N5 Trig – Sine/Cosine Rules

Applications

SPTA Mathematics – Topic Questions with Notes

Sine Rule

Formula. This formula is <u>given</u> on the National 5 Mathematics exam paper.

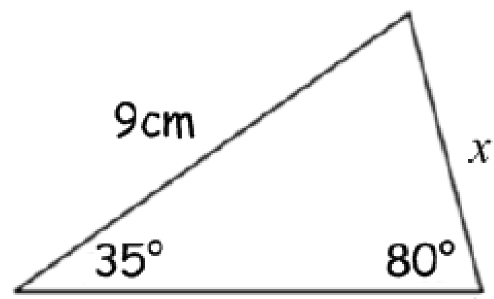
Sine rule $$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Where a, b and c are the lengths of the sides of the triangle, and A, B and C are the angles in the triangle. Side a is opposite angle A etc.

Important: to answer a question you do <u>not</u> use the formula as it is written. You only need the first two 'bits': $\frac{a}{\sin A} = \frac{b}{\sin B}$

<u>Example 1 – sine rule for lengths</u>

Find the length x in this triangle.



Solution

x cm is opposite 35°, so use $a = x$ and $A = 35°$

9cm is opposite 80°, so use $b = 9$ and $B = 80°$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{x}{\sin 35} = \frac{9}{\sin 80}$$

$$x = \frac{9 \sin 35}{\sin 80} \qquad \text{(moving the } \sin 35 \text{ to the other side)}$$

$$x = 5 \cdot 241822996...$$

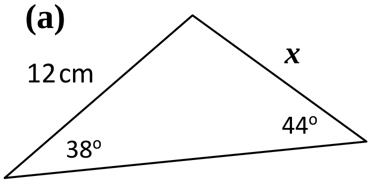
$$\underline{x = 5 \cdot 2\text{cm (1 d.p.)}}$$

If there are *two triangles* in the diagram, you have to use the sine rule (or cosine rule, or SOH CAH TOA) twice. These questions will normally have 4-6 marks.

Exercise 1

1. Use the *sine rule* to calculate the <u>side</u> marked ***x*** in each triangle below.

(a)



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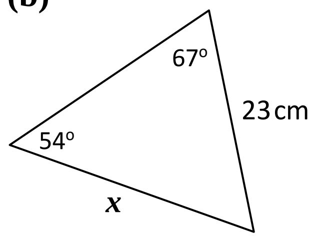

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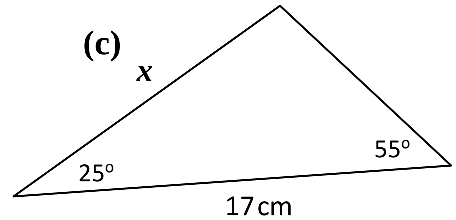

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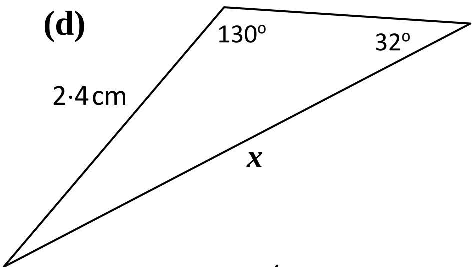

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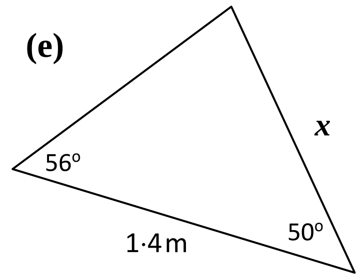

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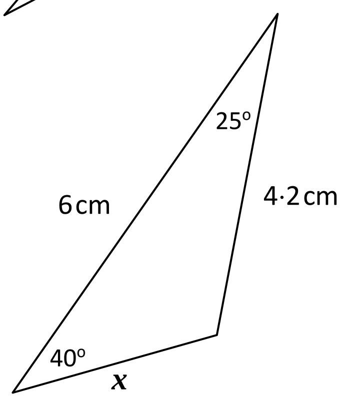

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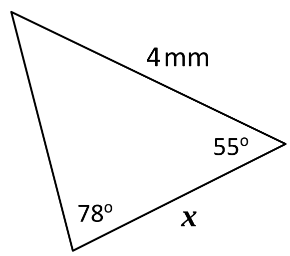

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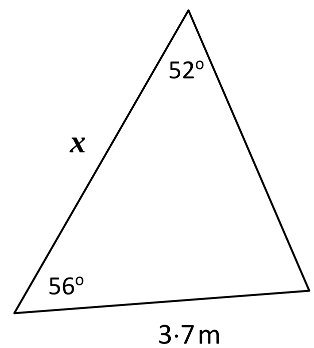
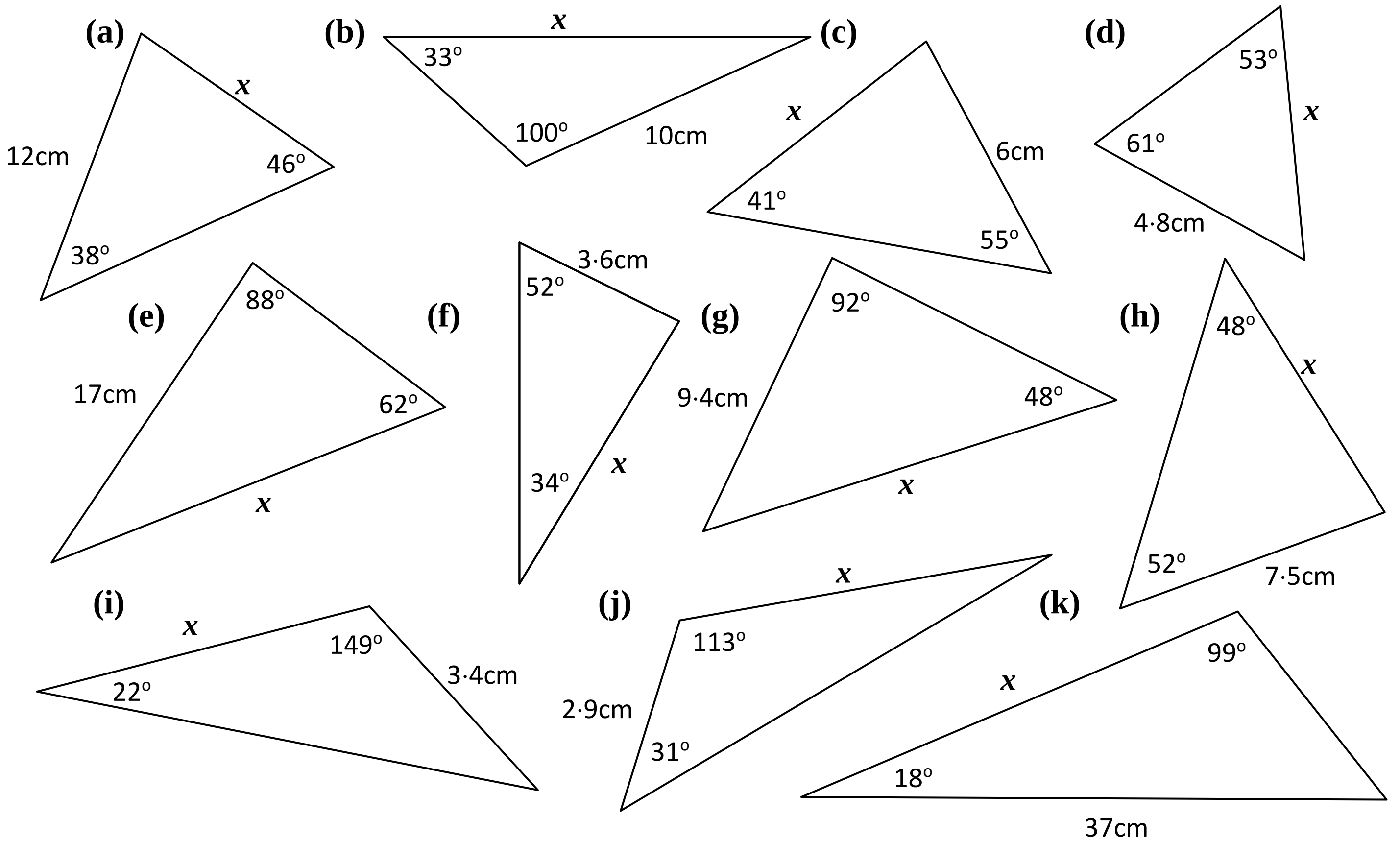
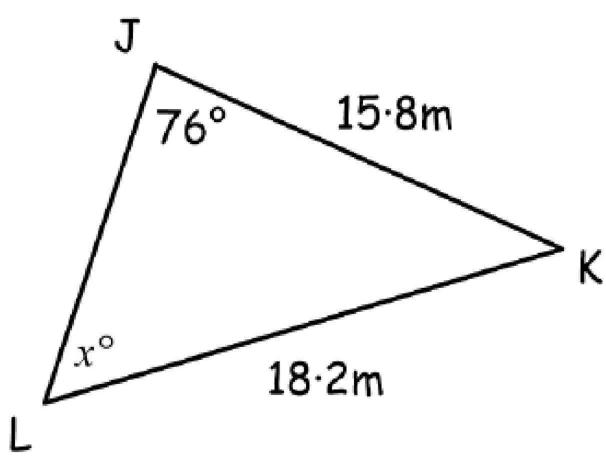


