



Expanding Brackets

Expressions & Formulae

SPTA Mathematics - Topic Questions with Notes



Single Brackets

At National 4, you learnt to multiply brackets (sometimes referred to as “expanding brackets”). At that level, there was only ever a number in front of a bracket.

Example 1 – numbers in front of brackets

Multiply out the brackets and simplify: $2(3x + 5) + 4(2x - 1)$

Solution

$$\begin{aligned} & 2(3x + 5) + 4(2x - 1) \\ &= 2 \times 3x + 2 \times 5 + 4 \times 2x + 4 \times (-1) \\ &= 6x + 10 + 8x - 4 \quad \text{(multiplying out both brackets)} \\ &= \underline{14x + 6} \quad \text{(collecting like terms)} \end{aligned}$$

At National 5, there may be letters in front of a bracket. There may also be *both* letters and numbers. The rule remains the same: multiply everything inside the bracket by the number(s) and/or letter(s) outside the bracket.

Examples 2 – letters in front of brackets

Expand the brackets: (the word “expand” makes no difference to the question)

(a) $x(x + 5)$

(b) $2a(3a + 4b)$

Solution

$$\begin{aligned} & x(x + 5) \\ &= x \times x + x \times 5 \\ &= \underline{x^2 + 5x} \end{aligned}$$

$$\begin{aligned} & 2a(3a + 4b) \\ &= 2a \times 3a + 2a \times 4b \\ &= \underline{6a^2 + 8ab} \end{aligned}$$

Exercise 1

1. Multiply out the brackets:

- | | | | | | | | |
|-----------|------------|------------|------------|------------|------------|------------|------------|
| a) | $3(x - 5)$ | (b) | $5(y + 7)$ | (c) | $8(a + 6)$ | (d) | $6(3 + t)$ |
| e) | $x(x + 9)$ | (f) | $y(3 - y)$ | (g) | $b(b - 4)$ | (h) | $p(5 + p)$ |
| i) | $a(b + c)$ | (j) | $x(x - y)$ | (k) | $p(q - r)$ | (l) | $a(a + x)$ |

2. Expand the brackets:

- | | | | | | | | |
|-----------|--------------|------------|--------------|------------|---------------|------------|--------------|
| a) | $4(2a + 5)$ | (b) | $7(3y - 4)$ | (c) | $2(12x + 11)$ | (d) | $9(4c - 7)$ |
| e) | $2a(a + 3)$ | (f) | $5x(x - 8)$ | (g) | $10y(3 - y)$ | (h) | $3t(t + 6)$ |
| i) | $3x(2x - 9)$ | (j) | $2y(7 - 5y)$ | (k) | $4b(3b - 8)$ | (l) | $5x(5x + 4)$ |

3. Expand and simplify:

- | | | | | | |
|-----------|-------------------|------------|-------------------|------------|------------------|
| a) | $3(3a - 1) + 2a$ | (b) | $2(5x + 3) - 3x$ | (c) | $8(b + 2) - 9$ |
| d) | $4(2h - 1) + 7$ | (e) | $5(3 - 4x) + 11x$ | (f) | $3(2c + 1) - 8$ |
| g) | $2(4t + 3) - 10t$ | (h) | $p(p + q) - 3pq$ | (i) | $7(1 - 3c) - 10$ |
| j) | $3 + 2(2x + 5)$ | (k) | $7a + 3(2a - 3)$ | (l) | $5 - 2(2x - 7)$ |
| m) | $6 + 5(3y - 2)$ | (n) | $9b - 2(4b - 1)$ | (o) | $8 - 3(5x + 7)$ |
| p) | $12x - 4(4x - 5)$ | (q) | $3c + 5(1 - 2c)$ | (r) | $7 - 2(5a - 12)$ |

Double Brackets

To multiply out double brackets, you have to multiply every term in the first bracket by *every* term in the second bracket. **You always need to simplify your answers – be very careful with negative signs.**

Example 1

Multiply out the brackets: $(x - 7)(x - 9)$

Solution

$$\begin{aligned}(x - 7)(x - 9) &= x \times x + x \times (-9) + (-7) \times x + (-7) \times (-9) \\&= x^2 - 9x - 7x + 63 \\&= \underline{x^2 - 16x + 63}\end{aligned}$$

Example 2

Multiply out the brackets: $(2y + 3)(y - 4)$

Solution

$$\begin{aligned}(2y + 3)(y - 4) &= 2y \times y + 2y \times (-4) + 3 \times y + 3 \times (-4) \\&= 2y^2 - 8y + 3y - 12 \\&= \underline{2y^2 - 5y - 12}\end{aligned}$$

