# N5 Vectors

Applications

`SPTA Mathematics – Topic Questions with Notes

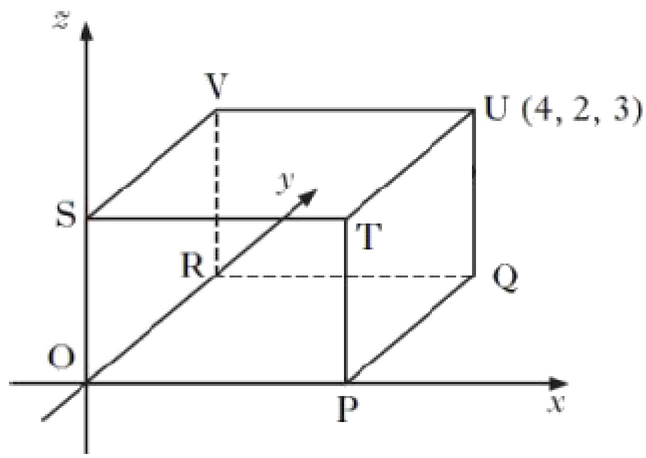
## 3d Coordinates

We can extend the traditional 2 dimensional Cartesian diagram into 3 dimensions by adding a third axis called the $z$ axis which is at right angles to both the $x$ axis and $y$ axis.

Example (diagram adapted from 2010 Higher exam paper)

**In the diagram on the right, the point U has coordinates (4, 2, 3).**

**State the coordinates of P, V and Q.**

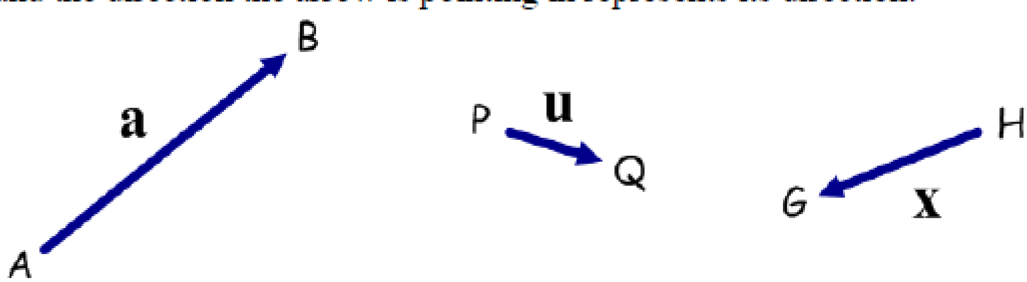



**Solution**

P is the point (4, 0, 0)
V is the point (0, 2, 3)
Q is the point (4, 2, 0)

## Definition of a Vector

A vector is a quantity that has both size and direction. It can be represented as an arrow, where the length of the arrow represents the vector's size (known as a **directed line segment**); and the direction the arrow is pointing in represents its direction.



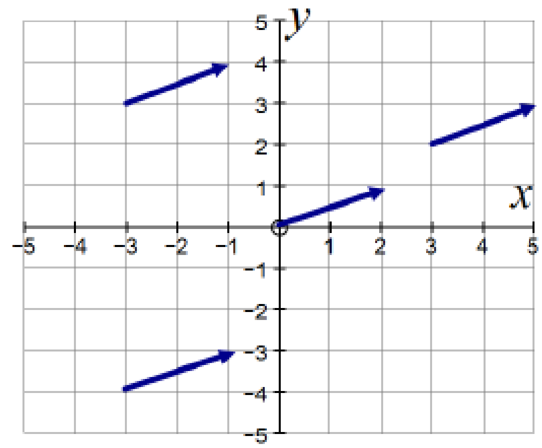

There are two ways of naming a vector:

- One way is to represent a vector by a single letter. For instance in the three examples above, the three vectors are called **a**, **u** and **x**. In print, we use a bold type letter to represent a vector, e.g. **a**. When handwriting, we use underlining in place of bold, e.g. a.
- Another way is to represent a vector using the start and end points. For instance the first vector above goes from A to B, and so it could be represented as $\overrightarrow{AB}$. The middle vector could be represented $\overrightarrow{PQ}$, and the final one would be represented $\overrightarrow{HG}$ (not $\overrightarrow{GH}$).

**Components of a vector**

A vector is described in terms of its **components**, which describe how far the vector moves in the $x$ and $y$ directions respectively. For a three-dimensional vector there would be three components, with the third component referring to the $z$ direction.

With vectors, the important thing is how the vector moves, not where it begins or starts. All the vectors in the diagram on the right represent the same vector **a**, as both move 2 units in the $x$ direction and 1 unit in the $y$ direction:




The components of a vector are written in a column. A 2-d vector would be written $\begin{pmatrix} x \\ y \end{pmatrix}$. A 3-d vector would be $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$. For example the vector $\begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix}$ is a 3-d vector moving 1 unit in the $x$ direction, 2 units in the $y$ direction and –3 units in the $z$ direction.

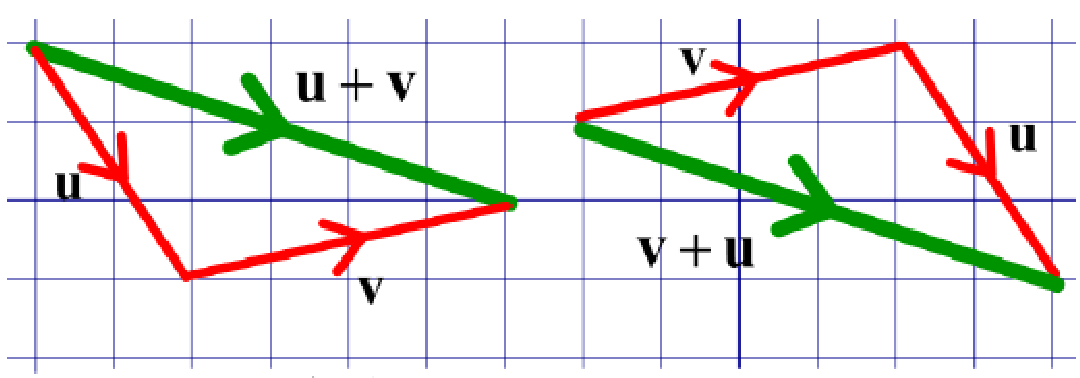
## Adding Vectors

We can add vectors to create a **resultant vector**. We can do this in two ways:

- **numerically** by adding their components.

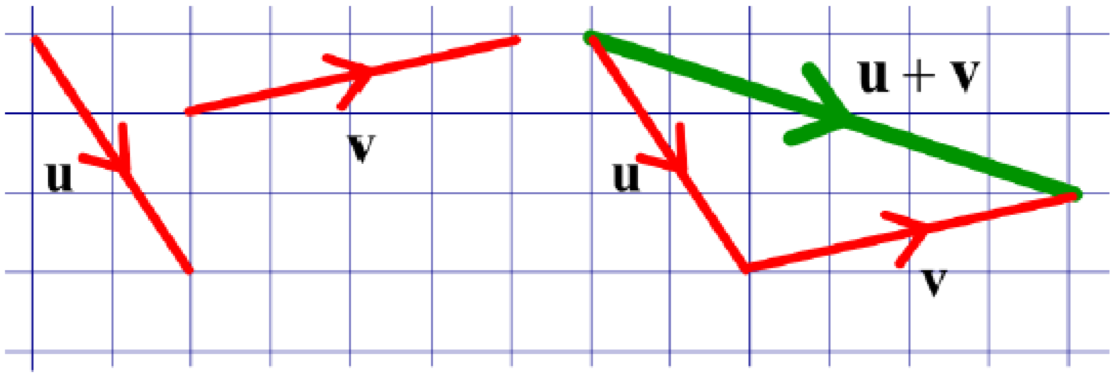
If we have two vectors, $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix}$, then the resultant vector $\mathbf{a} + \mathbf{b} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$.

- **In a diagram** by joining them ‘nose to tail’.




$$\mathbf{u} + \mathbf{v} = \begin{pmatrix} 6 \\ -2 \end{pmatrix} \qquad \mathbf{v} + \mathbf{u} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$$

It does not matter which order you add vectors in: $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$ .




$$\mathbf{u} = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad \mathbf{v} = \begin{pmatrix} 4 \\ 1 \end{pmatrix} \qquad \mathbf{u} + \mathbf{v} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$$

In real life, resultant vectors can be used to work out what the combined effect of more than one force pulling on an object will be.

<u>Example 1 – numerical</u>

**Three forces act on an object. The three forces are represented by the vectors a, b and c, where:**

$$\mathbf{a} = \begin{pmatrix} -1 \\ 3 \\ 2 \end{pmatrix} \quad \mathbf{b} = \begin{pmatrix} 0 \\ -5 \\ 6 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix}$$

**Find the resultant force.**

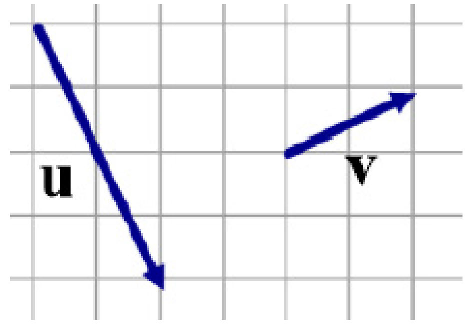
**Solution**

The resultant force is given b y $\mathbf{a} + \mathbf{b} + \mathbf{c}$ .

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \begin{pmatrix} (-1) + 0 + 4 \\ 3 + (-5) + 0 \\ 2 + 6 + 2 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ 10 \end{pmatrix}$$

Example 2 – from a diagram

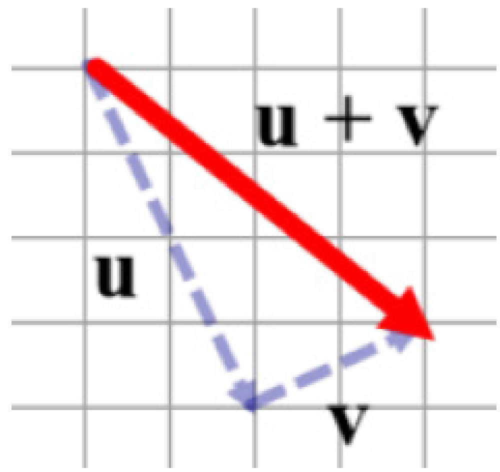
**The diagram on the right shows two directed line segments u and v. Draw the resultant vector u + v**




**Solution**

Too add the vectors, we join the 'tail' of **v** to the 'nose' (pointed end) of **u**:


We can now draw in the vector **u** + **v** going from the 'tail' of **u** to the 'nose' of **v**.




**Vector Pathways**

We can use the rules of adding and taking away vectors to express a vector $\overrightarrow{AB}$ in a diagram as a combination of other, known, vectors.

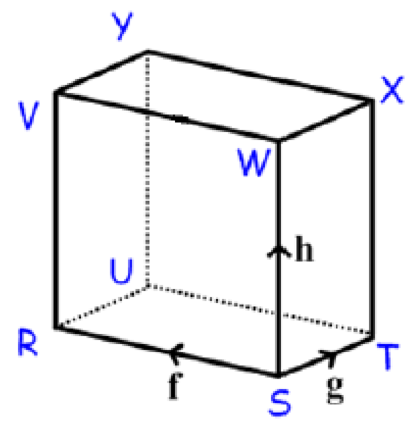
To do this, we identify a route, or pathway, between A and B, in which each step of the route can be expressed in terms of one of the other known pathways. We can choose *any* route we like, and the final answer, when simplified, will always be the same.

**Fact:** If we move backwards along a vector, we take that vector away.

Example 3 – vector pathways

**The diagram shows a cuboid. $\overrightarrow{SR}$ represents vector f, $\overrightarrow{ST}$ represents vector g and $\overrightarrow{SW}$ represents vector h.**

**Express $\overrightarrow{SU}$ and $\overrightarrow{TV}$ in terms of f, g and h.**




**Solution**

For $\overrightarrow{SU}$ :

Step one – identify a pathway from S to U.

One possible pathway is $\overrightarrow{SR}, \overrightarrow{RU}$

Step two – express each part of the pathway in terms of a known vector

$\overrightarrow{SR} = \mathbf{f}$, $\overrightarrow{RU} = \mathbf{g}$

Therefore $\overrightarrow{SU} = \mathbf{f} + \mathbf{g}$

For $\overrightarrow{TV}$ :

Step one – identify a pathway from T to V.

One possible pathway is $\overrightarrow{TS}, \overrightarrow{SW}, \overrightarrow{WV}$

Step two – express each part of the pathway in terms of a known vector

$\overrightarrow{TS}$ = backwards along **g**, $\overrightarrow{SW} = \mathbf{h}$, $\overrightarrow{WV} = \mathbf{f}$

Therefore $\overrightarrow{TV} = -\mathbf{g} + \mathbf{h} + \mathbf{f}$ (or $\mathbf{f} - \mathbf{g} + \mathbf{h}$ or any other equivalent expression)

## Multiplying a vector by a scalar

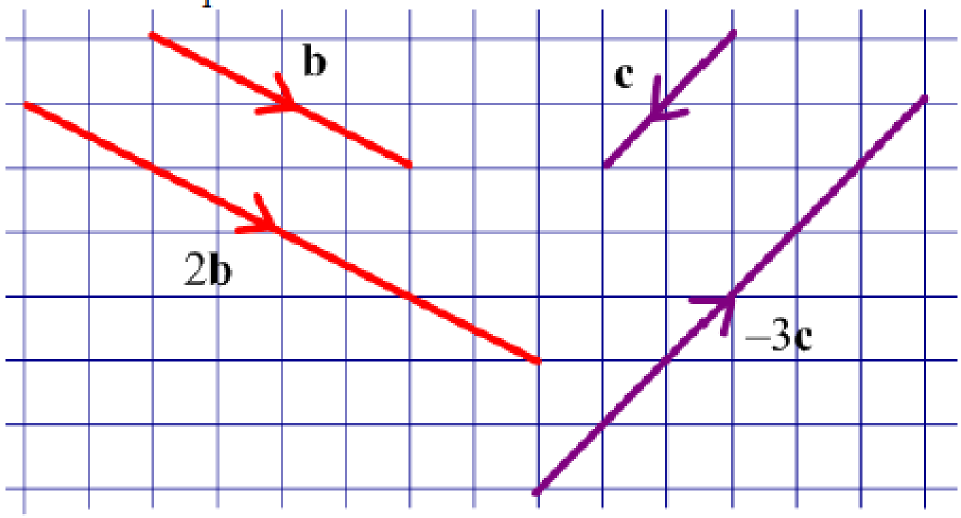
A **scalar** is a quantity that has size but no direction. 'Normal' numbers such as 2, –5 or 14·1 are scalars.

We can multiply a vector by a scalar in two ways:

- **numerically** by multiplying each component of the vector.

If we have a vectors, $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ and a scalar $k$, then $k\mathbf{a} = \begin{pmatrix} ka_1 \\ ka_2 \end{pmatrix}$.

- **In a diagram** by making a vector shorter or longer by a scale factor of $k$. The vector will still point in the same direction, but will be $k$ times longer (or shorter if $k < 1$). If $k$ is negative, the vector will point 'backwards'.




$$\mathbf{b} = \begin{pmatrix} 4 \\ -2 \end{pmatrix} \quad 2\mathbf{b} = \begin{pmatrix} 8 \\ -4 \end{pmatrix} \quad \mathbf{c} = \begin{pmatrix} -2 \\ -2 \end{pmatrix} \quad -3\mathbf{c} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$$

<u>Example</u>

**Given that** $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$ **and** $\mathbf{c} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$, **calculate** $3\mathbf{a} - 2\mathbf{b} + 4\mathbf{c}$

**Solution**

$$\begin{aligned} 3\mathbf{a} - 2\mathbf{b} + 4\mathbf{c} &= 3\begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} - 2\begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix} + 4\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} \\ &= \begin{pmatrix} 9 \\ 6 \\ -12 \end{pmatrix} - \begin{pmatrix} 10 \\ 0 \\ 2 \end{pmatrix} + \begin{pmatrix} 4 \\ -8 \\ 12 \end{pmatrix} \\ &= \begin{pmatrix} 9 - 10 + 4 \\ 6 - 0 + (-8) \\ (-12) - 2 + 12 \end{pmatrix} \\ &= \begin{pmatrix} 3 \\ -2 \\ -2 \end{pmatrix} \end{aligned}$$

# Magnitude

The **magnitude** of a vector is the length of a vector. The magnitude of the vector **a** is written using two vertical lines, $|\mathbf{a}|$.

