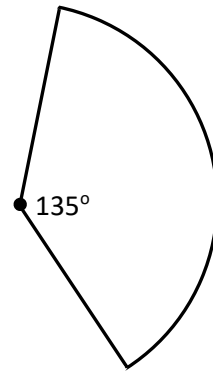
Calculate the area of the circle.



Exercise 3

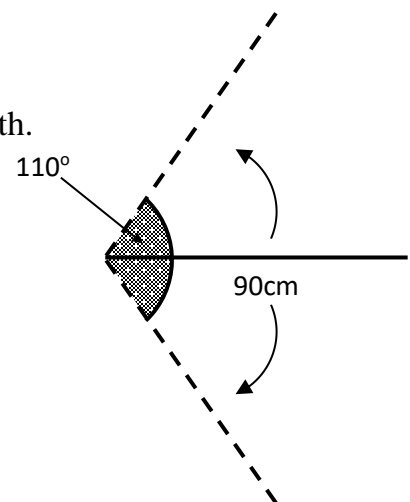
Give your answers correct to 3 significant figures unless otherwise stated.

1. Calculate the area of the sector shown in the diagram,
given that it has radius 6.8cm .



A table is in the shape of a sector of a circle with radius 1.6m .
The angle at the centre is 130° as shown in the diagram.
Calculate the perimeter of the table.

3. The door into a restaurant kitchen swings backwards and forwards through 110° .
The width of the door is 90cm .
Calculate the area swept out by the door as it swings back and forth.



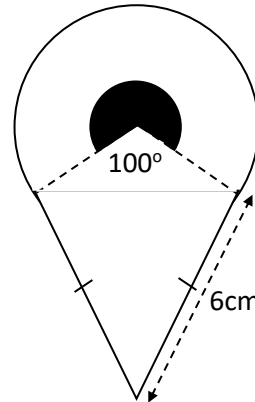
4. The YUMMY ICE CREAM Co uses this logo.



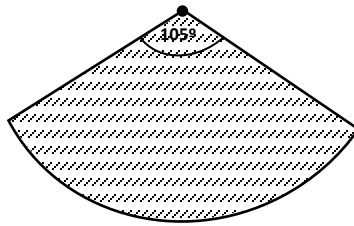
It is made up from an isosceles triangle and a sector of a circle as shown in the diagram.

- The equal sides of the triangle are 6cm
- The radius of the sector is 3.3cm.

Calculate the perimeter of the logo.



5. A sensor on a security system covers a horizontal area in the shape of a sector of a circle of radius 3.5m.



The sensor detects movement in an area with an angle of 105° .

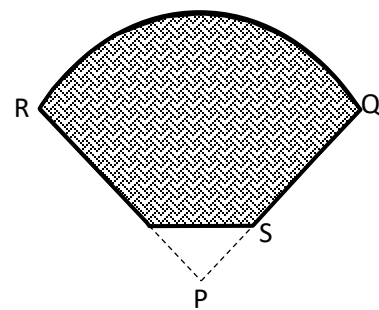
Calculate the area covered by the sensor.

6. A biscuit is in the shape of a sector of a circle with triangular part removed as shown in the diagram.

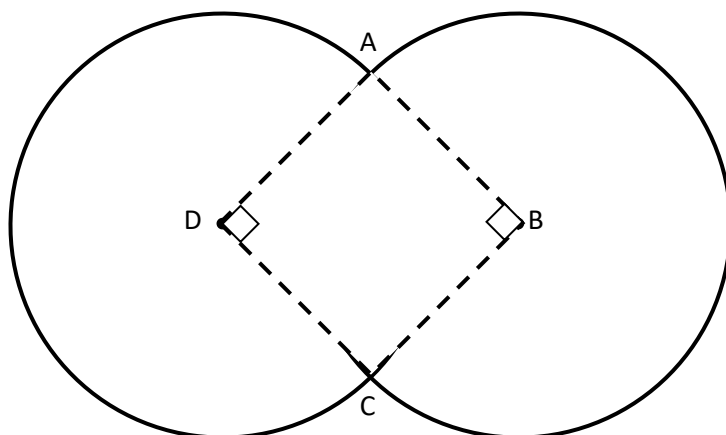
The radius of the circle, PQ, is 7 cm and PS = 1.5 cm.