Exercise 2

1. Multiply out the brackets:

a) $(x + 2)(x + 3)$

(b) $(y + 5)(y + 2)$

(c) $(a + 4)(a + 6)$

d) $(b + 3)(b + 4)$

(e) $(x + 9)(x + 5)$

(f) $(s + 3)(s + 8)$

g) $(y + 7)(y + 4)$

(h) $(b + 3)(b + 3)$

(i) $(c + 6)(c + 7)$

j) $(a + 8)(a + 4)$

(k) $(y + 4)(y + 2)$

(l) $(x + 9)(x + 8)$

m) $(p + 12)(p + 7)$

(n) $(c + 5)(c + 6)$

(o) $(t + 7)(t + 9)$

p) $(x + 4)(x + 9)$

(q) $(y + 12)(y + 5)$

(r) $(a + 11)(a + 9)$

2. Multiply out the brackets:

a) $(x - 1)(x - 5)$

(b) $(c - 4)(c - 2)$

(c) $(y - 3)(y - 7)$

d) $(b - 6)(b - 8)$

(e) $(x - 5)(x - 2)$

(f) $(s - 8)(s - 5)$

g) $(y - 2)(y - 9)$

(h) $(a - 4)(a - 4)$

(i) $(t - 3)(t - 6)$

j) $(x - 6)(x - 5)$

(k) $(b - 5)(b - 3)$

(l) $(c - 10)(c - 4)$

m) $(a - 3)(a - 9)$

(n) $(y - 8)(y - 7)$

(o) $(x - 12)(x - 3)$

p) $(s - 4)(s - 7)$

(q) $(d - 1)(d - 15)$

(r) $(b - 10)(b - 1)$

3. Multiply out the brackets:

a) $(x - 1)(x + 5)$

(b) $(a + 3)(a - 7)$

(c) $(t - 5)(t + 4)$

d) $(y + 8)(y - 4)$

(e) $(c + 2)(c - 7)$

(f) $(x - 6)(x + 1)$

g) $(b - 2)(b + 9)$

(h) $(p - 10)(p + 2)$

(i) $(y - 8)(y + 7)$

j) $(z + 4)(z - 6)$

(k) $(x + 1)(x - 1)$

(l) $(a + 2)(a - 15)$

m) $(c - 3)(c + 3)$

(n) $(p - 7)(p + 1)$

(o) $(b + 10)(b - 5)$

When squaring a bracket, it is important to realise that (for example) $(x + 3)^2$ is NOT $x^2 + 9$. Instead you have to rewrite $(x + 3)^2$ as $(x + 3)(x + 3)$ and then to multiply out the brackets using the double bracket method.

Example 3 – Squaring brackets

Expand the bracket: $(2a + 1)^2$

Solution

$$\begin{aligned}(2a + 1)^2 &= (2a + 1)(2a + 1) \\ &= 2a \times 2a + 2a \times 1 + 1 \times 2a + 1 \times 1 \\ &= 4a^2 + 2a + 2a + 1 \\ &= \underline{4a^2 + 4a + 1}\end{aligned}$$

Exercise 3

1. Multiply out the brackets:

- | | | | | | | | |
|-----------|--------------|------------|--------------|------------|--------------|------------|--------------|
| a) | $(x + 3)^2$ | (b) | $(w - 2)^2$ | (c) | $(a - 5)^2$ | (d) | $(c + 8)^2$ |
| e) | $(y - 4)^2$ | (f) | $(a + 6)^2$ | (g) | $(b + 1)^2$ | (h) | $(s + 7)^2$ |
| i) | $(b - 9)^2$ | (j) | $(x - 10)^2$ | (k) | $(c - 1)^2$ | (l) | $(y - 3)^2$ |
| m) | $(2x - 1)^2$ | (n) | $(5y + 2)^2$ | (o) | $(3x + 4)^2$ | (p) | $(4b - 5)^2$ |

2. Multiply out the brackets:

- | | | | | | |
|-----------|------------------|------------|------------------|------------|------------------|
| a) | $(a + b)(c + d)$ | (b) | $(2 + x)(3 + y)$ | (c) | $(a + 4)(b + 5)$ |
| d) | $(p - q)(r - s)$ | (e) | $(1 - a)(7 - b)$ | (f) | $(c - 6)(d + 8)$ |

In some questions, the second bracket may have three terms in it (a **trinomial**). In these examples, the basic method (multiply everything in the first bracket by everything in the second bracket) is still the same, however you will have to do more multiplications. The answer will often involve a term in x^3 (because $x \times x^2 = x^3$).