The magnitude of a two-dimensional vector is found using a version of Pythagoras' Theorem:

**Formula.** This formula is <u>not given</u> on the National 5 Mathematics exam paper.

The magnitude of the vector $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$ is given by $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2}$

There is also a three-dimensional equivalent of Pythagoras' theorem that can be used to find the magnitude of a 3-d vector when its components are known.

The magnitude of the vector $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ is given by $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

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Example 1 – magnitude

**Calculate the magnitude of the vector** $\mathbf{x} = \begin{pmatrix} 2 \\ -5 \\ 1 \end{pmatrix}$

**Solution**

$$\begin{aligned} |\mathbf{x}| &= \sqrt{2^2 + (-5)^2 + 1^2} \\ &= \sqrt{4 + 25 + 1} \\ &= \sqrt{30} \text{ units} \end{aligned}$$

Example 2

**Given that** $\mathbf{a} = \begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix}$**,** $\mathbf{b} = \begin{pmatrix} 5 \\ 0 \\ 1 \end{pmatrix}$ **and** $\mathbf{c} = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$**, calculate** $|2\mathbf{a} - 3\mathbf{c}|$

**Solution**

$$2\mathbf{a} - 3\mathbf{c} = 2\begin{pmatrix} 3 \\ 2 \\ -4 \end{pmatrix} - 3\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$$

$$= \begin{pmatrix} 6 \\ 4 \\ -8 \end{pmatrix} - \begin{pmatrix} 3 \\ -6 \\ 9 \end{pmatrix}$$

$$= \begin{pmatrix} 3 \\ 10 \\ -17 \end{pmatrix}$$

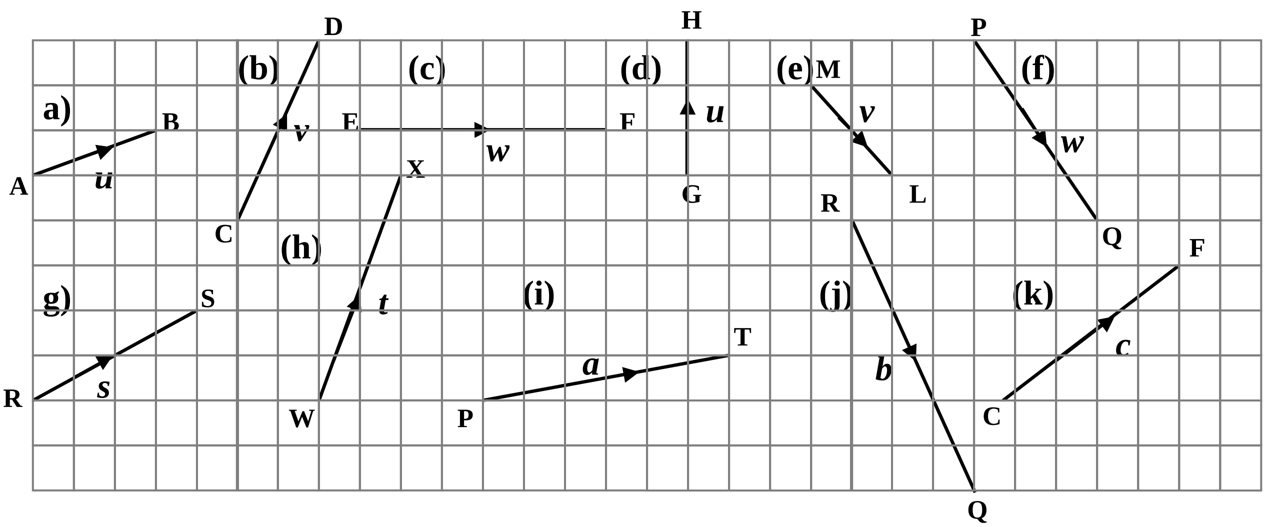
$$\text{so } |2\mathbf{a} - 3\mathbf{c}| = \sqrt{3^2 + 10^2 + (-17)^2}$$

$$= \sqrt{9 + 100 + 289}$$

$$= \sqrt{398}$$

## *Exercise 1*

**1.** Name the following vectors in 2 ways and write down the components:



**2.** Draw representations of the following vectors on squared paper.

**a)** $\boldsymbol{v} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$ **(b)** $\boldsymbol{w} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$ **(c)** $\boldsymbol{u} = \begin{pmatrix} -3 \\ 6 \end{pmatrix}$ **(d)** $\overrightarrow{AB} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$

**e)** $\overrightarrow{CD} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$ **(f)** $\overrightarrow{EF} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$ **(g)** $\boldsymbol{r} = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ **(h)** $\boldsymbol{p} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$

**i)** $\boldsymbol{q} = \begin{pmatrix} -5 \\ -6 \end{pmatrix}$ **(j)** $\overrightarrow{XY} = \begin{pmatrix} -4 \\ 3 \end{pmatrix}$ **(k)** $\overrightarrow{PQ} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$ **(l)** $\overrightarrow{ST} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

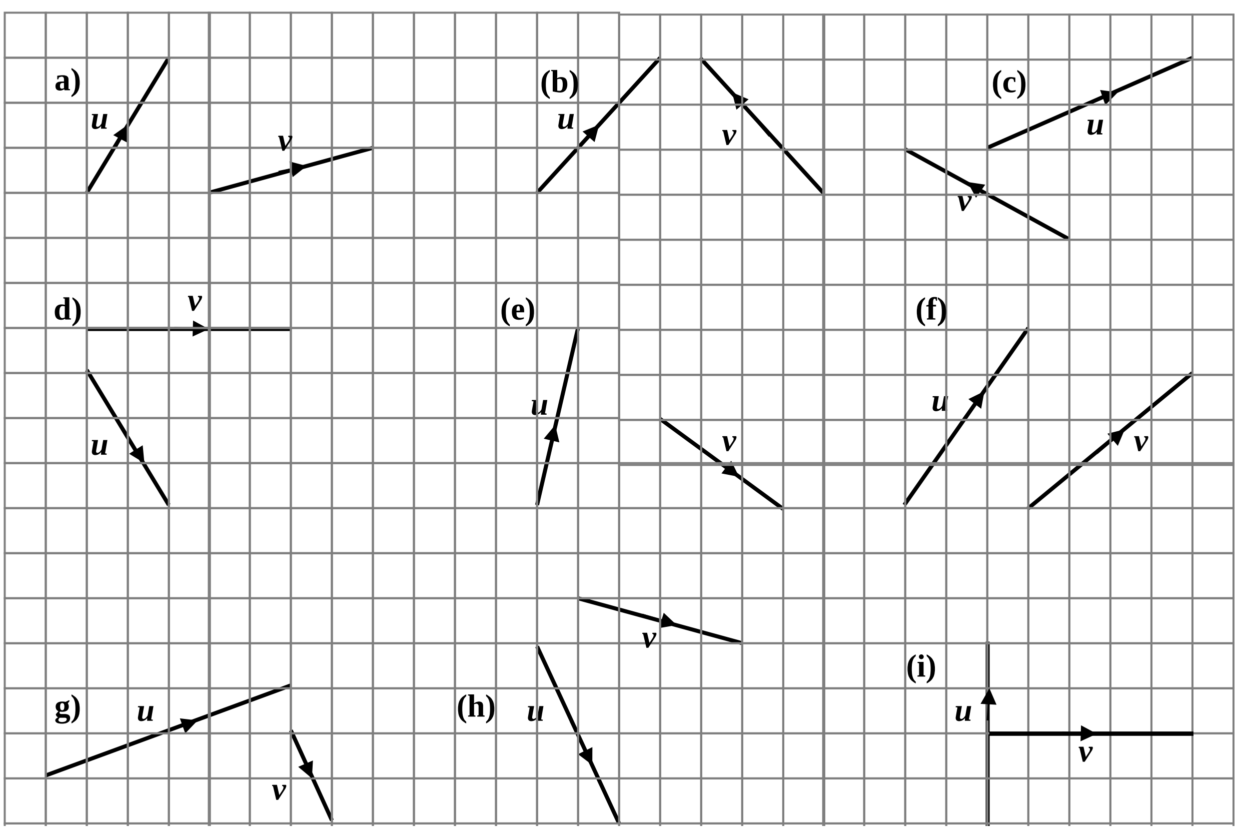
**3.** Calculate the magnitude of each of the vectors in questions 1 and 2 above leaving your answers as surds in their simplest form.

**4.** Find **a)** $\left|\begin{pmatrix} 3 \\ 4 \end{pmatrix}\right|$ **(b)** $\left|\begin{pmatrix} 7 \\ 24 \end{pmatrix}\right|$ **(c)** $\left|\begin{pmatrix} 12 \\ 5 \end{pmatrix}\right|$

**d)** $\left|\begin{pmatrix} -6 \\ -8 \end{pmatrix}\right|$ **(e)** $\left|\begin{pmatrix} -3 \\ 4 \end{pmatrix}\right|$ **(f)** $\left|\begin{pmatrix} 12 \\ -5 \end{pmatrix}\right|$

## *Exercise 2*

**1.** **(i)** Draw diagrams on squared paper to illustrate ***u*** + ***v*** for each pair of vectors given.
**(ii)** State the components of the resultant vector and calculate its magnitude leaving your answers as a surd in its simplest form



**2.** **i)** Draw diagrams on squared to illustrate ***a*** + ***b*** for each the following pairs of vectors.

**ii)** State the components of the resultant vector and calculate its magnitude.

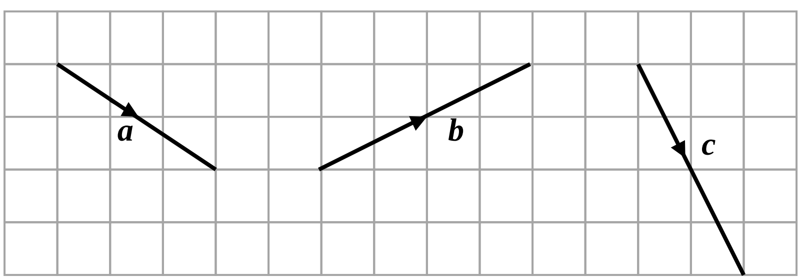
**a)** $\boldsymbol{a} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$ **(b)** $\boldsymbol{a} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} -9 \\ 3 \end{pmatrix}$

**c)** $\boldsymbol{a} = \begin{pmatrix} -4 \\ -2 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$ **(d)** $\boldsymbol{a} = \begin{pmatrix} 0 \\ -5 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} -3 \\ 3 \end{pmatrix}$

**e)** $\boldsymbol{a} = \begin{pmatrix} -6 \\ -4 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} -5 \\ 6 \end{pmatrix}$ **(f)** $\boldsymbol{a} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} 0 \\ -3 \end{pmatrix}$

**g)** $\boldsymbol{a} = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$ **(h)** $\boldsymbol{a} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} 2 \\ -4 \end{pmatrix}$

**3.** The diagram shows 3 vectors ***a***, ***b*** and ***c***.




**i)** Draw diagrams on squared paper to represent:

**ii)** For each resultant vector, state the components and calculate its magnitude correct to one decimal place.

**a)** ***a*** + ***b*** **(b)** ***a*** + ***c*** **(c)** ***b*** + ***c*** **(d)** (***a*** + ***b***) + ***c***

**e)** ***a*** + ( ***b*** + ***c***)

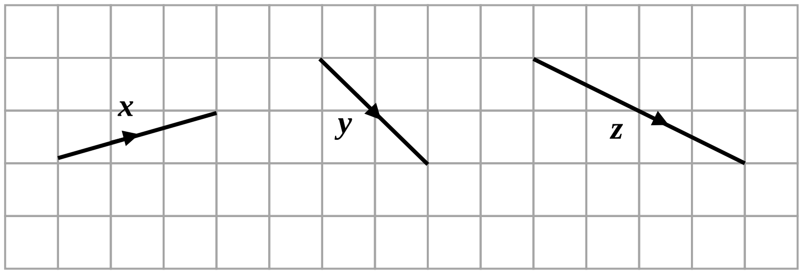
**4.** **i)** For the vectors in question 3 draw representations of these vectors.

**a)** 2***a*** **(b)** 3***b*** **(c)** 0·5***c*** **(d)** –2***b***

**e)** – 4***a*** **(f)** –***c*** **(g)** 3***a*** + 2***b*** **(h)** ***c*** + 4***a***

**ii)** State the components of each of the vectors above and calculate the magnitude leaving answers as a surd in its simplest form.

**5.** The diagram shows 3 vectors $\mathbf{x}$, $\mathbf{y}$ and $\mathbf{z}$.




**i)** Draw diagrams to represent:

**a)** $\mathbf{x} + \mathbf{y}$ **(b)** $\mathbf{x} + \mathbf{z}$ **(c)** $\mathbf{y} + \mathbf{z}$ **(d)** $(\mathbf{x} + \mathbf{y}) + \mathbf{z}$

**e)** $\mathbf{x} + (\mathbf{y} + \mathbf{z})$

**ii)** Calculate, correct to one decimal place:

**a)** $|\mathbf{x} + \mathbf{y}|$ **(b)** $|\mathbf{x} + \mathbf{z}|$ **(c)** $|\mathbf{y} + \mathbf{z}|$ **(d)** $|(\mathbf{x} + \mathbf{y}) + \mathbf{z}|$

**e)** $|\mathbf{x} + (\mathbf{y} + \mathbf{z})|$

**6.** For the vectors in question 5, calculate:

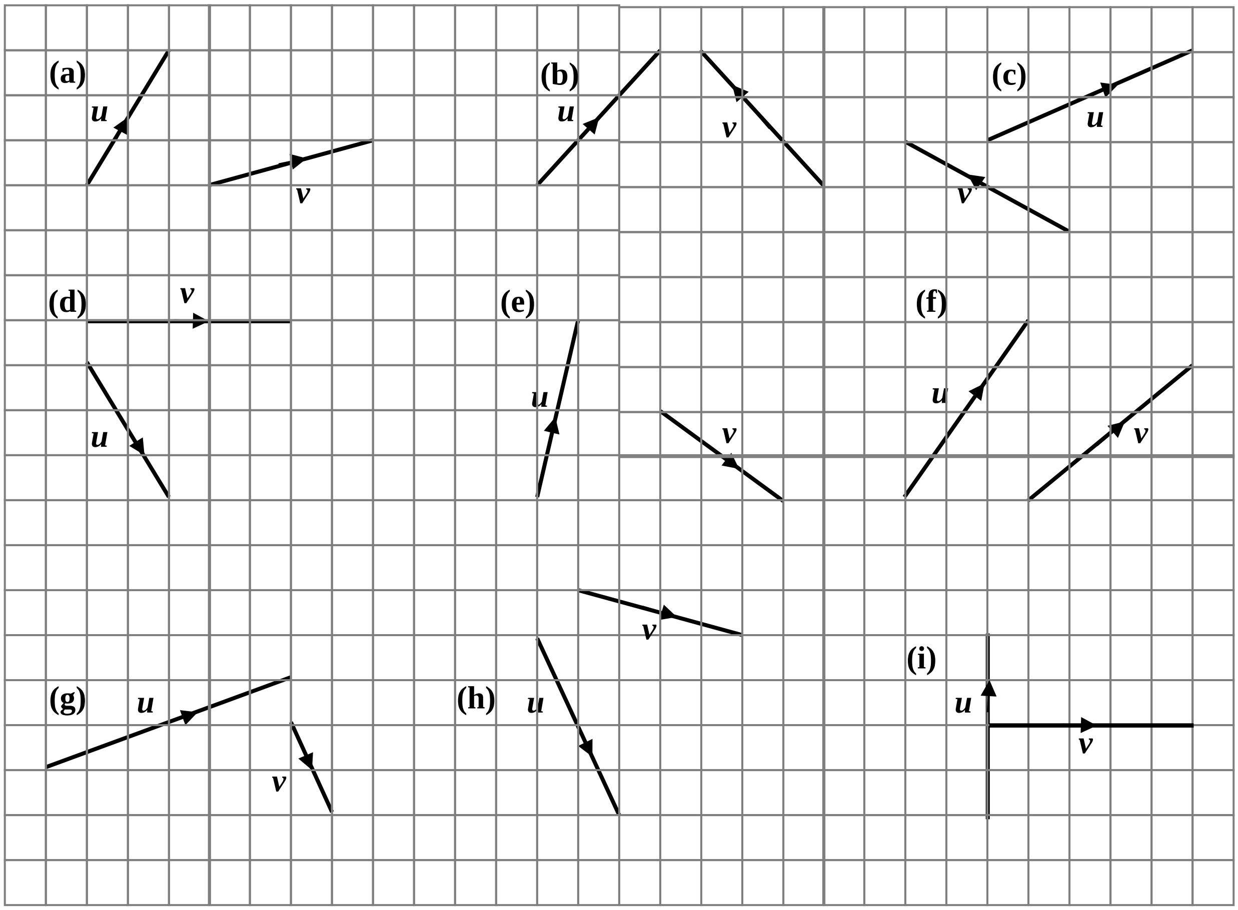
**a)** $|2\mathbf{x}|$ **(b)** $|3\mathbf{y}|$ **(c)** $|0 \cdot 5\mathbf{z}|$ **(d)** $|-2\mathbf{y}|$

**e)** $|-4\mathbf{x}|$ **(f)** $|-\mathbf{z}|$ **(g)** $|3\mathbf{x} + 2\mathbf{y}|$ **(h)** $|4\mathbf{y} + 3\mathbf{x}|$

*Exercise 3*

**1.** **i)** Draw diagrams on squared paper to illustrate ***u*** – ***v*** for each pair of vectors given.

**ii)** State the components of the resultant vector and calculate its magnitude leaving your answers as surds in their simplest form.