Angle QPR = 80° .

Calculate the area of the biscuit.



7. Two congruent circles overlap to form the symmetrical shape shown below. Each circle has a diameter of 12 cm and have centres at B and D.

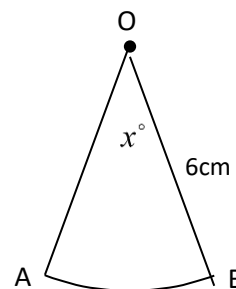


Calculate the area of the shape.

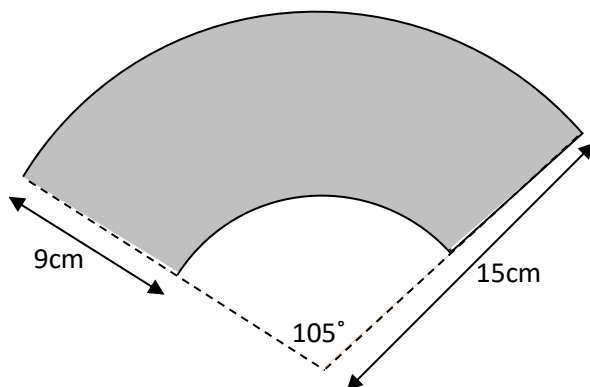
8. A sector of a circle with radius 6cm is shown opposite.

Angle $AOB = x^\circ$

If the exact **area** of the sector is 4π square centimetres, calculate the size of the angle marked x .

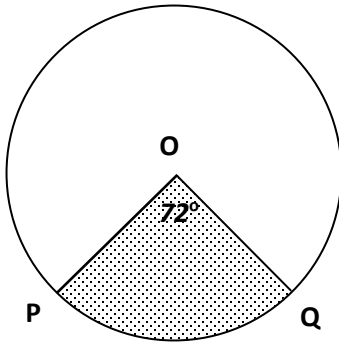


9. A hand fan is made of wooden slats with material on the outer edge.



- Calculate the area of material needed for the hand fan.
- Calculate the perimeter of the shaded area in the diagram above.

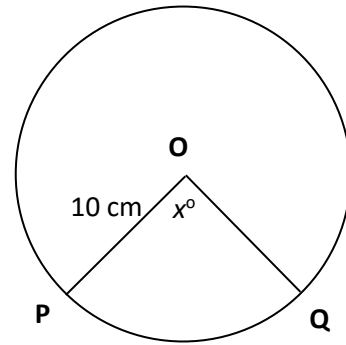
10.



The area of the shaded sector is 5.024 cm^2 .

Calculate the area of the circle.

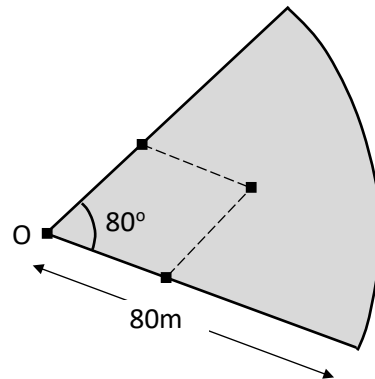
11. The area sector OPQ is 78.5 cm^2 .
Calculate the size of angle x° of the circle.



12. A school baseball field is in the shape of a sector of a circle as shown.

Given that O is the centre of the circle, calculate:

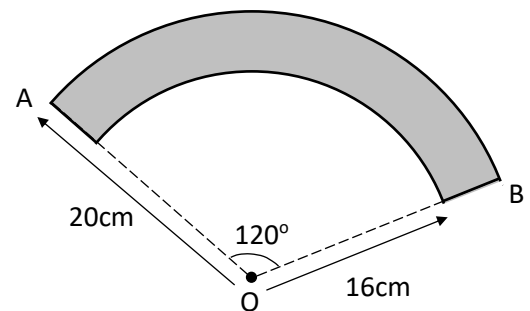
- (a) the perimeter of the playing field;
(b) the area of the playing field.