2. Use the sine rule to calculate the length of the side marked x in each of the triangles below.



<u>Example 2 – sine rule for angles</u>

Calculate the size of angle $x°$ in this triangle



Solution

$x°$ is opposite 15·8°, so use $a = 15\cdot8$ and $A = x°$

76° is opposite 18·2m, so use $b = 18\cdot2$ and $B = 76°$

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{15\cdot8}{\sin x} = \frac{18\cdot2}{\sin 76}$$

$$15\cdot8\sin 76 = 18\cdot2\sin x \qquad \text{(cross-multiplying)}$$

$$\sin x = \frac{15\cdot8\sin 76}{18\cdot2} \qquad \text{(dividing by } 18\cdot2 \text{ to make } \sin x \text{ the subject)}$$

$$\sin x = 0\cdot842344...$$

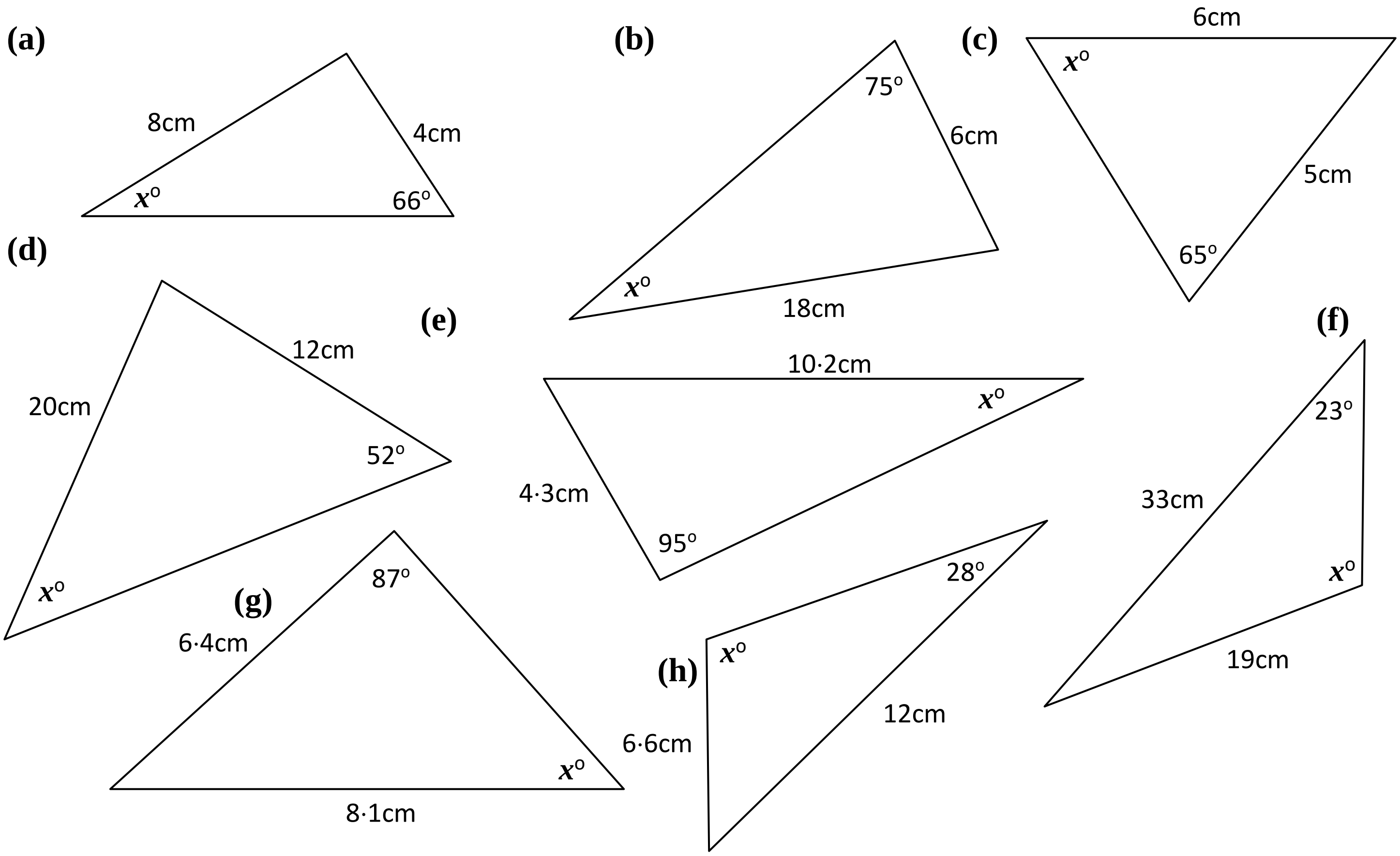
$$x = \sin^{-1} 0\cdot842344...$$

$$x = 57\cdot3885...$$

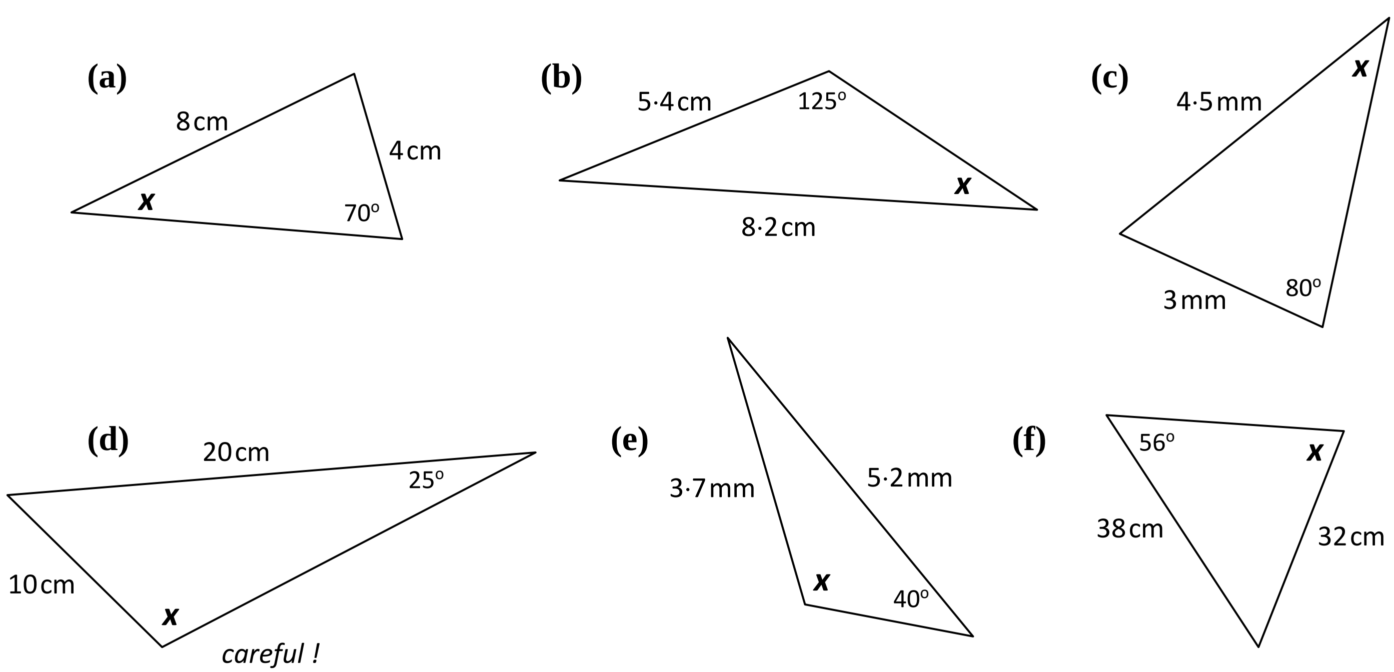
$$x = \underline{\underline{57\cdot4°}} \text{ (1 d.p.)}$$

Exercise 2

1. Use the sine rule to calculate the length of the angle marked $x°$ in each of the triangles below.



2. Use the *sine rule* to calculate the size of the angle marked x in each triangle below.



The Cosine Rule

Formula. This formula is given on the National 5 Mathematics exam paper.

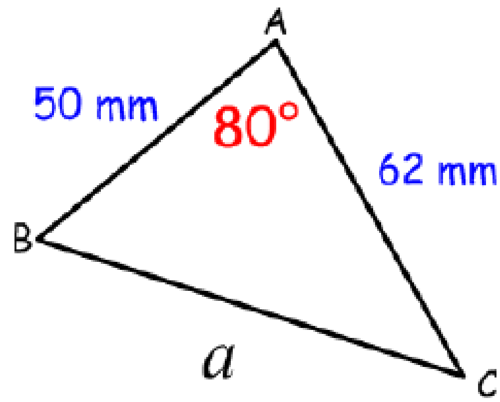
Cosine rule: $a^2 = b^2 + c^2 - 2bc\cos A$ or $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

where A is the angle between the two sides b and c, and a is the length opposite angle A. You use the first version of the formula to calculate a *length*, and the second to calculate an *angle*.