**2.** **i)** Draw diagrams on squared to illustrate ***a*** – ***b*** for each the following pairs of vectors.

**ii)** State the components of the resultant vector and calculate its magnitude correct to one decimal place.

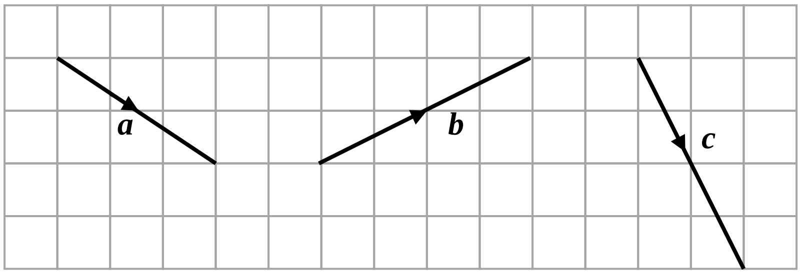
**a)** $\boldsymbol{a} = \begin{pmatrix} 9 \\ 7 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$ **(b)** $\boldsymbol{a} = \begin{pmatrix} -4 \\ -7 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}$ **(c)** $\boldsymbol{a} = \begin{pmatrix} 5 \\ 8 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

**d)** $\boldsymbol{a} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ **(e)** $\boldsymbol{a} = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} -2 \\ -6 \end{pmatrix}$ **(f)** $\boldsymbol{a} = \begin{pmatrix} 4 \\ -3 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

**g)** $\boldsymbol{a} = \begin{pmatrix} 0 \\ 7 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$ **(h)** $\boldsymbol{a} = \begin{pmatrix} 0 \\ -6 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$ **(i)** $\boldsymbol{a} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

**j)** $\boldsymbol{a} = \begin{pmatrix} 0 \\ 4 \end{pmatrix}$; $\boldsymbol{b} = \begin{pmatrix} -4 \\ 0 \end{pmatrix}$

**3.** The diagram shows 3 vectors $\boldsymbol{a}$, $\boldsymbol{b}$ and $\boldsymbol{c}$.




**i)** Draw diagrams on squared paper to represent:

**a)** $\boldsymbol{a} - \boldsymbol{b}$ **(b)** $\boldsymbol{a} - \boldsymbol{c}$ **(c)** $\boldsymbol{b} - \boldsymbol{c}$ **(d)** $(\boldsymbol{a} + \boldsymbol{b}) - \boldsymbol{c}$

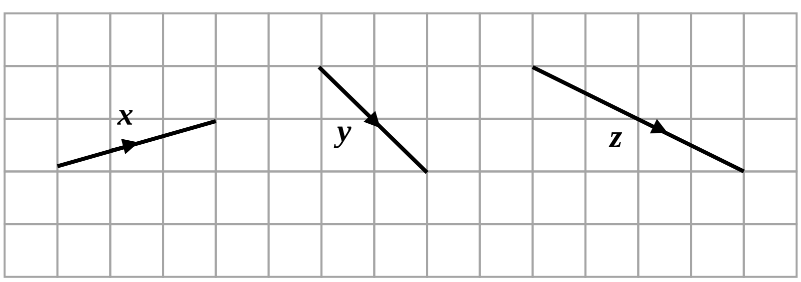
**e)** $\boldsymbol{a} - (\boldsymbol{b} - \boldsymbol{c})$

**ii)** Calculate, correct to two decimal places:

**a)** $|\boldsymbol{a} - \boldsymbol{b}|$ **(b)** $|\boldsymbol{a} - \boldsymbol{c}|$ **(c)** $|\boldsymbol{b} - \boldsymbol{c}|$ **(d)** $|(\boldsymbol{a} + \boldsymbol{b}) - \boldsymbol{c}|$

**e)** $|\boldsymbol{a} - (\boldsymbol{b} - \boldsymbol{c})|$

**4.** The diagram shows 3 vectors $\boldsymbol{x}$, $\boldsymbol{y}$ and $\boldsymbol{z}$.



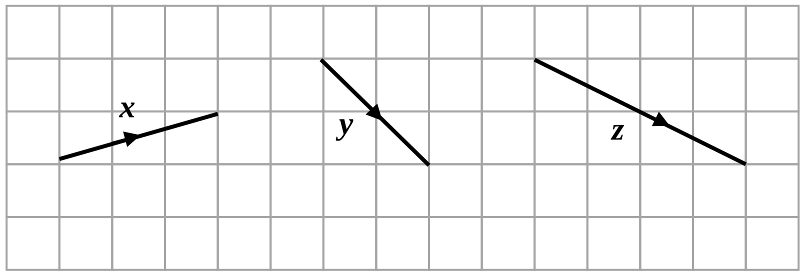

**i)** Draw diagrams to represent:

**ii)** For each resultant vector, state the components and calculate its magnitude correct to one decimal place.

**a)** $\boldsymbol{x} - \boldsymbol{y}$ **(b)** $\boldsymbol{x} - \boldsymbol{z}$ **(c)** $\boldsymbol{y} - \boldsymbol{z}$ **(d)** $(\boldsymbol{x} - \boldsymbol{y}) - \boldsymbol{z}$

**e)** $\boldsymbol{x} - (\boldsymbol{y} - \boldsymbol{z})$

**5.** The diagram shows 3 vectors ***x***, ***y*** and ***z***.




**i)** Draw diagrams on squared paper to show:

**ii)** State the components of each resultant vector above and calculate its magnitude correct to 3 significant figures.

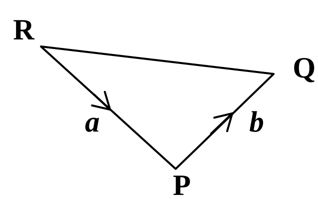
| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a)** | 2***x*** + ***y*** | **(b)** | 3***z*** + 2***y*** | **(c)** | 3***x*** + ***z*** | **(d)** | 2***z*** + 4***x*** |
| **e)** | 3***x*** – 4***y*** | **(f)** | 3***x*** – ***z*** | **(g)** | 3***y*** – 2***x*** | **(h)** | –3***y*** – 2***z*** (careful!) |

## *Exercise 4*

**1.** Express each of the following displacements in terms of vectors ***a*** and ***b***.

**(a)** $\overrightarrow{PQ}$ **(b)** $\overrightarrow{QP}$ **(c)** $\overrightarrow{PR}$

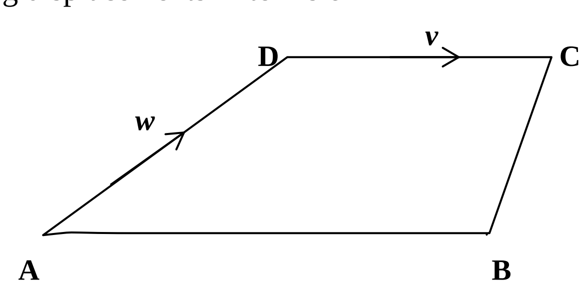
**(d)** $\overrightarrow{RQ}$ **(e)** $\overrightarrow{QR}$




**2.** In the diagram $\overrightarrow{AB} = 2\overrightarrow{DC}$. Express each of the following displacements in terms of vectors ***v*** and ***w***.

**(a)** $\overrightarrow{CD}$ **(b)** $\overrightarrow{CA}$ **(c)** $\overrightarrow{AB}$

**(d)** $\overrightarrow{CB}$ **(e)** $\overrightarrow{BD}$

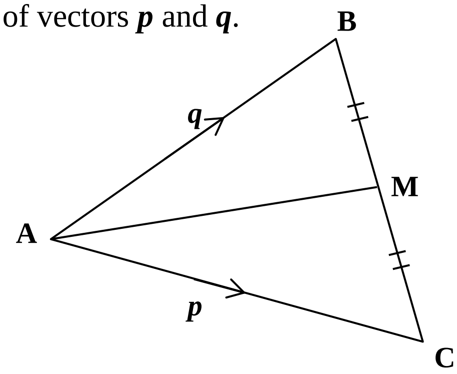


**3.** In the diagram 'M' is the mid – point of BC.

Express each of the following displacements in terms of vectors ***p*** and ***q***.

**(a)** $\overrightarrow{CB}$ **(b)** $\overrightarrow{BC}$ **(c)** $\overrightarrow{BM}$

**(d)** $\overrightarrow{AM}$

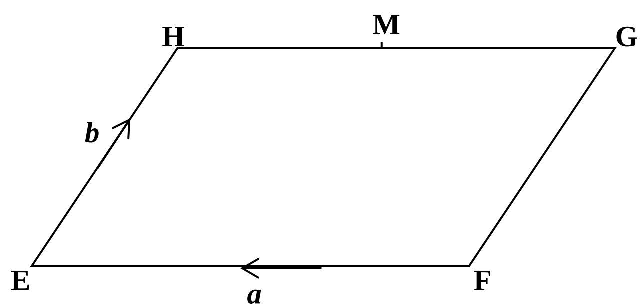



**4.** EFGH is a parallelogram. 'M' is the mid point of side HG.

Express each of the following displacements in terms of vectors ***a*** and ***b***.

**(a)** $\overrightarrow{FG}$ **(b)** $\overrightarrow{GH}$ **(c)** $\overrightarrow{GM}$

**(d)** $\overrightarrow{FM}$

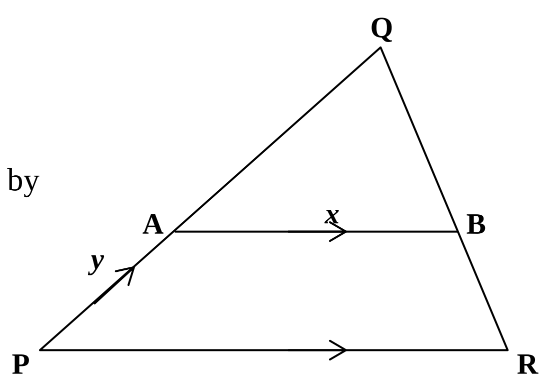



**5.** In the diagram AB is parallel to PR.

PA = 1 cm and PQ = 3 cm

Find in terms of ***x*** and/or ***y*** the vectors represented by

**(a)** AQ **(b)** QB



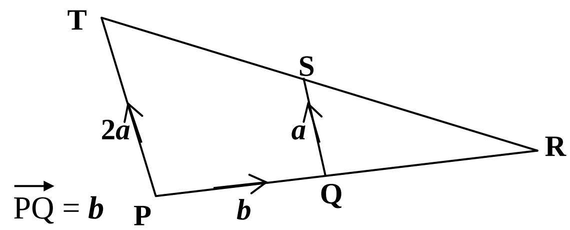
## *Exercise 5*

**1.** **a)** Express in terms of ***a*** and ***b***.

**(i)** $\overrightarrow{PS}$ **(ii)** $\overrightarrow{ST}$

**(b)** If $\overrightarrow{QR} = \frac{3}{2}\overrightarrow{PQ}$ , show that RS can be expressed as

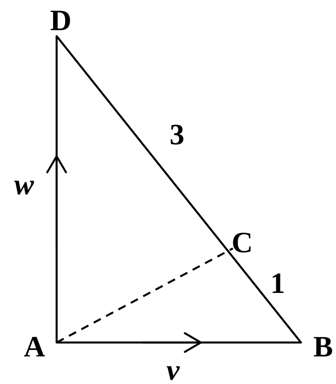
$\frac{1}{2}(2\boldsymbol{a} - 3\boldsymbol{b})$




**2.** Express in terms of vectors ***v*** and ***w***.

**a)** $\overrightarrow{BD}$ **(b)** $\overrightarrow{BC}$ **(c)** $\overrightarrow{AC}$

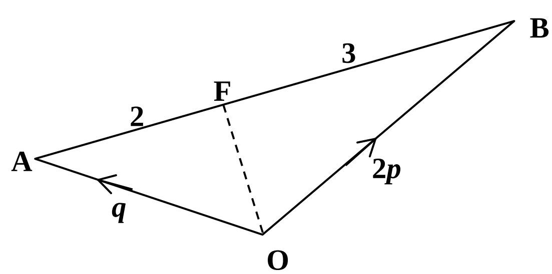
If $\boldsymbol{v} = \begin{pmatrix} 8 \\ 0 \end{pmatrix}$ and $\boldsymbol{w} = \begin{pmatrix} 0 \\ 12 \end{pmatrix}$, find the components of the displacement $\overrightarrow{AC}$.




**3.** Express in terms of ***p*** and ***q***.

**a)** $\overrightarrow{AB}$ **(b)** $\overrightarrow{AF}$ **(c)** $\overrightarrow{OF}$

If $\boldsymbol{p} = \begin{pmatrix} -10 \\ 5 \end{pmatrix}$ and $\boldsymbol{q} = \begin{pmatrix} 5 \\ 5 \end{pmatrix}$ find the components of $\overrightarrow{OF}$ and hence its magnitude correct to 1 decimal place .

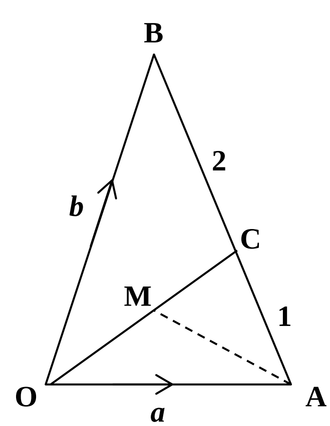



**4.** **a)** Express in terms of ***a*** and ***b***:-

**(i)** $\overrightarrow{AB}$ **(ii)** $\overrightarrow{AC}$ **(iii)** $\overrightarrow{OC}$

**b)** If M is the mid-point of OC show that:-

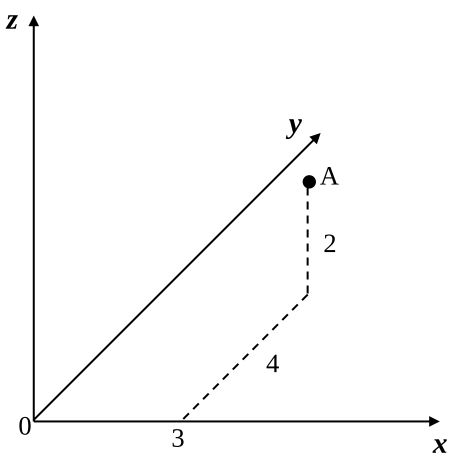
$\overrightarrow{AM} = \frac{1}{6}\boldsymbol{b} - \frac{2}{3}\boldsymbol{a} = \frac{1}{6}(\boldsymbol{b} - 4\boldsymbol{a})$



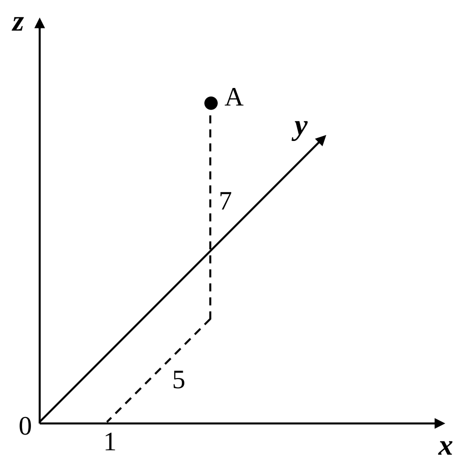
*Exercise 6*

**1.** For each diagram, write down the coordinates of the point A and the components of the vector $\overrightarrow{OA}$.