13. In the diagram opposite, O is the centre of two concentric circles with radii 16cm and 20cm as shown.

Angle $AOB = 120^\circ$. Calculate:

- a) The **perimeter** of the shaded shape.
b) The shaded area.



14. A Japanese paper fan is fully opened when angle $PQR = 150^\circ$ as shown.

- (a) Using the dimensions shown in *diagram 1*, calculate the approximate area of paper material in the fan.

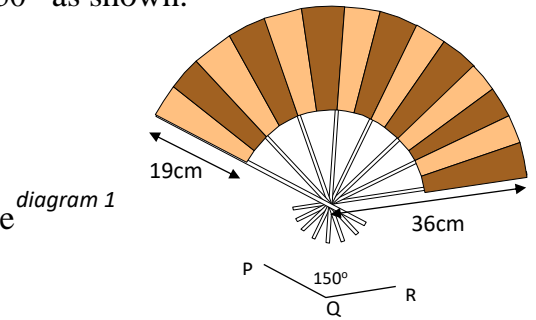
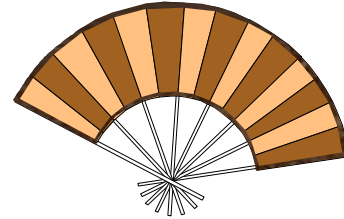


diagram 2

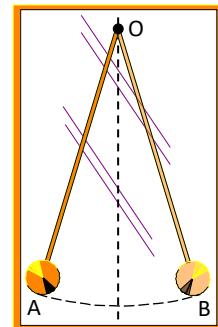


- (b) Decorative silk bands are placed along the edges as shown in *diagram 2*, calculate the approximate total length of this silk edging strip.

15. A grandfather clock has a pendulum which travels along an arc of a circle, centre O.

- The arm length of the pendulum is 60cm.
- The pendulum swings from position OA to OB.
- The length of the arc AB is 21cm.

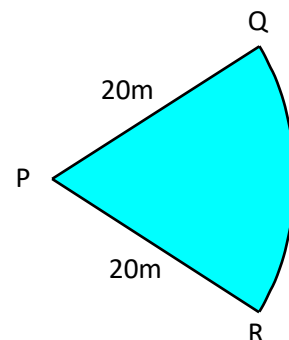
Calculate the size of angle AOB to the nearest degree.



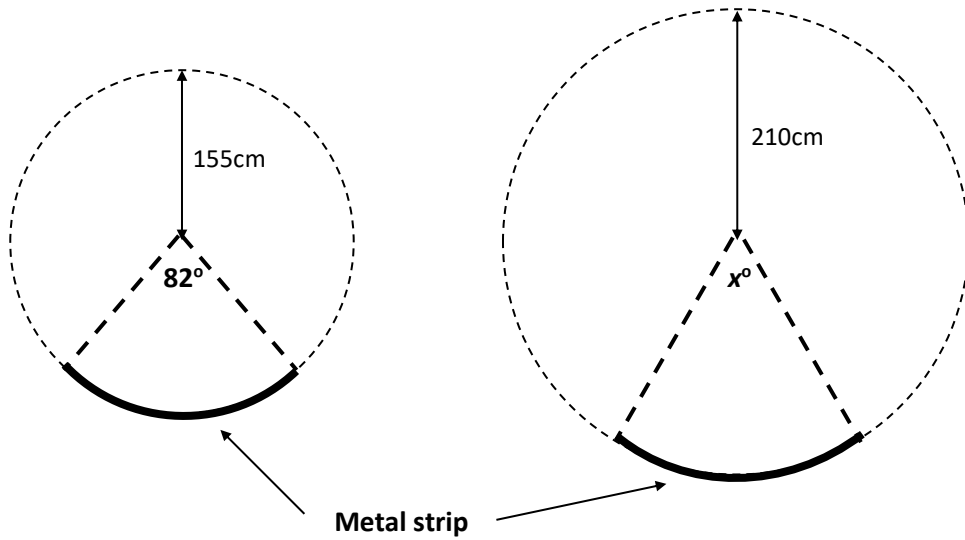
16. The shape opposite is the sector of a circle, centre P, radius 20m.