Example 4 – three terms in one bracket

Multiply out the bracket: $(x + 4)(3x^2 - 2x + 5)$

Solution

$$\begin{aligned} & (x + 4)(3x^2 - 2x + 5) \\ &= x \times 3x^2 + x \times (-2x) + x \times 5 + 4 \times 3x^2 + 4 \times (-2x) + 4 \times 5 \\ &= 3x^3 - 2x^2 + 5x + 12x^2 - 8x + 20 \\ &= \underline{3x^3 + 10x^2 - 3x + 20} \end{aligned}$$

Exercise 4

1. Multiply out the brackets:

- a)** $x(x^2 + x - 1)$ **(b)** $3(2x^2 - 3x + 5)$ **(c)** $x(3x^2 - 5x + 8)$
d) $2x(x^2 + 2x + 3)$ **(e)** $-5(x^2 - 8x + 2)$ **(f)** $x(x^2 - 4x - 7)$

2. Multiply out the brackets and simplify:

- | | |
|------------------------------------|-------------------------------------|
| a) $(x + 2)(x^2 + 3x + 1)$ | (b) $(x + 5)(x^2 + 4x + 2)$ |
| c) $(x + 1)(x^2 + 5x + 4)$ | (d) $(x + 3)(x^2 + x + 5)$ |
| e) $(x + 8)(x^2 + 2x + 3)$ | (f) $(x + 4)(x^2 + 7x + 6)$ |
| g) $(x + 12)(x^2 + x + 7)$ | (h) $(x + 10)(x^2 + 3x + 9)$ |
| i) $(x + 9)(x^2 + 12x + 7)$ | (j) $(x + 7)(x^2 + 9x + 1)$ |
| k) $(x + 3)(x^2 - 5x + 2)$ | (l) $(x - 6)(x^2 - x + 11)$ |
| m) $(x + 2)(x^2 - 8x + 3)$ | (n) $(x + 5)(x^2 - 6x + 7)$ |
| o) $(x + 10)(x^2 + 3x - 6)$ | (p) $(x + 9)(x^2 + 5x - 6)$ |
| q) $(x + 11)(x^2 + x - 2)$ | (r) $(x + 7)(x^2 + 8x - 3)$ |

3. Multiply out the brackets and simplify:

- | | |
|-----------------------------------|--------------------------------------|
| a) $(x - 1)(x^2 + x + 1)$ | (b) $(x - 7)(x^2 + 3x + 5)$ |
| c) $(x - 2)(x^2 + 4x + 3)$ | (d) $(x - 4)(x^2 + 6x + 1)$ |
| e) $(x - 3)(x^2 - 2x + 5)$ | (f) $(x - 6)(x^2 - 5x + 2)$ |
| g) $(x - 4)(x^2 - x + 2)$ | (h) $(x - 1)(x^2 - 2x + 7)$ |
| i) $(x - 9)(x^2 + 3x - 2)$ | (j) $(x - 5)(x^2 + 8x + 6)$ |
| k) $(x - 8)(x^2 + x - 7)$ | (l) $(x - 3)(x^2 + 9x - 12)$ |
| m) $(x - 5)(x^2 - 4x - 1)$ | (n) $(x - 10)(x^2 - 3x - 8)$ |
| o) $(x - 6)(x^2 - 7x - 2)$ | (p) $(x - 1)(x^2 - 17x - 13)$ |