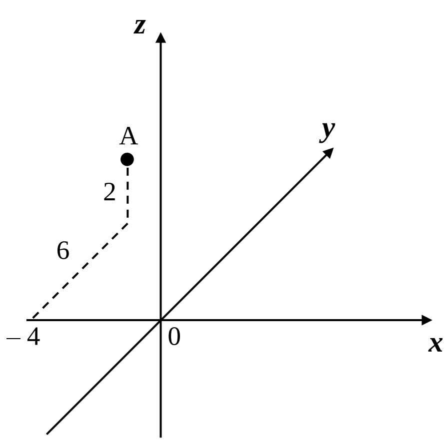
**a)**



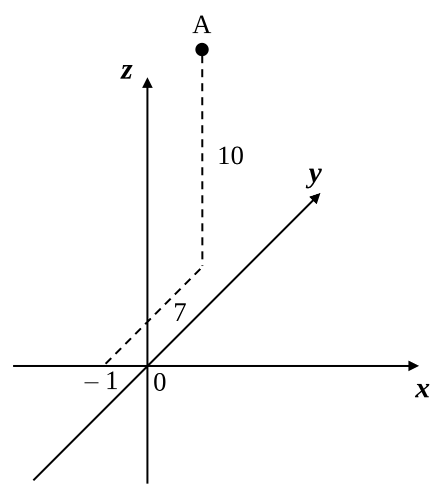

**(b)**



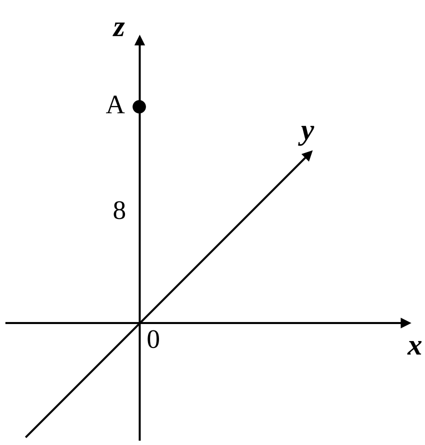

**c)**



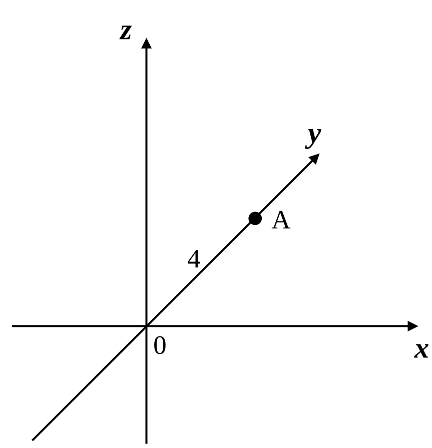

**(d)**



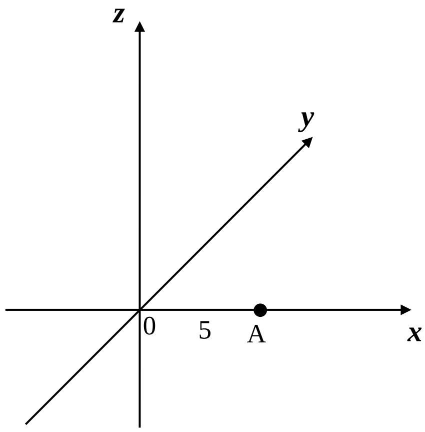

**e)**



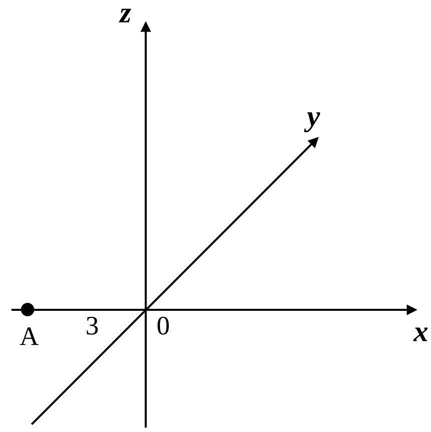

**(f)**



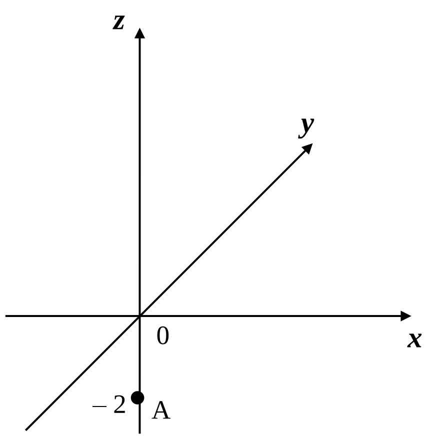
**g)**



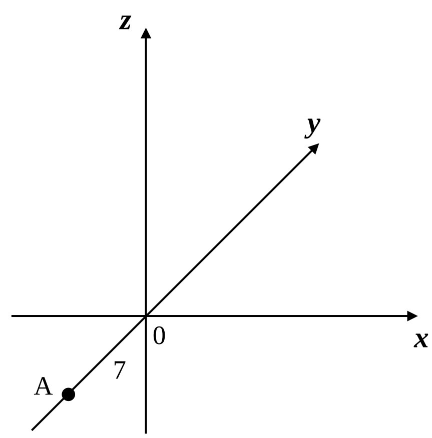

**(h)**



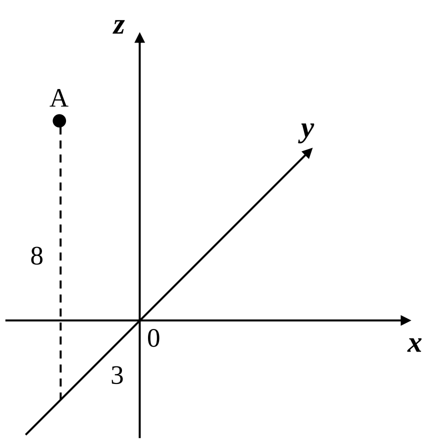

**i)**



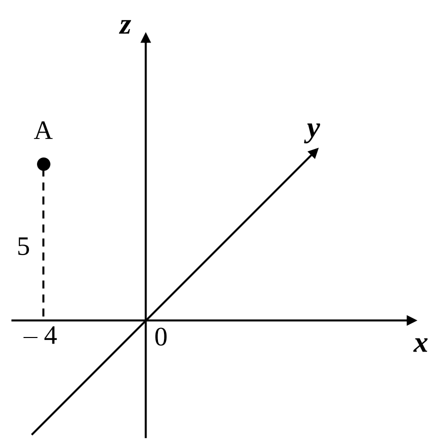

**(j)**



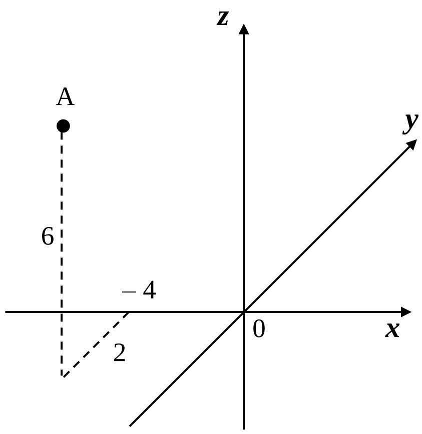

**k)**



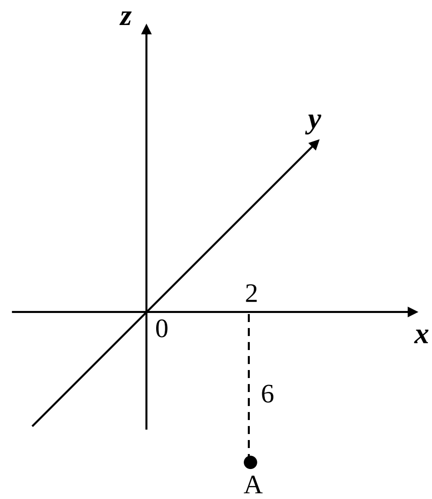

**(l)**



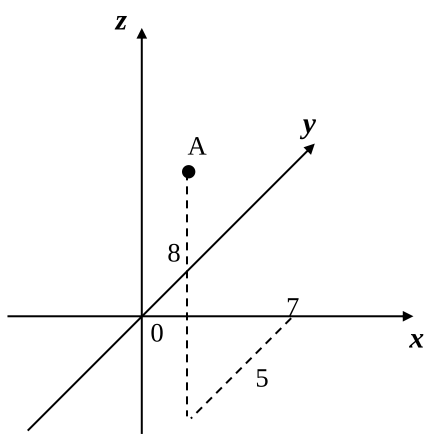
m)



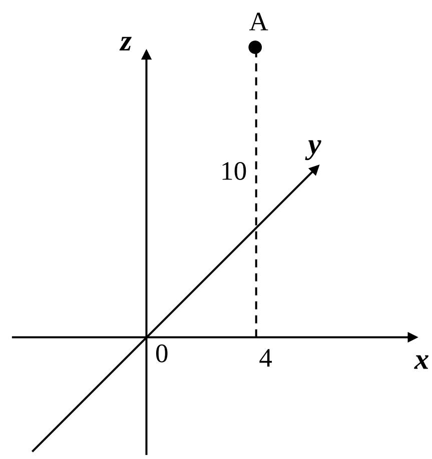

(n)



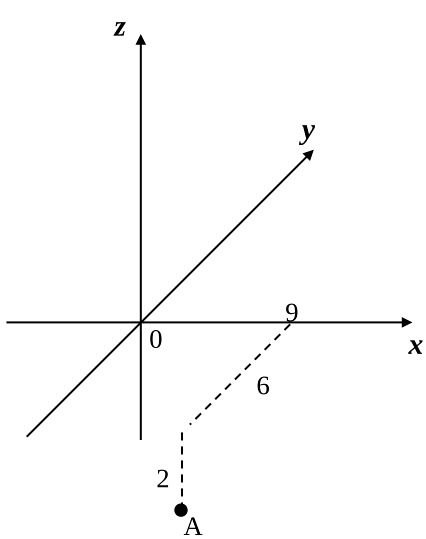

o)



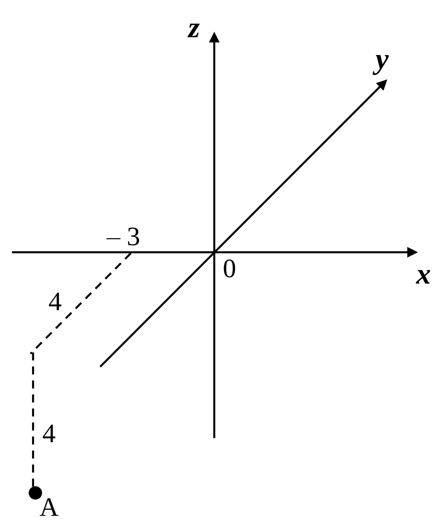

(p)




q)

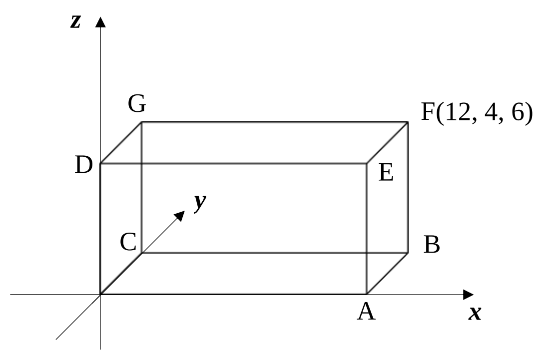



(r)

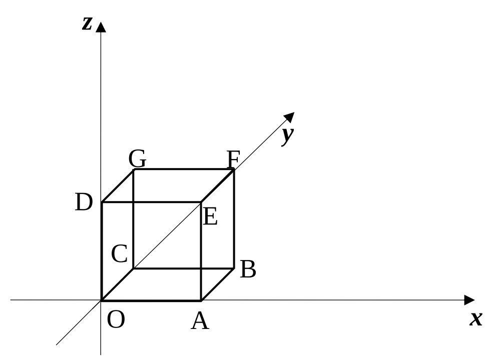


**2.** Calculate the magnitude of each of the vectors in question 1 correct to one decimal place.

**3.** State the coordinates of each vertex of the cuboid shown in the diagram.

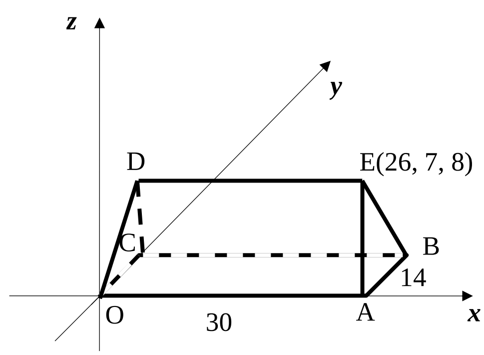



**4.** A cube of side 6 units is placed on coordinate axes as shown in the diagram. Write down the coordinates of each vertex of the cube.

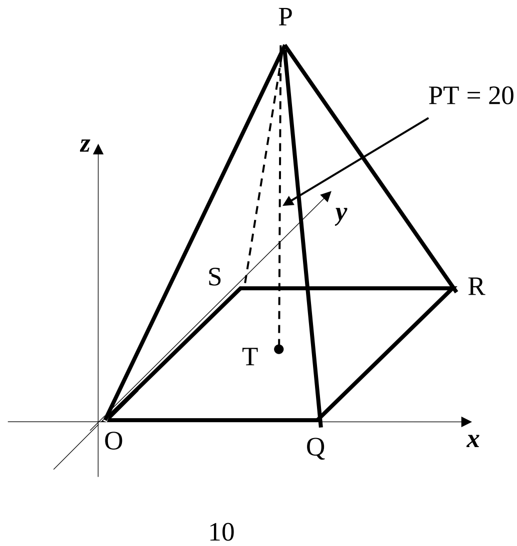



**5.** This shape is made up from 2 congruent trapezia and 2 congruent isosceles triangles.

From the information given in the diagram, write down the coordinates of each corner of the shape.



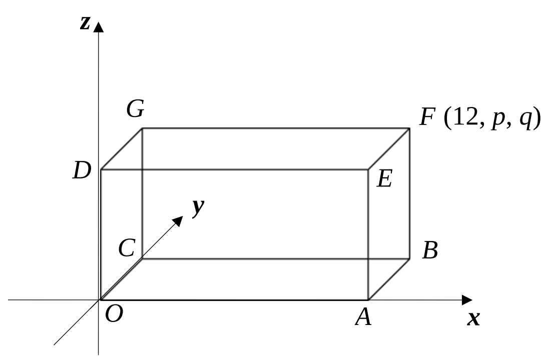
**6.** State the coordinates of each vertex of the **square based** pyramid shown in the diagram.




**7.** A cuboid is placed on coordinate axes as shown.

The dimensions of the cuboid are in the ratio $OA : AB : BF = 4 : 1 : 2$

The point $F$ has coordinates $(12, p, q)$ as shown.




Establish the values of $p$ and $q$ and write down the coordinates of all the vertices of the cuboid.