The area of the sector is 251.2 square metres.

Find the length of the arc QR.



17. A metal strip has been moulded into an arc of a circle of radius 155 centimetres which subtends an angle of 82° at the centre of the circle as shown in the diagram below.



The **same strip** of metal has now been remoulded to form an arc of a circle of radius 210 centimetres as shown.

Calculate the size of x , the angle now subtended by the metal strip.

18. Draw a diagram to help you answer these questions.

- (a) A circle, centre O, has an arc PQ of length 40cm.

If the diameter of the circle is 80cm, calculate the size of angle POQ correct to 1 d.p.

- (b) A circle, centre O, has a sector EOF with an area of 50cm^2 .

If the radius of the circle is 8cm, calculate the size of angle EOF correct to 1 d.p.

- (c) An arc AB on a circle, centre O, has a length of 16mm.

If angle AOB = 75° , calculate the radius of this circle.

- (d) A sector of a circle has an area of 12cm^2 . If the angle at the centre is 60° , calculate the diameter of the circle correct to 2-decimal places.

Answers

Exercise 1

- | | | | | | | | |
|-----------|------------|---------|------------|---------|------------|---------|--------------------|
| 1. | (a) | 12.57cm | (b) | 34.21mm | (c) | 1.22m | |
| 2. | (a) | 28.57cm | (b) | 62.21mm | (c) | 5.22m | |
| 3. | (a) | 7.85cm | (b) | 4.71cm | (c) | 18.85cm | (d) 3.67cm |
| | (e) | 4.89cm | (f) | 16.76cm | (g) | 20.94cm | (h) 12.57cm |
| 4. | (a) | 23.55cm | (b) | 14.13cm | (c) | 37.70cm | (d) 40.37cm |
| | (e) | 7.68cm | (f) | 58.64cm | (g) | 29.32cm | (h) 50.28cm |

Exercise 2

- | | | | | | | | |
|-----------|------------|---------------------|------------|---------------------|------------|---------------------|--------------------------------|
| 1. | (a) | 50.2cm ² | (b) | 239mm ² | (c) | 1.22m ² | |
| 2. | (a) | 19.6cm ² | (b) | 7.07cm ² | (c) | 84.8cm ² | (d) 12.8cm ² |
| | (e) | 4.88cm ² | (f) | 100cm ² | (g) | 83.7cm ² | (h) 62.8cm ² |
| 3. | (a) | 58.9cm ² | (b) | 21.2cm ² | (c) | 170cm ² | (d) 141cm ² |
| | (e) | 7.68cm ² | (f) | 352cm ² | (g) | 117cm ² | (h) 251cm ² |
| 4. | | 12.8cm ² | | | | | |

Exercise 3

- | | | | | | | | | | |
|------------|---------------------|---------------------|---------------------|--------------------|---------------------|---------------------|------------|------------|--------------------|
| 1. | 54.4cm ² | 2. | 6.83m | 3. | 7770cm ² | 4. | 27cm | 5. | 11.2m ² |
| 6. | 33.1cm ² | 7. | 206 cm ² | 8. | 40° | | | | |
| 9. | (a) | 173cm ² | (b) | 56.5cm | 10. | 25.1cm ² | 11. | 90° | |
| 12. | (a) | 272m | (b) | 4470m ² | 13. | (a) | 83.4cm | (b) | 151cm ² |
| 14. | (a) | 1320cm ² | (b) | 177cm | 15. | 20° | 16. | 25.1cm | |
| 17. | | 60.5° | | | | | | | |
| 18. | (a) | 57.3° | (b) | 89.6° | (c) | 12.2mm | (d) | 9.58cm | |