4. Multiply out the brackets and simplify:

a) $(x + 5)(2x^2 + 4x + 9)$

(b) $(x - 3)(5x^2 + x + 6)$

c) $(x - 2)(6x^2 - 5x + 7)$

(d) $(x + 7)(3x^2 + 9x - 2)$

e) $(x - 4)(5x^2 - x - 8)$

(f) $(x + 1)(7x^2 - 2x + 11)$

g) $(2x + 1)(3x^2 + 4x + 1)$

(h) $(3x + 4)(x^2 - 11x + 2)$

i) $(5x - 2)(2x^2 + 3x - 7)$

(j) $(4x - 3)(3x^2 - 5x - 4)$

Mixed Exercise

1. Expand and simplify each of the following expressions:

a) $3(x - 4) + (x + 2)^2$

(b) $(2x - 1)(x + 3) + 2x(x - 3)$

c) $(2x + 3)^2 - 4(x + 1)$

(d) $-(x + 2)^2 + 4x$

e) $-3(2x - 1)^2 + 12x^2$

(f) $(x - 3)(x + 2) - (x + 4)^2$

g) $3x(x - 4) - (x + 2)(x - 4)$

(h) $(x + 2)^2 + (2x - 1)^2 - (x + 3)$

i) $(2x - 3)^2 - 4(x - 3)(2x + 1)$

(j) $3x(x + 3)^2 + 2x(x - 3)$

k) $2x(x^2 - x + 2) + (x - 3)^2$

(l) $(x - 1)^2 - x(x + 1)^2$

Answers

Exercise 1

1. **a)** $3x - 15$ **(b)** $5y + 35$ **(c)** $8a + 48$ **(d)** $18 + 6t$
 e) $x^2 + 9x$ **(f)** $3y - y^2$ **(g)** $b^2 - 4b$ **(h)** $5p + p^2$
 i) $ab + ac$ **(j)** $x^2 - xy$ **(k)** $pq - pr$ **(l)** $a^2 + ax$
2. **a)** $8a + 20$ **(b)** $21y - 28$ **(c)** $24x + 22$ **(d)** $36c - 63$
 e) $2a^2 + 6a$ **(f)** $5x^2 - 40x$ **(g)** $30y - 10y^2$ **(h)** $3t^2 + 18t$
 i) $6x^2 - 27x$ **(j)** $14y - 10y^2$ **(k)** $12b^2 - 32b$ **(l)** $25x^2 + 20x$
3. **a)** $11a - 3$ **(b)** $7x + 6$ **(c)** $8b + 7$ **(d)** $8h + 3$
 e) $15 - 9x$ **(f)** $6c - 5$ **(g)** $-2t + 6$ **(h)** $p^2 - 2pq$
 i) $-3 - 21c$ **(j)** $13 + 4x$ **(k)** $13a - 9$ **(l)** $19 - 4x$
 m) $-4 + 15y$ **(n)** $b + 2$ **(o)** $-13 - 15x$ **(p)** $-4x + 20$
 q) $-7c + 5$ **(r)** $31 - 10a$

Exercise 2

1. **a)** $x^2 + 5x + 6$ **(b)** $y^2 + 7y + 10$ **(c)** $a^2 + 10a + 24$
 d) $b^2 + 7b + 12$ **(e)** $x^2 + 14x + 45$ **(f)** $s^2 + 11s + 24$
 g) $y^2 + 11y + 28$ **(h)** $b^2 + 6b + 9$ **(i)** $c^2 + 13c + 42$
 j) $a^2 + 12a + 32$ **(k)** $y^2 + 6y + 8$ **(l)** $x^2 + 17x + 72$
 m) $p^2 + 19p + 84$ **(n)** $c^2 + 11c + 30$ **(o)** $t^2 + 16t + 63$
 p) $x^2 + 13x + 36$ **(q)** $y^2 + 17y + 60$ **(r)** $a^2 + 20a + 19$

2. **a)** $x^2 - 6x + 5$ **(b)** $c^2 - 6c + 8$ **(c)** $y^2 - 10y + 21$
- d)** $b^2 - 14b + 48$ **(e)** $x^2 - 7x + 10$ **(f)** $s^2 - 13s + 40$
- g)** $y^2 - 11y + 18$ **(h)** $a^2 - 8a + 16$ **(i)** $t^2 - 9t + 18$
- j)** $x^2 - 11x + 30$ **(k)** $b^2 - 8b + 15$ **(l)** $c^2 - 14c + 40$
- m)** $a^2 - 12a + 27$ **(n)** $y^2 - 15y + 56$ **(o)** $x^2 - 15x + 36$
- p)** $s^2 - 11s + 28$ **(q)** $d^2 - 16d + 15$ **(r)** $b^2 - 11b + 10$

3. **a)** $x^2 + 4x - 5$ **(b)** $a^2 - 4a - 21$ **(c)** $t^2 - t - 20$
- d)** $y^2 + 4y - 32$ **(e)** $c^2 - 5c - 14$ **(f)** $x^2 - 5x - 6$
- g)** $b^2 + 7b - 18$ **(h)** $p^2 - 8p - 20$ **(i)** $y^2 - y - 56$
- j)** $z^2 - 2z - 24$ **(k)** $x^2 - 1$ **(l)** $a^2 - 13a - 30$
- m)** $c^2 - 9$ **(n)** $p^2 - 6p - 7$ **(o)** $b^2 + 5b - 50$