To find a **length** using the cosine rule, you must know the other two sides and the angle in between. It does not matter which side is called b and which is called c.

Example 1

Find the length of *a* in this diagram



Solution

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$= 50^2 + 62^2 - 2 \times 50 \times 62 \times \cos 80$$
$$= 6344 - 6200\cos 80$$
$$= 5267 \cdot 3881298...$$
$$a = \sqrt{5267 \cdot 388...}$$
$$= 72 \cdot 5767...$$
$$= \underline{72 \cdot 6\text{mm}} \text{ (1 d.p.)}$$

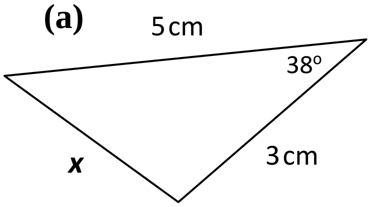
To find an **angle** using the cosine rule, you must know the lengths of all three sides to be able to use this formula. To find an angle, you use the second version of the formula.

In the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$, it is crucial that a must be the side opposite the angle you are finding. It does not matter which way around b and c go.

Exercise 3

1. Use the *cosine rule* to calculate the <u>side</u> marked ***x*** in each triangle below.

(a)



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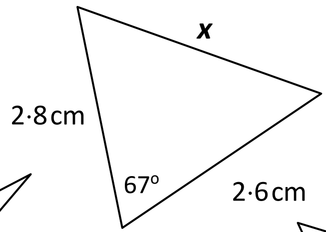

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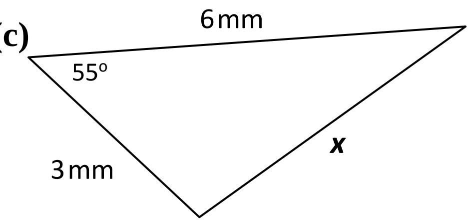

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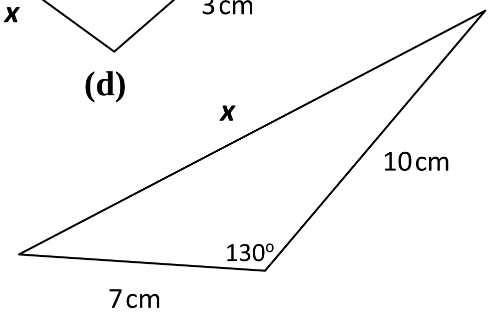

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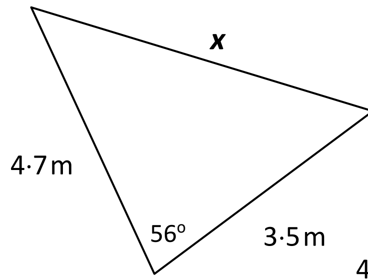

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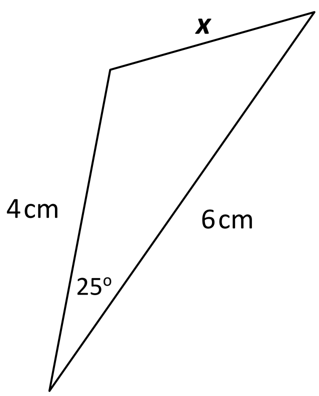

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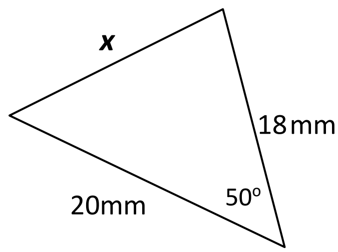