**8.** Write in component form:

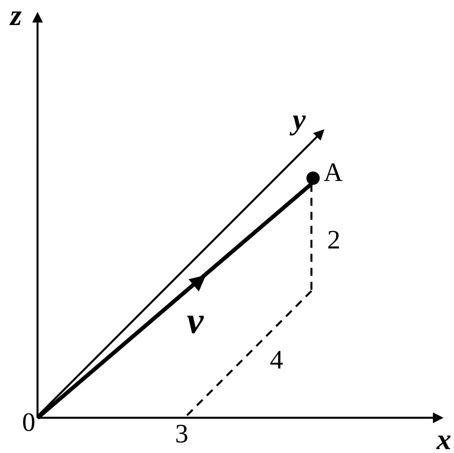
**a)** $\boldsymbol{v} = 2\boldsymbol{i} + 3\boldsymbol{j} - 4\boldsymbol{k}$ **(b)** $\boldsymbol{w} = 3\boldsymbol{i} - 6\boldsymbol{j} + 2\boldsymbol{k}$

**c)** $\boldsymbol{u} = 6\boldsymbol{i} - 3\boldsymbol{k}$ **(d)** $\boldsymbol{a} = -3\boldsymbol{j} - 4\boldsymbol{k}$

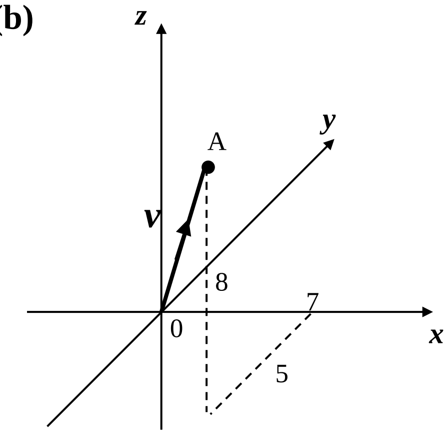
**e)** $\boldsymbol{b} = 7\boldsymbol{i} - 2\mathbf{j}$ **(f)** $\boldsymbol{c} = 6\boldsymbol{j}$

**9.** For each of these diagrams express ***v*** in terms of ***i***, ***j*** and ***k***.

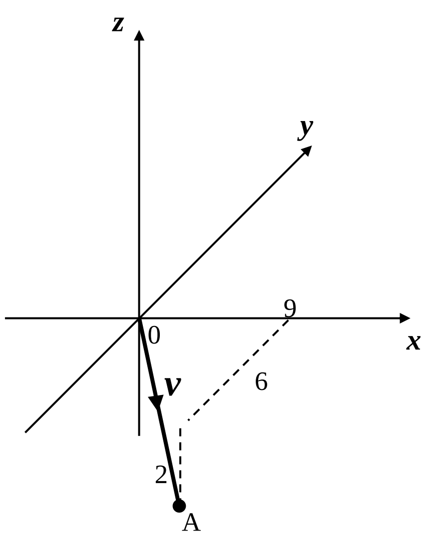
**a)**



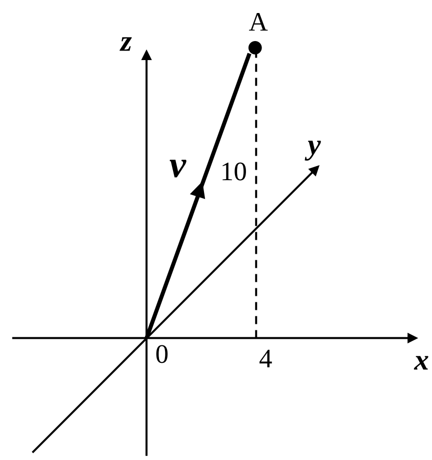

**(b)**




**c)**




**(d)**



## *Exercise 7*

**1.** For each pair of vectors:

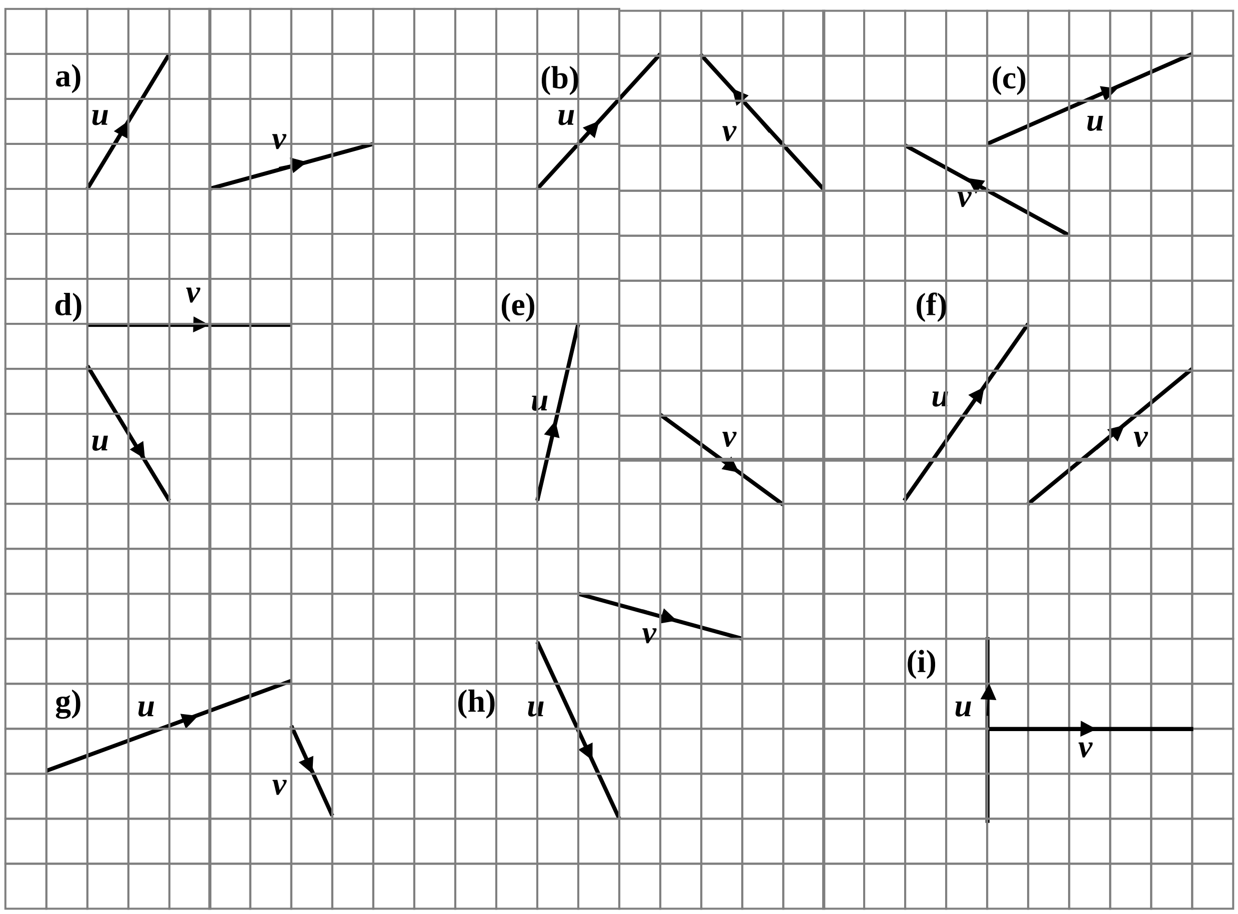
**i)** Write down the components of ***u*** and ***v***.

**ii)** Find the components of the resultant vector ***u*** + ***v***

**iii)** Find the components of the resultant vector ***v*** – ***u***

**iv)** Find the components of the resultant vector 2***v*** + 3***u***

**v)** Find the components of the resultant vector 3***v*** – 4***u***




**2.** ***u***, ***v*** and ***w*** are 3 vectors with components $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$, $\begin{pmatrix} -4 \\ 5 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ -3 \end{pmatrix}$ respectively.

Find the components of the following:

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **a)** | 2***u*** + 3***v*** | **(b)** | 3***u*** – 6**v** | **(c)** | 3***w*** + 2***v*** | **(d)** | 4***u*** – 2***w*** |
| **e)** | – 3***u*** – 4***v*** | **(f)** | 3***w*** – 4***u*** | **(g)** | 3***u*** – 6***v*** + 2***w*** | **(h)** | 2***u*** + 3***v*** – 4***w*** |
| **i)** | 3***u*** – 2***v*** + ***w*** | | | | | | |

**3.** Calculate the magnitude of each of these vectors giving answers to one decimal place:

**a)** $\boldsymbol{p} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$ **(b)** $\boldsymbol{v} = \begin{pmatrix} 3 \\ 4 \\ -7 \end{pmatrix}$ **(c)** $\boldsymbol{r} = \begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix}$ **(d)** $\boldsymbol{t} = \begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$

**e)** $\boldsymbol{u} = \begin{pmatrix} 6 \\ -1 \\ -4 \end{pmatrix}$ **(f)** $\boldsymbol{q} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ **(g)** $\boldsymbol{a} = \begin{pmatrix} 2 \\ -1 \\ -2 \end{pmatrix}$ **(h)** $\boldsymbol{b} = \begin{pmatrix} 5 \\ -12 \\ 0 \end{pmatrix}$

**4.** ***u***, ***v*** and ***w*** are 3 vectors with components $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ 8 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 5 \\ -1 \end{pmatrix}$ respectively.

**i)** Find the components of the following:

**ii)** Calculate the magnitude of each resultant vector above giving answers to 1 decimal place.

**a)** $2\boldsymbol{u} + 3\boldsymbol{v}$ **(b)** $3\boldsymbol{u} - 6\boldsymbol{v}$ **(c)** $3\boldsymbol{w} + 2\boldsymbol{v}$ **(d)** $4\boldsymbol{u} - 2\boldsymbol{w}$

**e)** $-3\boldsymbol{u} - 4\boldsymbol{v}$ **(f)** $3\boldsymbol{w} - 4\boldsymbol{u}$ **(g)** $3\boldsymbol{u} - 6\boldsymbol{v} + 2\boldsymbol{w}$ **(h)** $2\boldsymbol{u} + 3\boldsymbol{v} - 4\boldsymbol{w}$

**5.** **i)** If $\boldsymbol{p} = 4\boldsymbol{i} + 2\boldsymbol{j} - 5\boldsymbol{k}$ and $\boldsymbol{q} = \boldsymbol{i} - 3\boldsymbol{j} + \boldsymbol{k}$, express the following in component form:

**ii)** Calculate the magnitude of each resultant vector above giving answers to 1 decimal place.

**a)** $\boldsymbol{p} + \boldsymbol{q}$ **(b)** $\boldsymbol{p} - \boldsymbol{q}$ **(c)** $\boldsymbol{q} - 2\boldsymbol{p}$ **(d)** $3\boldsymbol{p} + \boldsymbol{q}$

**e)** $3\boldsymbol{p} - 2\boldsymbol{q}$ **(f)** $2\boldsymbol{q} - 3\boldsymbol{p}$ **(g)** $3\boldsymbol{p} + 4\boldsymbol{q}$ **(h)** $-2\boldsymbol{q} - 2\boldsymbol{p}$

**6.** Calculate the magnitude of these vectors , leaving you answer a surd in its in simplest form.

**a)** $\boldsymbol{u} = \begin{pmatrix} -5 \\ 3 \\ 2 \end{pmatrix}$ **(b)** $AB = \begin{pmatrix} -1 \\ 1 \\ 5 \end{pmatrix}$ **(c)** $\boldsymbol{t} = 3\boldsymbol{i} - 2\boldsymbol{j} + 5\boldsymbol{k}$

**d)** ***t*** where point T has coordinates$(\sqrt{3}, \sqrt{5}, 2\sqrt{2})$ **(e)** $\boldsymbol{v} = \sqrt{3}\boldsymbol{k} + \boldsymbol{j} - 7\boldsymbol{i}$

**7.** Given that $\boldsymbol{v} = 2\boldsymbol{k} - 3\boldsymbol{i} + 4\boldsymbol{k}$, $\boldsymbol{u} = 5\boldsymbol{i} + a\boldsymbol{j} - \boldsymbol{k}$ have the same magnitude, calculate the value of $a$ if $a > 0$.

**8.** A skater is suspended by three wires with forces $\begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 4 \\ 8 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 5 \\ -1 \end{pmatrix}$ acting on them.

Calculate the resultant force and its magnitude correct to 3 significant figures where necessary.

**9.** If $\boldsymbol{u} = \begin{pmatrix} -4 \\ 1 \\ 3 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 2 \\ 2 \\ -5 \end{pmatrix}$, solve each vector equation for $\boldsymbol{x}$.

**a)** $\boldsymbol{u} + \boldsymbol{x} = \boldsymbol{v}$ **(b)** $2\boldsymbol{u} + \boldsymbol{x} = 2\boldsymbol{v}$ **(c)** $2\boldsymbol{x} + 3\boldsymbol{v} = 4\boldsymbol{u} - \boldsymbol{x}$

**10.** **i)** If $\boldsymbol{r} = \begin{pmatrix} 2 \\ 6 \\ -3 \end{pmatrix}$, $\boldsymbol{s} = \begin{pmatrix} 6 \\ 6 \\ -1 \end{pmatrix}$ and $\boldsymbol{t} = \begin{pmatrix} -4 \\ 0 \\ 1 \end{pmatrix}$, express these in component form:

**a)** $2\boldsymbol{r} + \boldsymbol{s}$ **(b)** $3\boldsymbol{t} - 2\boldsymbol{s}$ **(c)** $(\boldsymbol{r} - \boldsymbol{s}) + \boldsymbol{t}$ **(d)** $\boldsymbol{r} - (\boldsymbol{s} + \boldsymbol{t})$

**ii)** Find: **a)** $|2\boldsymbol{r} + \boldsymbol{s}|$ **(b)** $|3\boldsymbol{t} - 2\boldsymbol{s}|$ **(c)** $|(\boldsymbol{r} - \boldsymbol{s}) + \boldsymbol{t}|$ **(d)** $|\boldsymbol{r} - (\boldsymbol{s} + \boldsymbol{t})|$

**11.** Two forces are represented by the vectors $F_1 = 2\underset{\sim}{i} + \underset{\sim}{j} - 3\underset{\sim}{k}$ and $F_2 = \underset{\sim}{i} + 4\underset{\sim}{k}$ .
Find the magnitude of the resultant force $F_1 + F_2$ .

**12.** Two vectors are defined as $V_1 = 4\underset{\sim}{i} + \underset{\sim}{j} + \sqrt{8}\underset{\sim}{k}$ and $V_2 = 8\underset{\sim}{i} + \sqrt{24}\underset{\sim}{j} + a\sqrt{3}\underset{\sim}{k}$ where $a$ is a constant and <u>all</u> coefficients of $\underset{\sim}{i}$ , $\underset{\sim}{j}$ and $\underset{\sim}{k}$ are greater than zero.
Given that $|V_2| = 2|V_1|$ , calculate the value of $a$.

**13.** Vector $\boldsymbol{a}$ has components $\boldsymbol{a} = \begin{pmatrix} 3 \\ -2 \\ k \end{pmatrix}$. If $|\boldsymbol{a}| = 4$, calculate the value(s) of $k$.

**14.** Calculate the length of vector $\boldsymbol{a}$ defined as $\boldsymbol{a} = 4\boldsymbol{i} + 2\sqrt{3}\boldsymbol{j} - 2\sqrt{2}\boldsymbol{k}$ .

**15.** Vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are defined by $\boldsymbol{a} = \boldsymbol{i} + \boldsymbol{j} + 2\boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} - \boldsymbol{j}$.
Find the components of $2\boldsymbol{a} - \boldsymbol{b}$ and calculate its magnitude.

*Answers*

*Exercise 1*

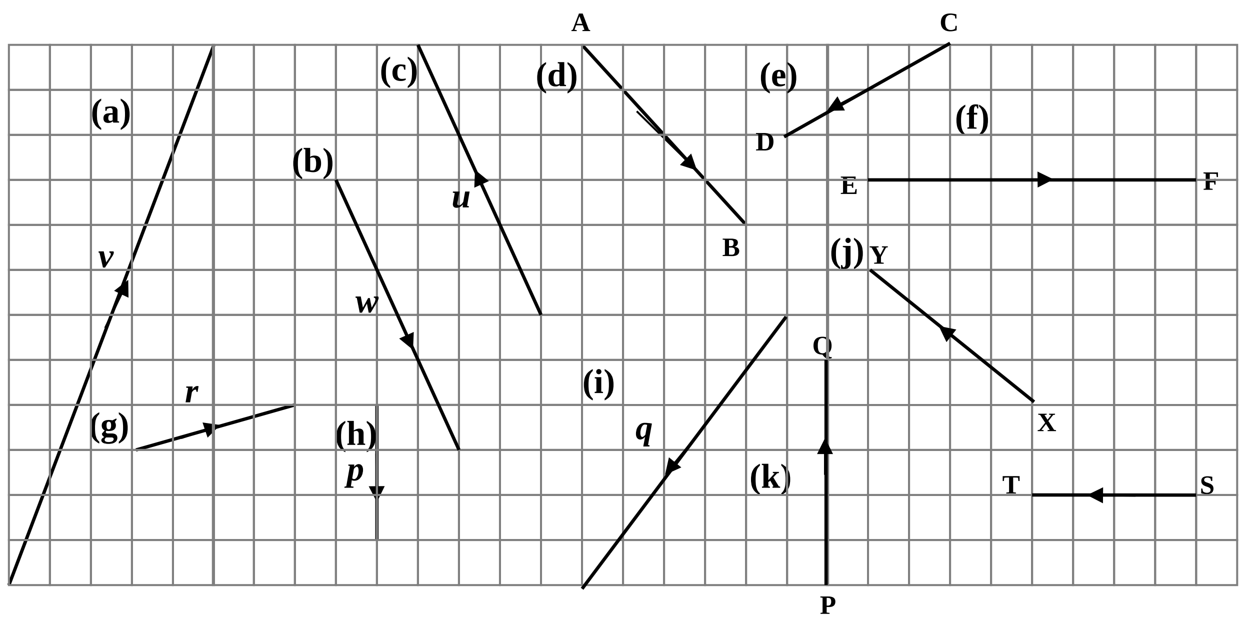
**1.** **(a)** $\overrightarrow{AB} = \boldsymbol{u} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ **(b)** $\overrightarrow{CD} = \boldsymbol{v} = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$ **(c)** $\overrightarrow{EF} = \boldsymbol{w} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$

**(d)** $\overrightarrow{GH} = \boldsymbol{u} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$ **(e)** $\overrightarrow{ML} = \boldsymbol{v} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$ **(f)** $\overrightarrow{PQ} = \boldsymbol{w} = \begin{pmatrix} 3 \\ -4 \end{pmatrix}$