Exercise 3

1. **a)** $x^2 + 6x + 9$ **(b)** $w^2 - 4w + 4$ **(c)** $a^2 - 10a + 25$
- d)** $c^2 + 16c + 64$ **(e)** $y^2 - 8y + 16$ **(f)** $a^2 + 12a + 36$
- g)** $b^2 + 2b + 1$ **(h)** $s^2 + 14s + 49$ **(i)** $b^2 - 18b + 81$
- j)** $x^2 - 20x + 100$ **(k)** $c^2 - 2c + 1$ **(l)** $y^2 - 6y + 9$
- m)** $4x^2 - 4x + 1$ **(n)** $25y^2 + 20y + 4$ **(o)** $9x^2 + 24x + 16$
- p)** $16b^2 - 40b + 25$
2. **a)** $ac + bc + ad + bd$ **(b)** $6 + 3x + 2y + xy$ **(c)** $ab + 4b + 5a + 20$
- d)** $pr - qr - ps + qs$ **(e)** $7 - 7a - b + ab$ **(f)** $cd - 6d + 8c - 48$

Exercise 4

1. **a)** $x^3 + x^2 - x$ **(b)** $6x^2 - 9x + 15$ **(c)** $3x^3 - 5x^2 + 8x$
 d) $2x^3 + 4x^2 + 6x$ **(e)** $-5x^2 + 40x - 10$ **(f)** $x^3 - 4x^2 - 7x$

2. **a)** $x^3 + 5x^2 + 7x + 2$ **(b)** $x^3 + 9x^2 + 22x + 10$
 c) $x^3 + 6x^2 + 9x + 4$ **(d)** $x^3 + 4x^2 + 8x + 15$
 e) $x^3 + 10x^2 + 19x + 24$ **(f)** $x^3 + 11x^2 + 34x + 24$
 g) $x^3 + 13x^2 + 19x + 84$ **(h)** $x^3 + 13x^2 + 39x + 90$
 i) $x^3 + 21x^2 + 115x + 63$ **(j)** $x^3 + 16x^2 + 64x + 7$
 k) $x^3 - 2x^2 - 13x + 6$ **(l)** $x^3 - 7x^2 + 17x - 66$
 m) $x^3 - 6x^2 - 13x + 6$ **(n)** $x^3 - x^2 - 23x + 35$
 o) $x^3 + 13x^2 + 24x - 60$ **(p)** $x^3 + 14x^2 + 39x - 54$
 q) $x^3 + 12x^2 + 9x - 22$ **(r)** $x^3 + 15x^2 + 53x - 21$

3. **a)** $x^3 - 1$ **(b)** $x^3 - 4x^2 - 16x - 35$
 c) $x^3 + 2x^2 - 5x - 6$ **(d)** $x^3 + 2x^2 - 23x - 4$
 e) $x^3 - 5x^2 + 11x - 15$ **(f)** $x^3 - 11x^2 + 32x - 12$
 g) $x^3 - 5x^2 + 6x - 8$ **(h)** $x^3 - 3x^2 + 9x - 7$
 i) $x^3 - 6x^2 - 29x + 18$ **(j)** $x^3 + 3x^2 - 34x - 30$
 k) $x^3 - 7x^2 - 15x + 56$ **(l)** $x^3 + 6x^2 - 39x + 36$
 m) $x^3 - 9x^2 + 19x + 5$ **(n)** $x^3 - 13x^2 + 22x + 80$
 o) $x^3 - 13x^2 + 40x + 12$ **(p)** $x^3 - 18x^2 + 4x + 13$

4. **a)** $2x^3 + 14x^2 + 29x + 45$ **(b)** $5x^3 - 14x^2 + 3x - 18$
 c) $6x^3 - 17x^2 + 17x - 14$ **(d)** $3x^3 + 30x^2 + 61x - 14$
 e) $5x^3 - 21x^2 - 4x + 32$ **(f)** $7x^3 + 5x^2 + 9x + 11$
 g) $6x^3 + 11x^2 + 6x + 1$ **(h)** $3x^3 - 29x^2 - 38x + 8$
 i) $10x^3 + 11x^2 - 41x + 14$ **(j)** $12x^3 - 29x^2 - x + 12$

Mixed Exercise

1. **a)** $x^2 + 7x - 8$ **(b)** $4x^2 - x - 3$ **(c)** $4x^2 + 8x + 5$
 d) $-x^2 - 4$ **(e)** $12x - 3$ **(f)** $-9x - 22$
 g) $2x^2 - 10x + 8$ **(h)** $5x^2 - x + 2$ **(i)** $21 + 8x - 4x^2$
 j) $3x^3 + 20x^2 + 21x$ **(k)** $2x^3 - x^2 - 2x + 9$ **(l)** $1 - 3x - x^2 - x^3$