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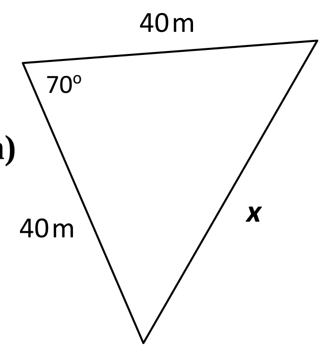
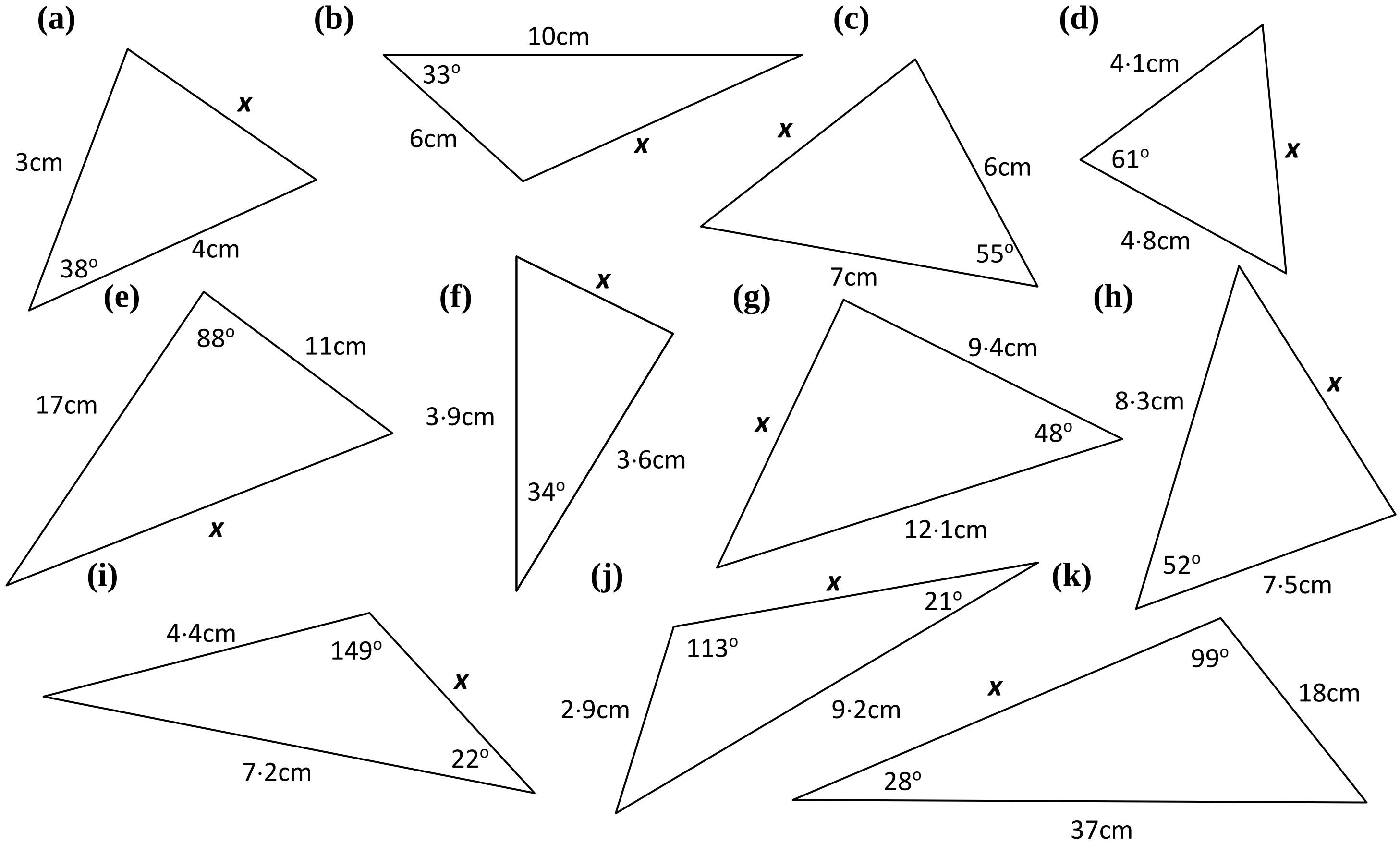
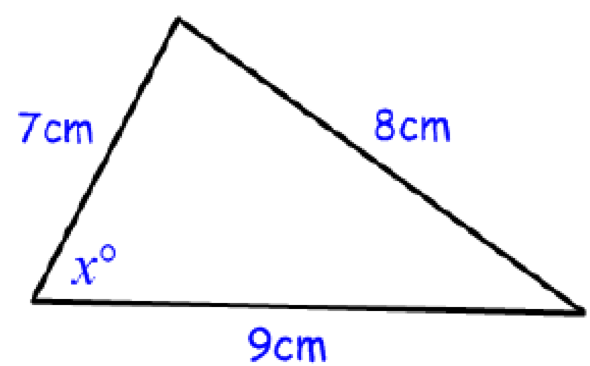


2. Use the cosine rule to calculate the length of the side marked ***x*** in each of the triangles below.



<u>Example 2 – finding an angle</u>

Find the size of angle $x°$ in this diagram



Solution

Length 'a' has to be the side opposite the angle we are finding, so $a = 8$.

It does not matter which way around b and c go, so we will say $b = 7$ and $c = 9$.

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos x = \frac{7^2 + 9^2 - 8^2}{2 \times 7 \times 9}$$

$$= \frac{66}{126}$$

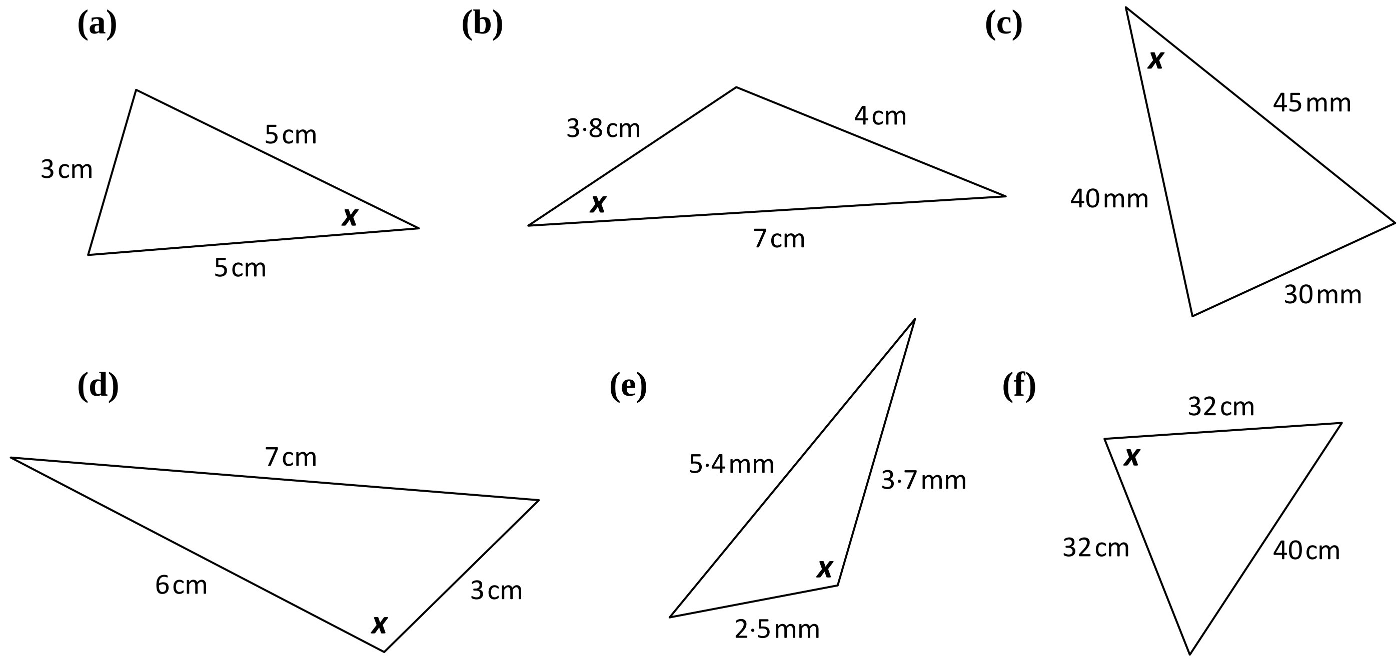
$$x = \cos^{-1}\left(\tfrac{66}{126}\right)$$