**(g)** $\overrightarrow{RS} = \boldsymbol{s} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$ **(h)** $\overrightarrow{WX} = \boldsymbol{t} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$ **(i)** $\overrightarrow{PT} = \boldsymbol{a} = \begin{pmatrix} 6 \\ 1 \end{pmatrix}$

**(j)** $\overrightarrow{RQ} = \boldsymbol{b} = \begin{pmatrix} 3 \\ -6 \end{pmatrix}$ **(k)** $\overrightarrow{CF} = \boldsymbol{c} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$

**2.**




**3.** For question 1

**(a)** $\sqrt{10}$ **(b)** $2\sqrt{5}$ **(c)** 6 **(d)** 3

**(e)** $2\sqrt{2}$ **(f)** 5 **(g)** $2\sqrt{5}$ **(h)** $\sqrt{29}$

**(i)** $\sqrt{37}$ **(j)** $3\sqrt{5}$ **(k)** 5

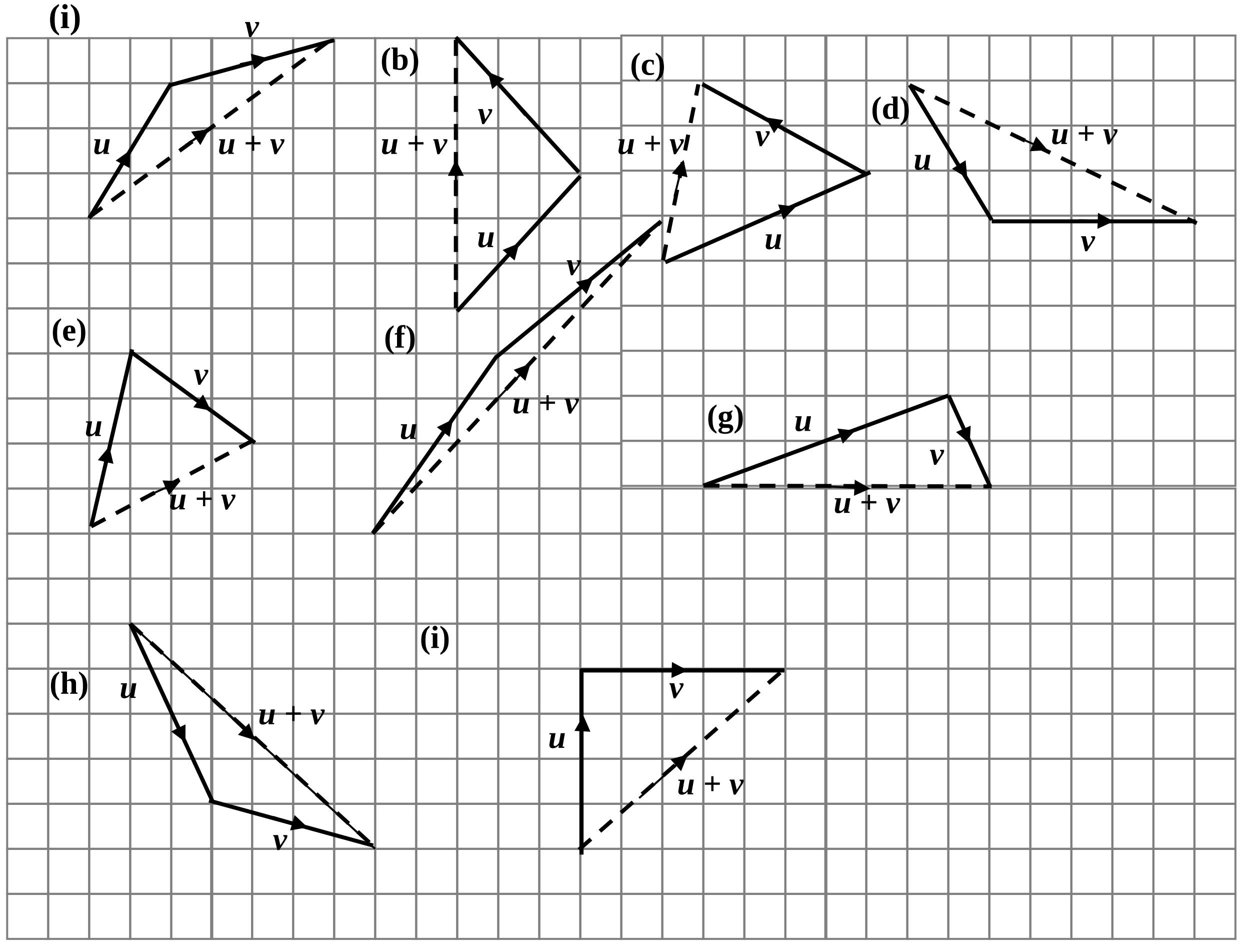
For question 2

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| **(a)** | 13 | **(b)** | $3\sqrt{5}$ | **(c)** | $3\sqrt{5}$ | **(d)** | $4\sqrt{2}$ |
| **(e)** | $2\sqrt{5}$ | **(f)** | 8 | **(g)** | $\sqrt{17}$ | **(h)** | 3 |
| **(i)** | $\sqrt{61}$ | **(j)** | 5 | **(k)** | 5 | **(l)** | 4 |

**4.** **(a)** 5 **(b)** 25 **(c)** 13 **(d)** 10 **(e)** 5 **(f)** 13

## *Exercise 2*

**1.** **(i)**

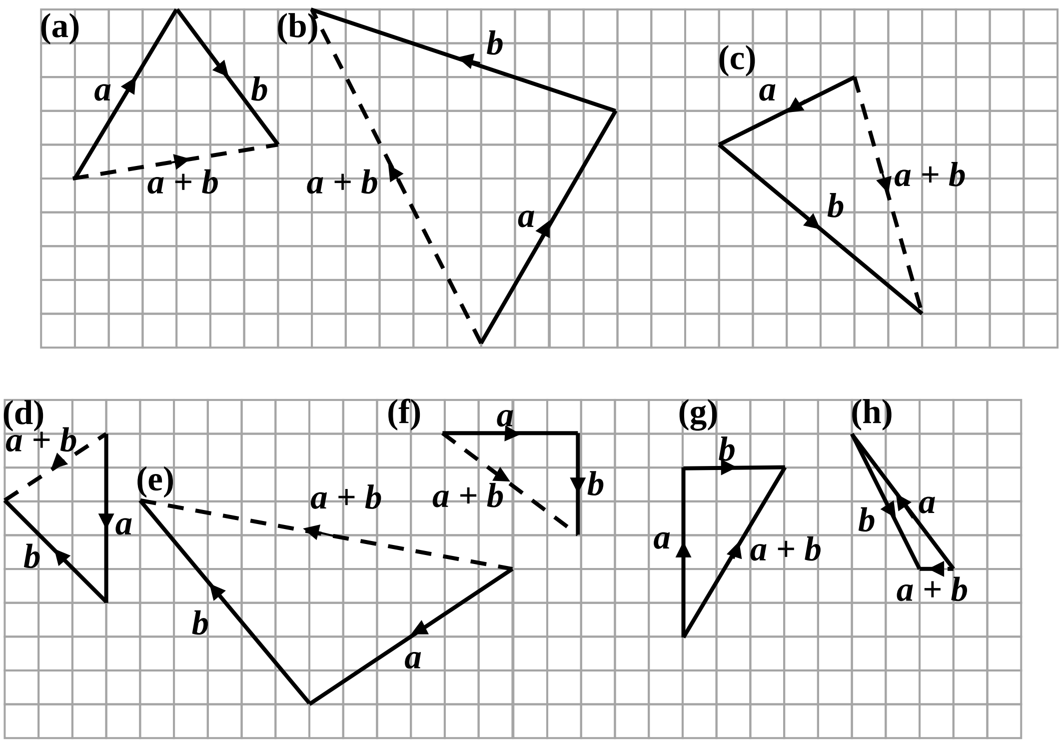


**(ii)** **(a)** $\begin{pmatrix}6\\4\end{pmatrix}$; $2\sqrt{13}$ **(b)** $\begin{pmatrix}0\\6\end{pmatrix}$; 6 **(c)** $\begin{pmatrix}1\\4\end{pmatrix}$; $\sqrt{17}$

**(d)** $\begin{pmatrix}7\\-3\end{pmatrix}$; $\sqrt{58}$ **(e)** $\begin{pmatrix}4\\2\end{pmatrix}$; $2\sqrt{5}$ **(f)** $\begin{pmatrix}7\\7\end{pmatrix}$; $7\sqrt{2}$

**(g)** $\begin{pmatrix}7\\0\end{pmatrix}$; 7 **(h)** $\begin{pmatrix}6\\-5\end{pmatrix}$; $\sqrt{61}$ **(i)** $\begin{pmatrix}5\\4\end{pmatrix}$; $\sqrt{41}$

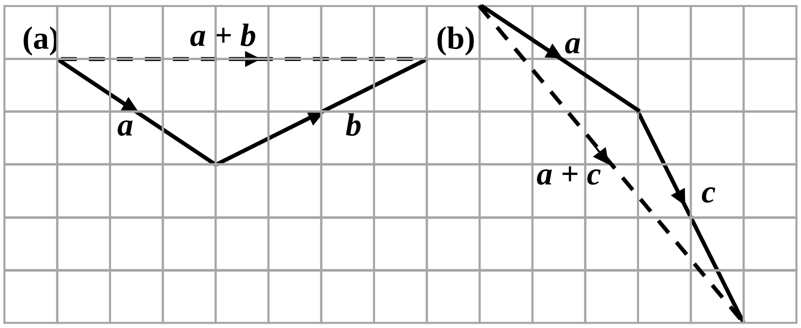
**2.** **(i)**

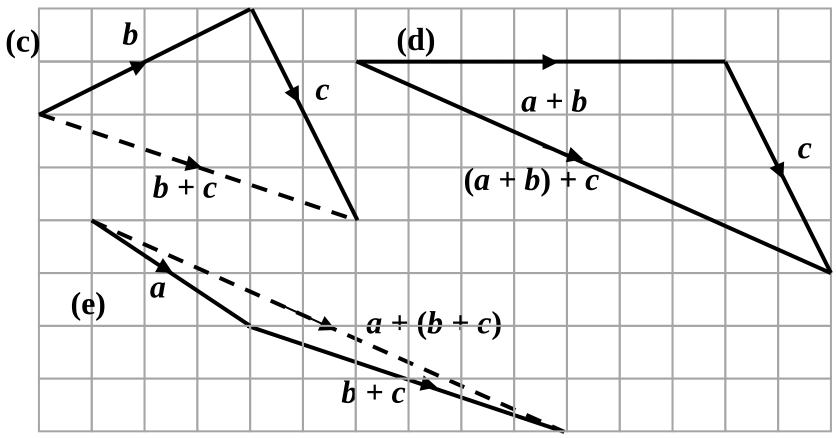



**(ii)** **(a)** $\begin{pmatrix}6\\1\end{pmatrix}$; $\sqrt{37}$ **(b)** $\begin{pmatrix}-5\\10\end{pmatrix}$; $5\sqrt{5}$ **(c)** $\begin{pmatrix}2\\-7\end{pmatrix}$; $\sqrt{53}$ **(d)** $\begin{pmatrix}-3\\-2\end{pmatrix}$; $\sqrt{13}$

**(e)** $\begin{pmatrix}-11\\2\end{pmatrix}$; $5\sqrt{5}$ **(f)** $\boldsymbol{a} = \begin{pmatrix}4\\-3\end{pmatrix}$; 5 **(g)** $\begin{pmatrix}3\\5\end{pmatrix}$; $\sqrt{34}$ **(h)** $\begin{pmatrix}-1\\0\end{pmatrix}$; 1

**3.** **(i)**

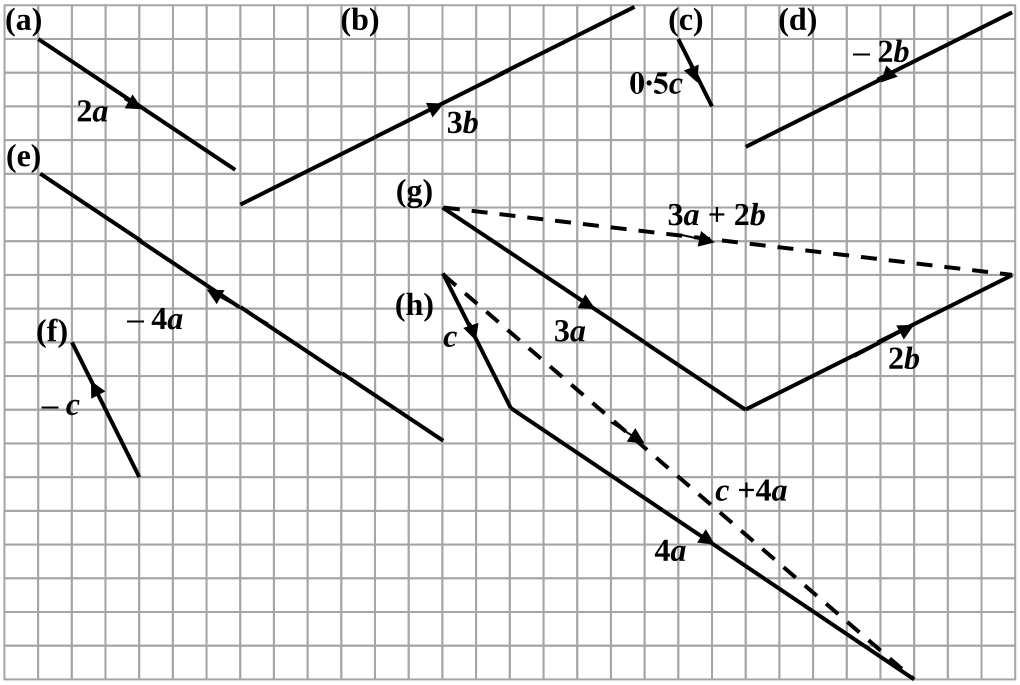



**(ii)** **(a)** $\begin{pmatrix} 7 \\ 0 \end{pmatrix}$; 7·0 **(b)** $\begin{pmatrix} 5 \\ -6 \end{pmatrix}$; 7·8 **(c)** $\begin{pmatrix} 6 \\ -2 \end{pmatrix}$; 6·3

**(d)** $\begin{pmatrix} 9 \\ -4 \end{pmatrix}$; 9·8 **(e)** $\begin{pmatrix} 9 \\ -4 \end{pmatrix}$; 9·8

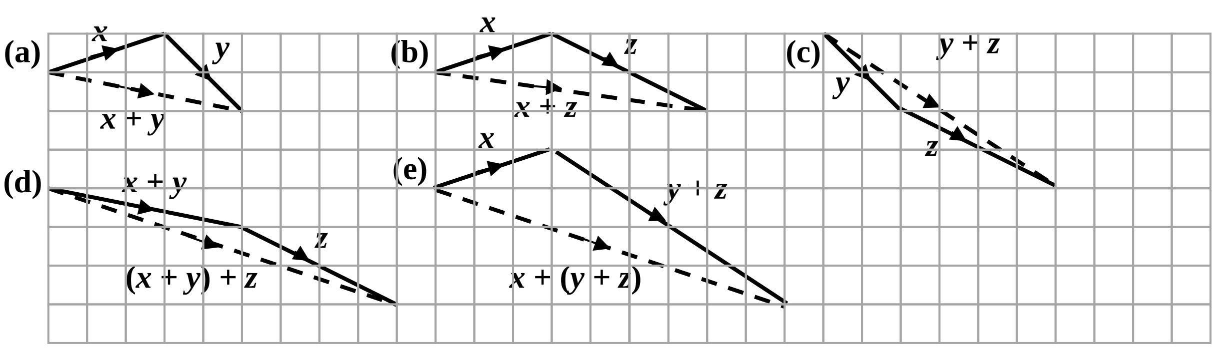
**4.** **(i)**




. **(ii)** **(a)** $\begin{pmatrix} 6 \\ -4 \end{pmatrix}$; $2\sqrt{13}$ **(b)** $\begin{pmatrix} 12 \\ 6 \end{pmatrix}$; $6\sqrt{5}$ **(c)** $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$; $\sqrt{5}$ **(d)** $\begin{pmatrix} -8 \\ -4 \end{pmatrix}$; $4\sqrt{5}$

**(e)** $\begin{pmatrix} -12 \\ 8 \end{pmatrix}$; $4\sqrt{13}$ **(f)** $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$; $2\sqrt{5}$ **(g)** $\begin{pmatrix} 17 \\ -2 \end{pmatrix}$; $\sqrt{293}$ **(h)** $\begin{pmatrix} 14 \\ -12 \end{pmatrix}$; $2\sqrt{85}$