$$= 58 \cdot 411864...$$

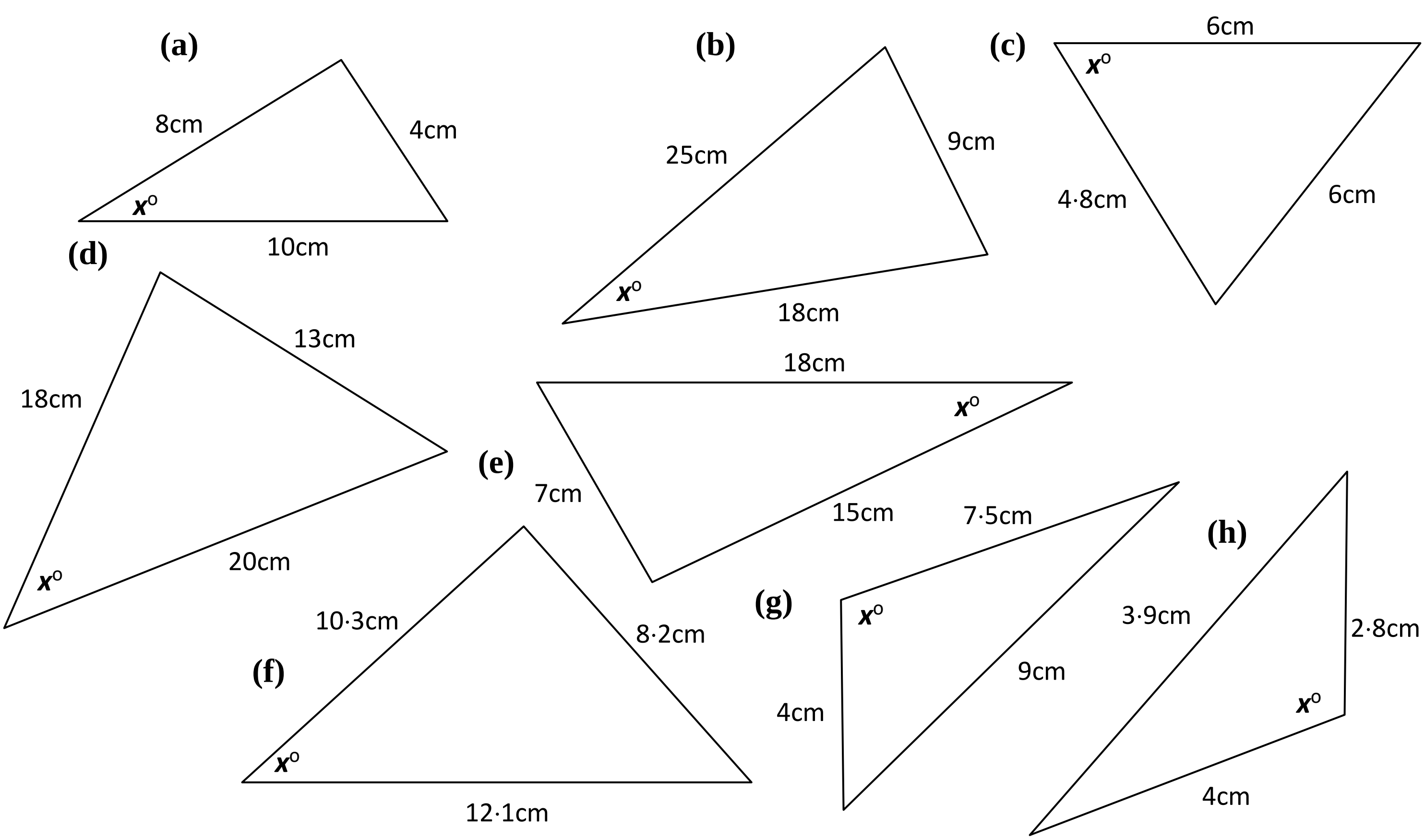
$$= \underline{58 \cdot 4°} \text{ (1 d.p.)}$$

Exercise 4

1. Use the 2^{nd} form of the *cosine rule* to calculate the size of the <u>angle</u> marked ***x*** below.



2. Use the cosine rule to calculate the angle marked ***x***° in each of the triangles below.



Choosing the Formula

Use the key words in the question to decide if you are being asked to calculate the area, an angle, or a length. Use the diagram to see what information you are being given.

Keywords: what is the question asking you to find?

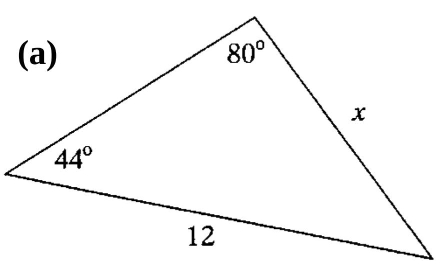
- **AREA:** use $A = \frac{1}{2}ab\sin C$
- **ANGLE:**
  - if you know all three sides use the **cosine rule for angles** $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$
  - if you only know two sides, use the **sine rule** $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$
- **LENGTH (distance, how far, how long, etc):**
  - if you know two sides and the angle in between, use the **cosine rule** $(a^2 = b^2 + c^2 - 2bc\cos A)$
  - if you know at least two angles and at least one other length, use the **sine rule** $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

As a rough rule, if you know (or can work out) all of the angles in the triangle, you are probably going to use the sine rule.

Exercise 5

1. Calculate the value of *x* in each triangle below.

(a)



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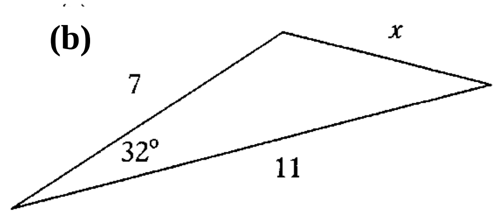

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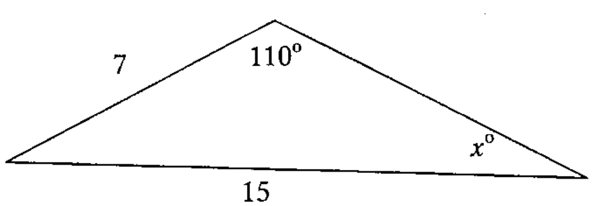
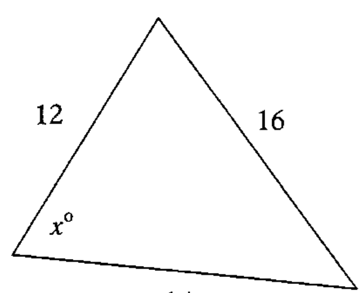


(d)

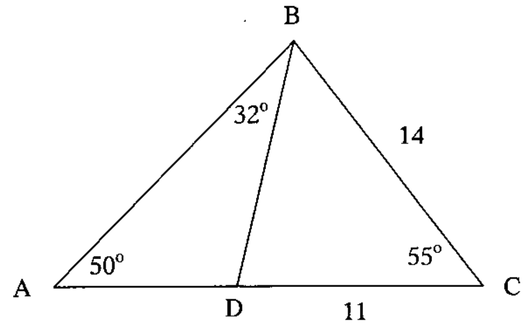


2. Calculate the area of the triangle with sides measuring 12 cm, 14 cm and 20 cm.

3. **(a)** Calculate the length of BD.

(b) Calculate the length of AD.

(c) Calculate the area of triangle ABC



4. From the framework opposite:

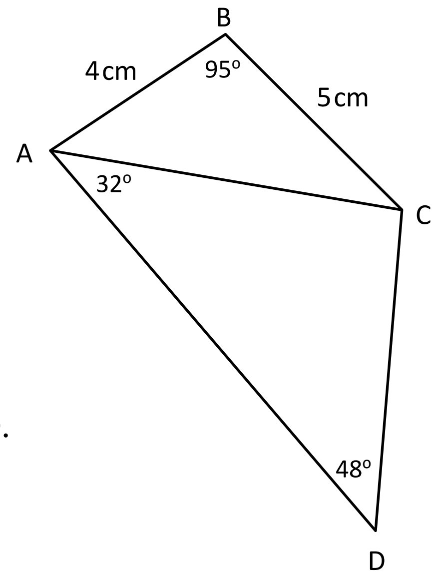
(a) Calculate the length of AC.

(b) Calculate the size of $\angle$BAC.

(c) Write down the size of $\angle$ACD.

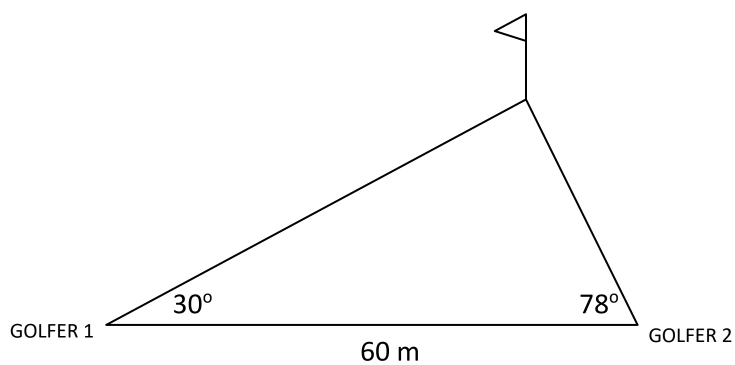
(d) Calculate the length of AD.

(e) Calculate the area of the quadrilateral ABCD.



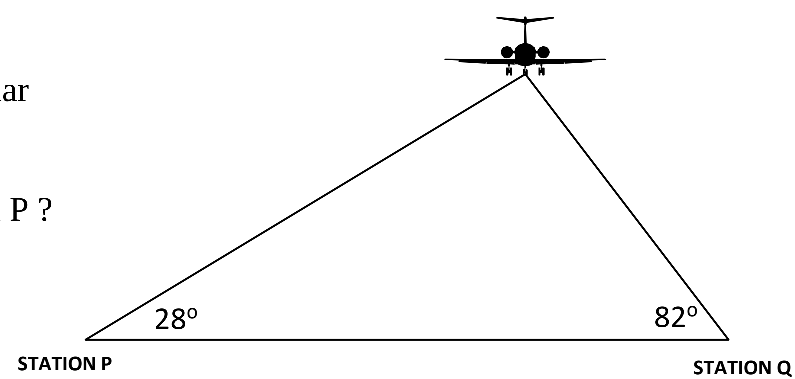
5. Two golfers are aiming for the green.
The golfers are 60 m apart and the angles are as shown in the diagram.

What distance will each golfer have to hit the ball in order to reach the pin?

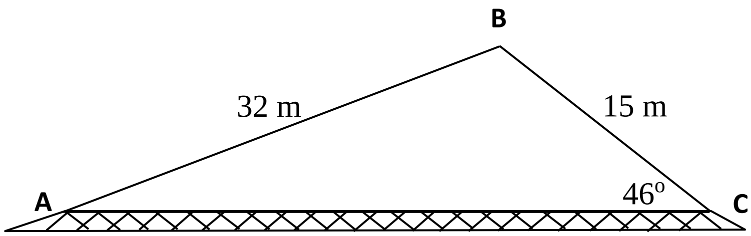


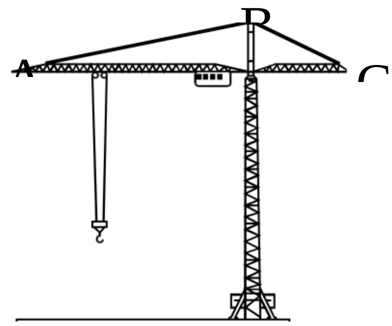
6. An aircraft is picked up by two radar stations, P and Q, 120 km apart.

How far is the aircraft from station P ?



7. A large crane is being used in the construction of a block of flats. The crossbeam is supported by two metal stays.

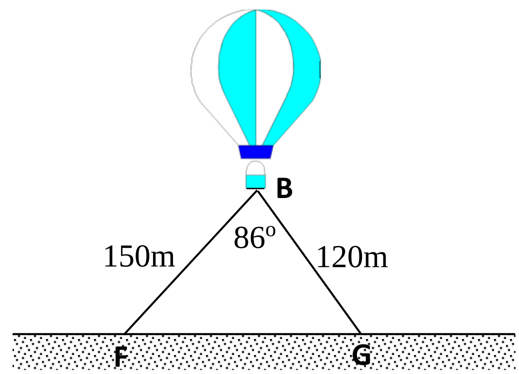




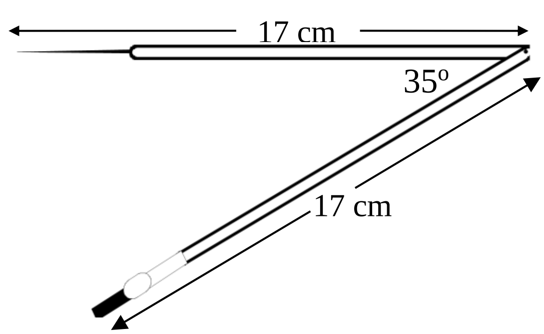
The length of AB is 32 m and the length of BC is 15 m. ∠BCA is 46°.
Calculate the size of ∠BAC and the length of the crossbeam AC.

8. A hot air balloon B is fixed to the ground at F and G by 2 ropes 120m and 150 m long.

If ∠FBG is 86°, how far apart are F and G?



9.



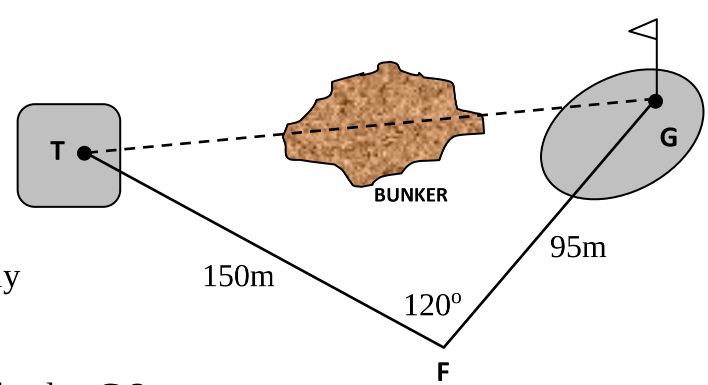
A set of compasses is shown where the angle between the arms is set at 35°

Calculate the diameter of the circle which could be drawn with the arms in this position.