**5.** **(i)**



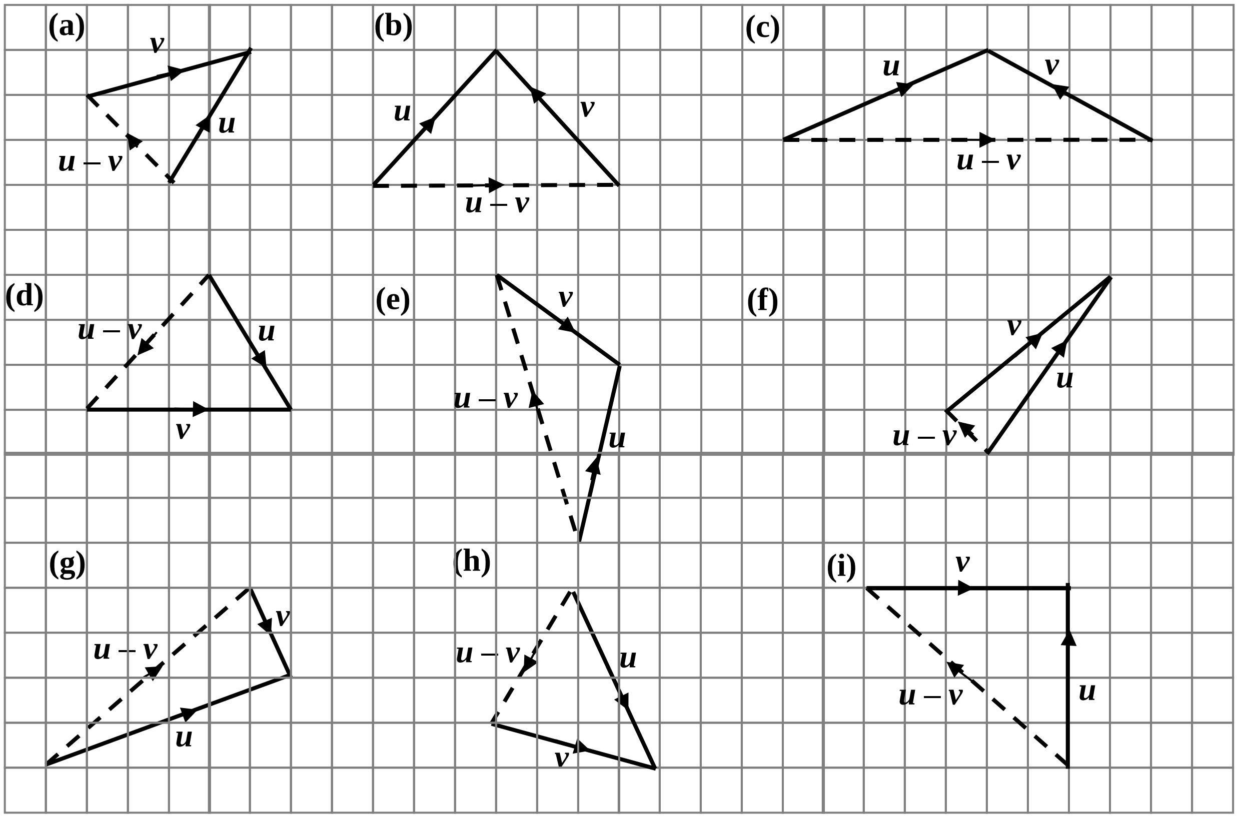

**(ii)** **(a)** 5·1 **(b)** 7·1 **(c)** 7·2 **(d)** 9·5 **(e)** 9·5

**6.** **(a)** 6·3 **(b)** 8·5 **(c)** 2·2 **(d)** 5·7

**(e)** 12·6 **(f)** 4·5 **(g)** 13·0 **(h)** 17·7

*Exercise 3*

**1.** **(i)**

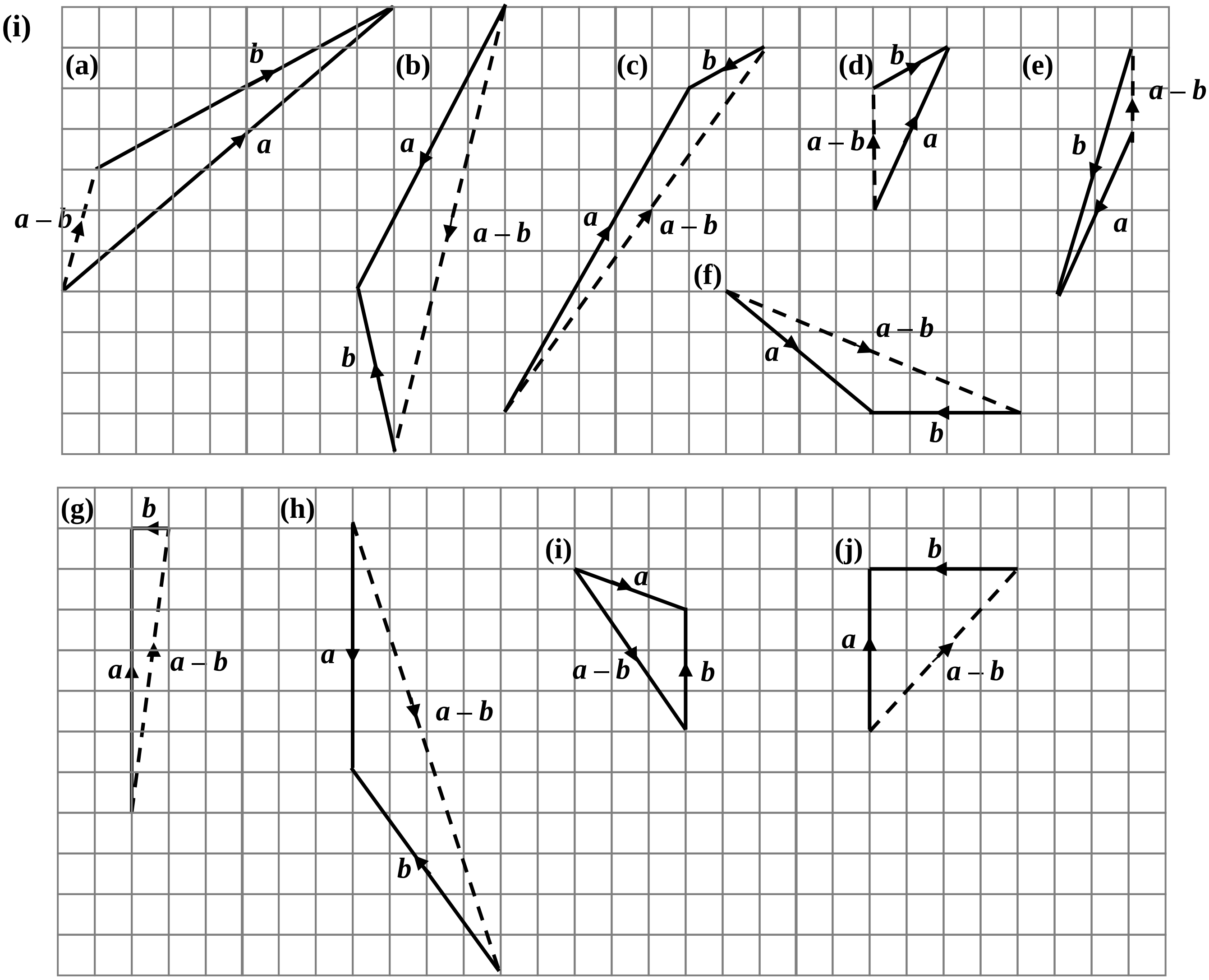



**(ii)** **(a)** $\begin{pmatrix}-2\\2\end{pmatrix}$; $2\sqrt{2}$ **(b)** $\begin{pmatrix}6\\0\end{pmatrix}$; $6$ **(c)** $\begin{pmatrix}9\\0\end{pmatrix}$; $9$ **(d)** $\begin{pmatrix}-3\\-3\end{pmatrix}$; $3\sqrt{2}$

**(e)** $\begin{pmatrix}-2\\6\end{pmatrix}$; $2\sqrt{10}$ **(f)** $\begin{pmatrix}-1\\1\end{pmatrix}$; $\sqrt{2}$ **(g)** $\begin{pmatrix}5\\4\end{pmatrix}$; $\sqrt{41}$ **(h)** $\begin{pmatrix}-2\\-3\end{pmatrix}$; $\sqrt{13}$

**(i)** $\begin{pmatrix}-5\\4\end{pmatrix}$; $\sqrt{41}$

**2.** **(i)**



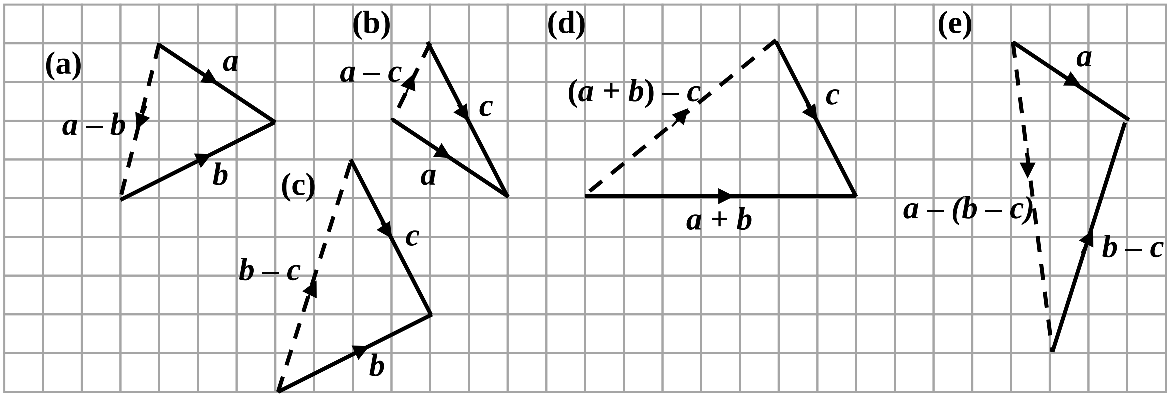

**(ii)**

**(a)** $\begin{pmatrix}1\\3\end{pmatrix}$; 3·1 **(b)** $\begin{pmatrix}-3\\-11\end{pmatrix}$; 11·4 **(c)** $\begin{pmatrix}7\\9\end{pmatrix}$; 11·4 **(d)** $\begin{pmatrix}0\\3\end{pmatrix}$; 3

**(e)** $\begin{pmatrix}0\\2\end{pmatrix}$; 2 **(f)** $\begin{pmatrix}8\\-3\end{pmatrix}$; 8·5 **(g)** $\begin{pmatrix}1\\7\end{pmatrix}$; 7·1 **(h)** $\begin{pmatrix}4\\-11\end{pmatrix}$; 11·7

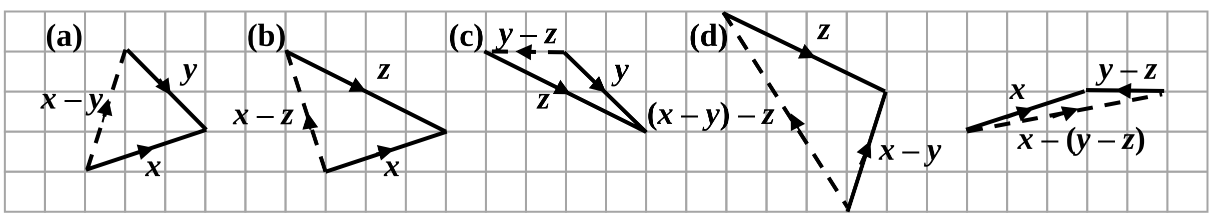
**(i)** $\begin{pmatrix}3\\-4\end{pmatrix}$; 5 **(j)** $\begin{pmatrix}4\\4\end{pmatrix}$; 5·7

**3.** **(i)**



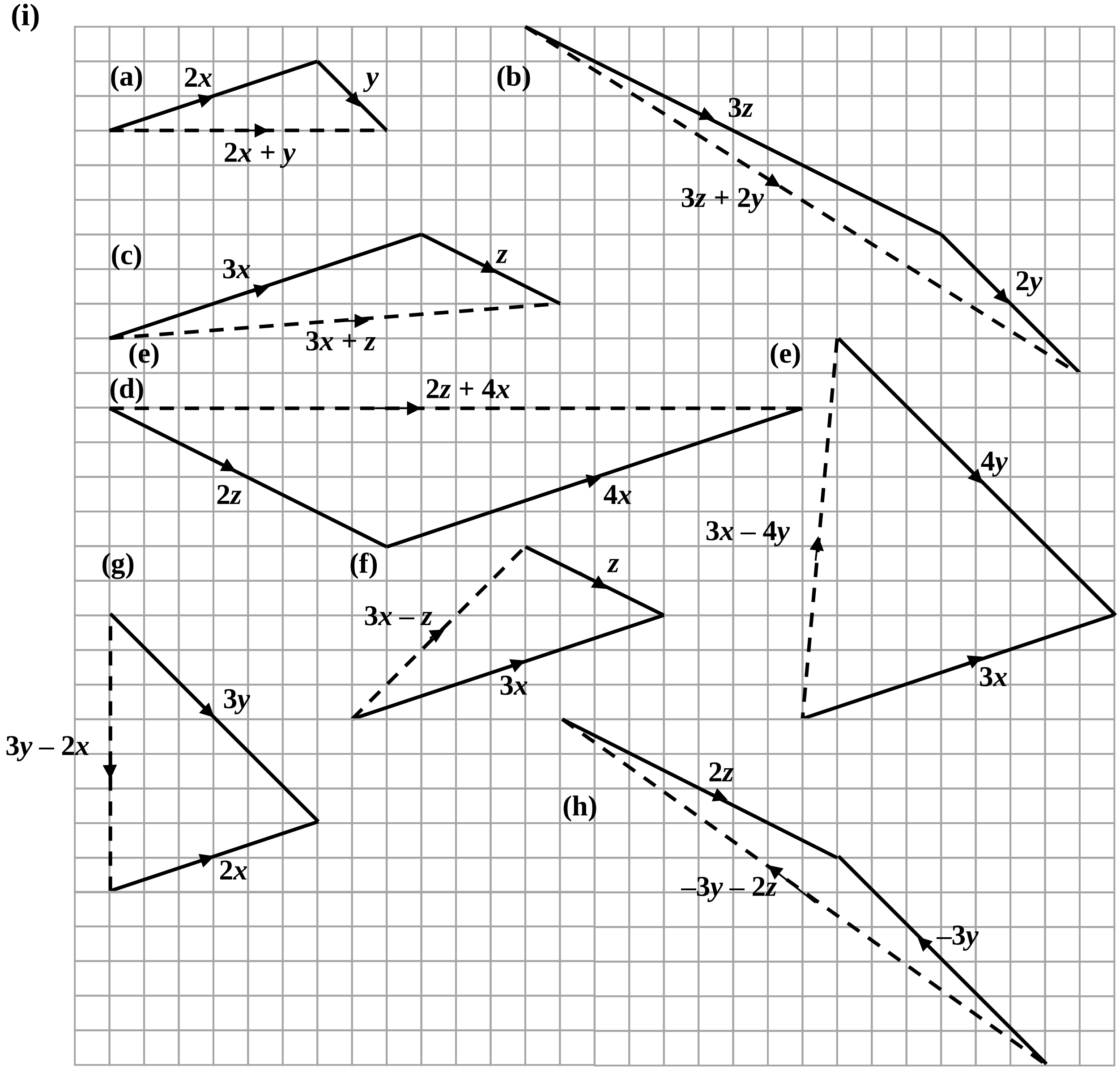

**(ii)** **(a)** 4·12 **(b)** 2·24 **(c)** 6·32 **(d)** 6·40 **(e)** 8·06

**4.** **(i)**




**(ii)** **(a)** $\begin{pmatrix}1\\3\end{pmatrix}$; 3·2 **(b)** $\begin{pmatrix}-1\\3\end{pmatrix}$; 3·2 **(c)** $\begin{pmatrix}-2\\0\end{pmatrix}$; 2 **(d)** $\begin{pmatrix}-3\\5\end{pmatrix}$; 5·8 **(e)** $\begin{pmatrix}5\\1\end{pmatrix}$; 5·1

**5.** **(i)**




**(ii)**

**(a)** $\begin{pmatrix} 8 \\ 0 \end{pmatrix}$; 8·00 **(b)** $\begin{pmatrix} 16 \\ -10 \end{pmatrix}$; 18·9 **(c)** $\begin{pmatrix} 13 \\ 1 \end{pmatrix}$; 13·0

**(d)** $\begin{pmatrix} 20 \\ 0 \end{pmatrix}$; 20 **(e)** $\begin{pmatrix} 1 \\ 11 \end{pmatrix}$; 11·0 **(f)** $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$; 7·07