10. During a golf match, Ian discovers that he has forgotten his sand wedge, so to avoid the bunker he plays a shot from T to F and then from F to G.

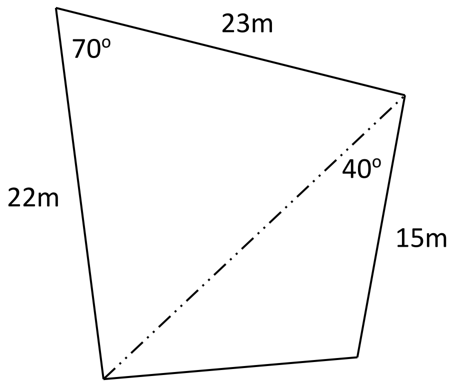
His opponent Fred decides to play directly from T to G.

How far will Fred need to hit his shot to land at G ?



Mixed Exercise

1. The sketch below shows a plot of land purchased to build a house on.

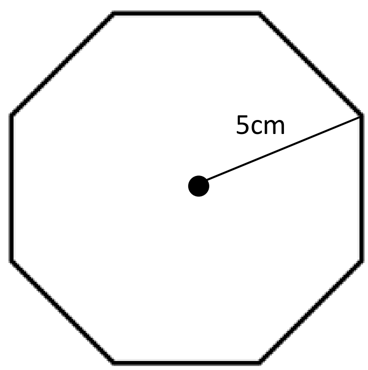


At present the land is valued £280 per square metre.

Calculate the value of the plot shown to the nearest £10.

2. The distance from the centre of a regular octagon to one of its vertexes is 5 cm.

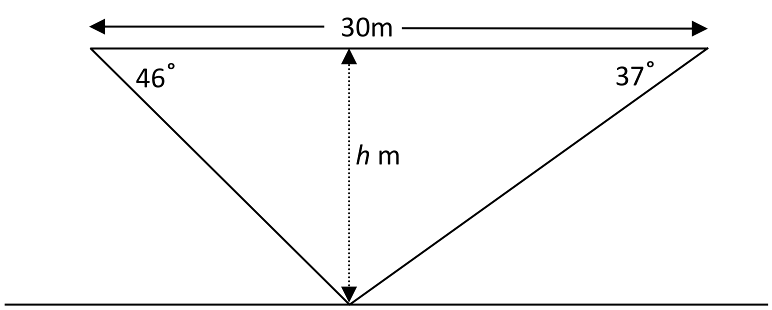
Calculate the area of the octagon.



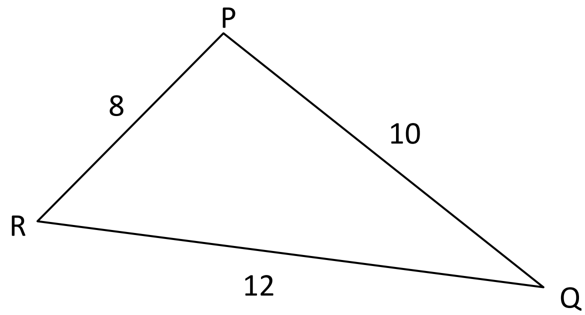
3. Two security cameras are positioned on a beam in a warehouse 30 metres apart.

One camera has an angle of depression of 37° and the other camera has an angle of depression of 46°.

Calculate the height, *h* metres, of the beam above the ground.



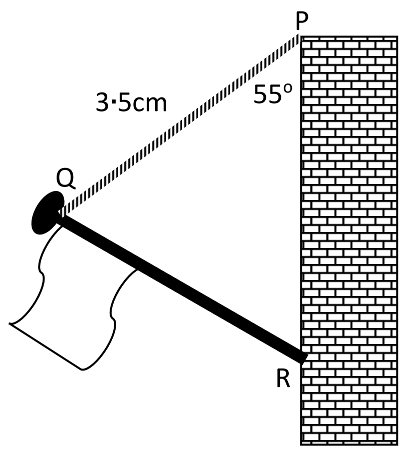
4. Triangle PQR has sides with lengths, in centimetres, as shown.



Show clearly that cos PQR $= 0 \cdot 75$.

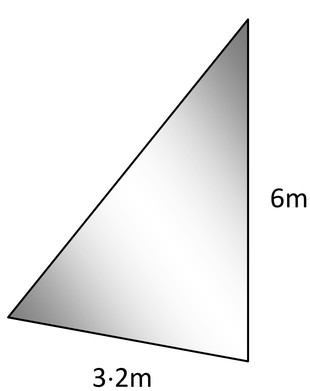
5. A flagpole is attached to a wall and is supported by a wire PQ as shown in the diagram.

The wire is 3·5 metres long and makes an angle of 55° with the vertical wall.



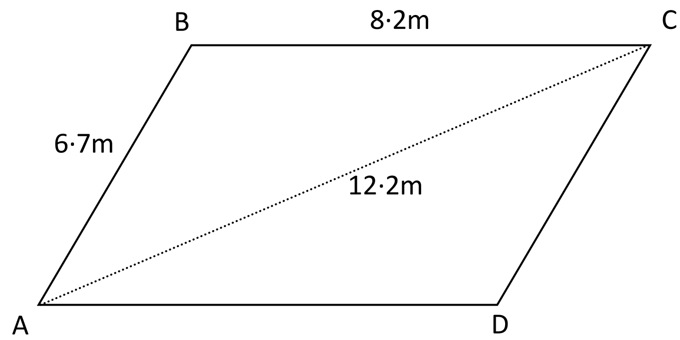
Given that the point P is 4·5metres above R in the diagram, calculate the length of the flagpole.

6. A triangular sail designed for a racing yacht is shown below.
Two of its edges measure 6 metres and 3·2 metres.



Given that the sail has a **perimeter** of 15·5 metres, calculate the **area** of the sail.

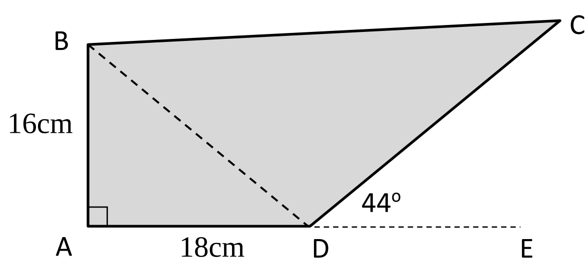
7. A sketch of Lee's garden is shown below.



(a) Calculate the size of angle ABC.

(b) Hence, or otherwise, calculate the area of the garden.

8. The diagram below shows a steel plate ABCD.



$AB = 16\text{cm}, \; AD = 18\text{cm}$

$\angle DAB = 90^{o}$

(a) Calculate the length of BD correct to 1 decimal place.

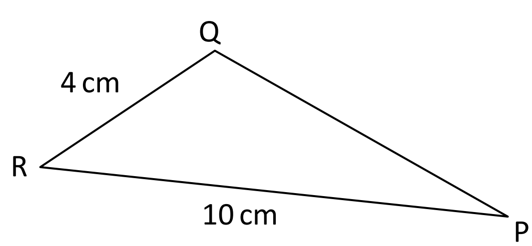
(b) Find the size of angle BDC correct to the nearest degree.

(c) Hence calculate the length of BC given that $DC = 25\text{cm}$.