**(g)** $\begin{pmatrix} 0 \\ -8 \end{pmatrix}$; 8·00 **(h)** $\begin{pmatrix} -14 \\ 10 \end{pmatrix}$; 17·2

## *Exercise 4*

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| **1.** | **(a)** | $\boldsymbol{b}$ | **(b)** | $-\boldsymbol{b}$ | **(c)** | $-\boldsymbol{a}$ | **(d)** | $\boldsymbol{a}+\boldsymbol{b}$ | **(e)** | $-(\boldsymbol{a}+\boldsymbol{b})$ |
| **2.** | **(a)** | $-\boldsymbol{v}$ | **(b)** | $-\boldsymbol{v}-\boldsymbol{w}$ | **(c)** | $2\boldsymbol{v}$ | **(d)** | $\boldsymbol{v}-\boldsymbol{w}$ | **(e)** | $\boldsymbol{w}-2\boldsymbol{v}$ |
| **3.** | **(a)** | $\boldsymbol{q}-\boldsymbol{p}$ | **(b)** | $\boldsymbol{p}-\boldsymbol{q}$ | **(c)** | $\frac{1}{2}(\boldsymbol{p}-\boldsymbol{q})$ | **(d)** | $\frac{1}{2}(\boldsymbol{p}+\boldsymbol{q})$ | | |
| **4.** | **(a)** | $\boldsymbol{b}$ | **(b)** | $\boldsymbol{a}$ | **(c)** | $\frac{1}{2}\boldsymbol{a}$ | **(d)** | $\boldsymbol{b}+\frac{1}{2}\boldsymbol{a}$ | | |
| **5.** | **(a)** | $2\boldsymbol{y}$ | **(b)** | $\boldsymbol{x}-2\boldsymbol{y}$ | | | | | | |

## *Exercise 5*

**1.** **(a)** **(i)** $\boldsymbol{b}+\boldsymbol{a}$ **(ii)** $\boldsymbol{a}-\boldsymbol{b}$ **(b)** proof

**2.** **(a)** $\boldsymbol{w}-\boldsymbol{v}$ **(b)** $\frac{1}{4}(\boldsymbol{w}-\boldsymbol{v})$ **(c)** $\frac{1}{4}(\boldsymbol{w}+3\boldsymbol{v})$ ; $\begin{pmatrix} 6 \\ 3 \end{pmatrix}$

**3.** **(a)** $2\boldsymbol{p}-\boldsymbol{q}$ **(b)** $\frac{2}{5}(2\boldsymbol{p}-\boldsymbol{q})$ **(c)** $\frac{1}{5}(4\boldsymbol{p}+3\boldsymbol{q})$ $\begin{pmatrix} -5 \\ 7 \end{pmatrix}$; $8{\cdot}6$

**4.** **(a)** **(i)** $\boldsymbol{b}-\boldsymbol{a}$ **(ii)** $\frac{1}{3}(\boldsymbol{b}-\boldsymbol{a})$ **(iii)** $\frac{1}{3}(2\boldsymbol{a}+\boldsymbol{b})$ **(b)** Proof

*Exercise 6*

**1.**

**(a)** (3, 4, 2); $\begin{pmatrix} 3 \\ 4 \\ 2 \end{pmatrix}$ **(b)** (1, 5, 7); $\begin{pmatrix} 1 \\ 5 \\ 7 \end{pmatrix}$ **(c)** (– 4, 6, 2); $\begin{pmatrix} -4 \\ 6 \\ 2 \end{pmatrix}$

**(d)** (– 1, 7, 10); $\begin{pmatrix} -1 \\ 7 \\ 10 \end{pmatrix}$ **(e)** (0, 0, 8); $\begin{pmatrix} 0 \\ 0 \\ 8 \end{pmatrix}$ **(f)** (0, 4, 0); $\begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$

**(g)** (5, 0, 0); $\begin{pmatrix} 5 \\ 0 \\ 0 \end{pmatrix}$ **(h)** (– 3, 0, 0); $\begin{pmatrix} -3 \\ 0 \\ 0 \end{pmatrix}$ **(i)** (0, 0, – 2); $\begin{pmatrix} 0 \\ 0 \\ -2 \end{pmatrix}$

**(j)** (0, – 7, 0); $\begin{pmatrix} 0 \\ -7 \\ 0 \end{pmatrix}$ **(k)** (0, –3, 8); $\begin{pmatrix} 0 \\ -3 \\ 8 \end{pmatrix}$ **(l)** (– 4, 0, 5); $\begin{pmatrix} -4 \\ 0 \\ 5 \end{pmatrix}$

**(m)** (– 4, –2, 6); $\begin{pmatrix} -4 \\ -2 \\ 6 \end{pmatrix}$ **(n)** (2, 0, – 6); $\begin{pmatrix} 2 \\ 0 \\ -6 \end{pmatrix}$ **(o)** (7, – 5, 8); $\begin{pmatrix} 7 \\ -5 \\ 8 \end{pmatrix}$

**(p)** (4, 0, 10); $\begin{pmatrix} 4 \\ 0 \\ 10 \end{pmatrix}$ **(q)** (9, – 6, – 2); $\begin{pmatrix} 9 \\ -6 \\ -2 \end{pmatrix}$ **(r)** (– 3, – 4, – 4); $\begin{pmatrix} -3 \\ -4 \\ -4 \end{pmatrix}$

**2.**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| **(a)** | 5·4 | **(b)** | 8·7 | **(c)** | 7·5 | **(d)** | 12·2 | **(e)** | 8 | **(f)** | 4 |
| **(g)** | 5 | **(h)** | 3 | **(i)** | 2 | **(j)** | 7 | **(k)** | 8·5 | **(l)** | 6·4 |
| **(m)** | 7·5 | **(n)** | 6·3 | **(o)** | 11·7 | **(p)** | 10·8 | **(q)** | 11 | **(q)** | 6·4 |

**3.** O(0, 0, 0); A(12, 0, 0); B(12, 4, 0); C(0, 4, 0);

D(0, 0, 6); E(12, 0, 6); F(12, 4, 6); G(0, 4, 6)

**4.** O(0, 0, 0); A(6, 0, 0); B(6, 6, 0); C(0, 6, 0);

D(0, 0, 6); E(6, 0, 6); F(6, 6, 6); G(0, 6, 6)

**5.** O(0, 0, 0); A(30, 0, 0); B(30, 14, 0); C(0, 14, 0);

D(4, 7, 8); E(26, 7, 8)

**6.** O(0, 0, 0); P(5, 5, 20); Q(10, 0, 0); R(10, 10, 0); S(0, 10, 0)

**7.** $p = 3$; $q = 6$

O(0, 0, 0); A(12, 0, 0); B(12, 3, 0); C(0, 3, 0);

D(0, 0, 6); E(12, 0, 6); F(12, 3, 6); G(0, 3, 6)

**8.** **(a)** $\begin{pmatrix} 2 \\ 3 \\ -4 \end{pmatrix}$ **(b)** $\begin{pmatrix} 3 \\ -6 \\ 2 \end{pmatrix}$ **(c)** $\begin{pmatrix} 6 \\ 0 \\ -3 \end{pmatrix}$ **(d)** $\begin{pmatrix} 0 \\ -3 \\ -4 \end{pmatrix}$ **(e)** $\begin{pmatrix} 7 \\ -2 \\ 0 \end{pmatrix}$ **(f)** $\begin{pmatrix} 0 \\ 6 \\ 0 \end{pmatrix}$

**9.** **(a)** $\boldsymbol{v} = 3\boldsymbol{i} + 4\boldsymbol{j} + 2\boldsymbol{k}$ **(b)** $\boldsymbol{v} = 7\boldsymbol{i} - 5\boldsymbol{j} + 8\boldsymbol{k}$

**(c)** $\boldsymbol{v} = 9\boldsymbol{i} - 6\boldsymbol{j} - 2\boldsymbol{k}$ **(d)** $\boldsymbol{v} = 4\boldsymbol{i} + 10\boldsymbol{k}$

*Exercise 7*

**1.** **(a)** **(i)** $\boldsymbol{u} = \begin{pmatrix}2\\3\end{pmatrix}\boldsymbol{v} = \begin{pmatrix}4\\1\end{pmatrix}$ **(ii)** $\begin{pmatrix}6\\4\end{pmatrix}$ **(iii)** $\begin{pmatrix}2\\-2\end{pmatrix}$

**(iv)** $\begin{pmatrix}14\\11\end{pmatrix}$ **(v)** $\begin{pmatrix}4\\-9\end{pmatrix}$

**(b)** **(i)** $\boldsymbol{u} = \begin{pmatrix}3\\3\end{pmatrix}\boldsymbol{v} = \begin{pmatrix}-3\\3\end{pmatrix}$ **(ii)** $\begin{pmatrix}0\\6\end{pmatrix}$ **(iii)** $\begin{pmatrix}-6\\0\end{pmatrix}$ **(iv)** $\begin{pmatrix}3\\15\end{pmatrix}$ **(v)** $\begin{pmatrix}-21\\-3\end{pmatrix}$

**(c)** **(i)** $\boldsymbol{u} = \begin{pmatrix}5\\2\end{pmatrix}\boldsymbol{v} = \begin{pmatrix}-4\\2\end{pmatrix}$ **(ii)** $\begin{pmatrix}1\\4\end{pmatrix}$ **(iii)** $\begin{pmatrix}-9\\0\end{pmatrix}$ **(iv)** $\begin{pmatrix}7\\10\end{pmatrix}$ **(v)** $\begin{pmatrix}-32\\-2\end{pmatrix}$

**(d)** **(i)** $\boldsymbol{u} = \begin{pmatrix}2\\-3\end{pmatrix}\boldsymbol{v} = \begin{pmatrix}5\\0\end{pmatrix}$ **(ii)** $\begin{pmatrix}7\\-3\end{pmatrix}$ **(iii)** $\begin{pmatrix}3\\3\end{pmatrix}$ **(iv)** $\begin{pmatrix}16\\-9\end{pmatrix}$ **(v)** $\begin{pmatrix}7\\12\end{pmatrix}$

**(e)** **(i)** $\boldsymbol{u} = \begin{pmatrix}1\\4\end{pmatrix}\boldsymbol{v} = \begin{pmatrix}3\\-2\end{pmatrix}$ **(ii)** $\begin{pmatrix}4\\2\end{pmatrix}$ **(iii)** $\begin{pmatrix}2\\-6\end{pmatrix}$ **(iv)** $\begin{pmatrix}9\\8\end{pmatrix}$ **(v)** $\begin{pmatrix}5\\-22\end{pmatrix}$

**(f)** **(i)** $\boldsymbol{u} = \begin{pmatrix}3\\4\end{pmatrix}\boldsymbol{v} = \begin{pmatrix}4\\3\end{pmatrix}$ **(ii)** $\begin{pmatrix}7\\7\end{pmatrix}$ **(iii)** $\begin{pmatrix}1\\-1\end{pmatrix}$ **(iv)** $\begin{pmatrix}17\\18\end{pmatrix}$ **(v)** $\begin{pmatrix}0\\-7\end{pmatrix}$

**(g)** **(i)** $\boldsymbol{u} = \begin{pmatrix}6\\2\end{pmatrix}\boldsymbol{v} = \begin{pmatrix}1\\-2\end{pmatrix}$ **(ii)** $\begin{pmatrix}7\\0\end{pmatrix}$ **(iii)** $\begin{pmatrix}-5\\-4\end{pmatrix}$ **(iv)** $\begin{pmatrix}20\\2\end{pmatrix}$ **(v)** $\begin{pmatrix}-21\\-14\end{pmatrix}$

**(h)** **(i)** $\boldsymbol{u} = \begin{pmatrix}2\\-4\end{pmatrix}\boldsymbol{v} = \begin{pmatrix}4\\-1\end{pmatrix}$ **(ii)** $\begin{pmatrix}6\\-5\end{pmatrix}$ **(iii)** $\begin{pmatrix}2\\3\end{pmatrix}$ **(iv)** $\begin{pmatrix}14\\-14\end{pmatrix}$ **(v)** $\begin{pmatrix}4\\13\end{pmatrix}$

**(i)** **(i)** $\boldsymbol{u} = \begin{pmatrix}0\\4\end{pmatrix}\boldsymbol{v} = \begin{pmatrix}5\\0\end{pmatrix}$ **(ii)** $\begin{pmatrix}5\\4\end{pmatrix}$ **(iii)** $\begin{pmatrix}5\\-4\end{pmatrix}$ **(iv)** $\begin{pmatrix}10\\12\end{pmatrix}$ **(v)** $\begin{pmatrix}15\\-16\end{pmatrix}$

**2.** **(a)** $\begin{pmatrix}-8\\21\end{pmatrix}$ **(b)** $\begin{pmatrix}30\\-21\end{pmatrix}$ **(c)** $\begin{pmatrix}-11\\1\end{pmatrix}$ **(d)** $\begin{pmatrix}10\\18\end{pmatrix}$

**(e)** $\begin{pmatrix}10\\-29\end{pmatrix}$ **(f)** $\begin{pmatrix}-11\\-21\end{pmatrix}$ **(g)** $\begin{pmatrix}28\\-27\end{pmatrix}$ **(h)** $\begin{pmatrix}-4\\33\end{pmatrix}$ **(i)** $\begin{pmatrix}13\\-4\end{pmatrix}$

**3.** **(a)** 5·4 **(b)** 8·6 **(c)** 3·7 **(d)** 5

**(e)** 7·3 **(f)** 1·7 **(g)** 3 **(e)** 13

**4.** **(i)** **(a)** $\begin{pmatrix}16\\30\\8\end{pmatrix}$ **(b)** $\begin{pmatrix}-18\\-39\\12\end{pmatrix}$ **(c)** $\begin{pmatrix}2\\31\\-3\end{pmatrix}$ **(d)** $\begin{pmatrix}12\\2\\18\end{pmatrix}$

**(e)** $\begin{pmatrix}-22\\-41\\-12\end{pmatrix}$ **(f)** $\begin{pmatrix}-14\\3\\-19\end{pmatrix}$ **(g)** $\begin{pmatrix}-22\\-29\\10\end{pmatrix}$ **(h)** $\begin{pmatrix}24\\10\\12\end{pmatrix}$

**(ii)** **(a)** 34·9 **(b)** 44·6 **(c)** 31·2 **(d)** 21·7

**(e)** 48·1 **(f)** 23·8 **(g)** 37·7 **(h)** 28·6

**5.** **(i)** **(a)** $\begin{pmatrix}5\\-1\\-4\end{pmatrix}$ **(b)** $\begin{pmatrix}3\\5\\-6\end{pmatrix}$ **(c)** $\begin{pmatrix}-7\\-7\\11\end{pmatrix}$ **(d)** $\begin{pmatrix}13\\3\\-14\end{pmatrix}$

**(e)** $\begin{pmatrix}10\\12\\-17\end{pmatrix}$ **(f)** $\begin{pmatrix}-10\\-12\\17\end{pmatrix}$ **(g)** $\begin{pmatrix}16\\-6\\-11\end{pmatrix}$ **(h)** $\begin{pmatrix}-10\\2\\8\end{pmatrix}$

**(ii)** **(a)** 6·5 **(b)** 8·4 **(c)** 14·8 **(d)** 19·3

**(e)** 23·1 **(f)** 23·1 **(g)** 20·3 **(h)** 13·0

**6.** **(a)** $\sqrt{38}$ **(b)** $3\sqrt{3}$ **(c)** $\sqrt{38}$ **(d)** 4 **(e)** $\sqrt{53}$

**7.** $a = \sqrt{3}$

**8.** $\begin{pmatrix}4\\16\\3\end{pmatrix}$; 16·8

**9.** **(a)** $\begin{pmatrix}6\\1\\-8\end{pmatrix}$ **(b)** $\begin{pmatrix}12\\2\\-16\end{pmatrix}$ **(c)** $\begin{pmatrix}\frac{-22}{3}\\\frac{-2}{3}\\-1\end{pmatrix}$

**10.** **(i)** **(a)** $\begin{pmatrix} 10 \\ 18 \\ -7 \end{pmatrix}$ **(b)** $\begin{pmatrix} -24 \\ -12 \\ 5 \end{pmatrix}$ **(c)** $\begin{pmatrix} -8 \\ 0 \\ -1 \end{pmatrix}$ **(d)** $\begin{pmatrix} 0 \\ 0 \\ -3 \end{pmatrix}$

**(ii)** **(a)** 21·7 **(b)** 27·3 **(c)** 8·1 **(d)** 3

**11.** $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$; $\sqrt{11}$

**12.** $a = 2$

**13.** $k = \pm\sqrt{3}$

**14.** 6

**15.** $\begin{pmatrix} -1 \\ 3 \\ 4 \end{pmatrix}$; $\sqrt{26}$ or 5·1