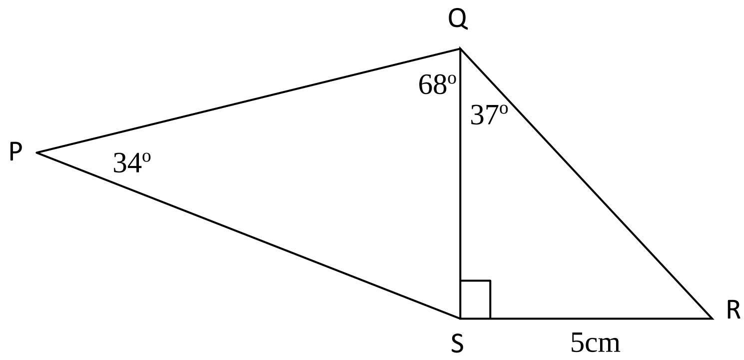
9. In triangle PQR, PR = 10 cm QR = 4 cm.

The perimeter of the triangle is 22 cm.

Find the size of angle PQR.

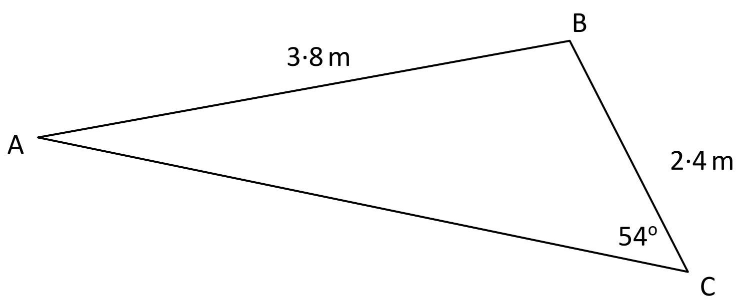


10. In the diagram shown SR = 5cm, angle SQR = 37°, angle QPS = 34° and angle PQS = 68°.



Calculate the length of PS.

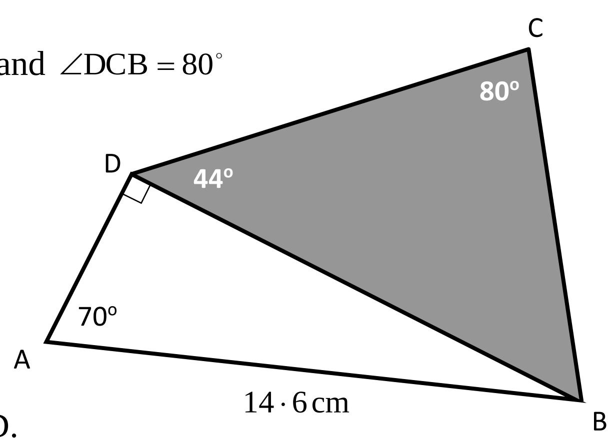
11. Calculate the size of angle BAC in this triangle.



12. In the diagram ABCD represents a steel framework with BCD being a triangular steel plate.

Angle ADB is a right angle.

$AB = 14 \cdot 6\,\text{cm}$, $\angle BAC = 70^\circ$, $\angle BDC = 44^\circ$ and $\angle DCB = 80^\circ$



(a) Find the length of DB.

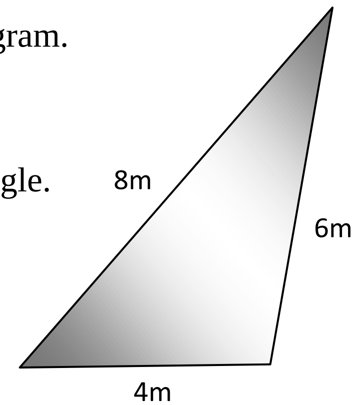
(b) Calculate the area of triangle BCD.

13. A triangular sail has measurements as shown in the diagram.

All lengths are in metres.

(a) Calculate the size of the largest angle in the triangle.

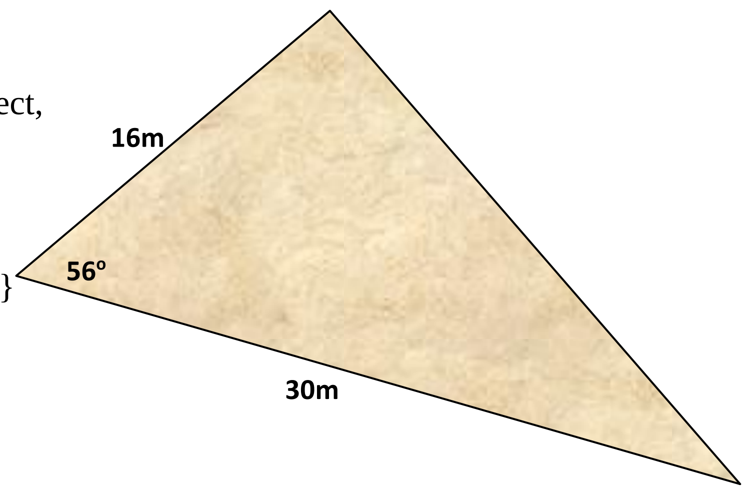
(b) Calculate the area of the sail in square metres.



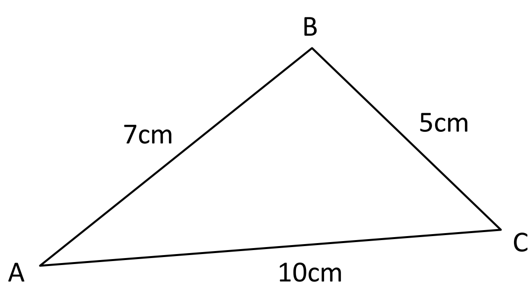
14. A building company has to fence off a triangular piece of waste ground.

The plan of the ground is shown below. All lengths are in metres.

If the fence costs £18.50 per metre to erect, how much will the company have to pay in total to fence off this piece of ground?
{Fencing is priced in whole metres only}



15. **(a)** Calculate the value of cos ABC in this triangle.



(b) Without actually calculating the size of the angle a pupil was able to say that angle ABC was obtuse.

By referring to your answer in (a), explain why the pupil was able to do this.

Answers

Exercise 1

1.	**a)**	10·6cm	**(b)**	26·2cm	**(c)**	14·1cm	**(d)**	3·5cm
	e)	1·2m	**(f)**	2·8cm	**(g)**	3mm	**(h)**	4·5m
2.	**a)**	10·3 cm	**(b)**	18·1 cm	**(c)**	7·5 cm	**(d)**	5·3 cm
	e)	19·2 cm	**(f)**	5·1 cm	**(g)**	12·6 cm	**(h)**	8·0 cm
	i)	4·7 cm	**(j)**	2·5 cm	**(k)**	33·4 cm		

Exercise 2

1.	**a)**	$27{\cdot}2^{o}$	**(b)**	$18{\cdot}8^{o}$	**(c)**	$49{\cdot}0^{o}$	**(d)**	$28{\cdot}2^{o}$
	e)	$24{\cdot}8^{o}$	**(f)**	$137{\cdot}3^{o}$	**(g)**	$52{\cdot}1^{o}$	**(h)**	121^{o}
2.	**a)**	28^{o}	**(b)**	$32{\cdot}6^{o}$	**(c)**	41^{o}	**(d)**	$122{\cdot}3^{o}$
	e)	$115{\cdot}4^{o}$	**(f)**	$79{\cdot}9^{o}$				

Exercise 3

1.	**a)**	3·2cm	**(b)**	3cm	**(c)**	4·9mm	**(d)**	15·5cm
	e)	4m	**(f)**	2·9cm	**(g)**	16·2mm	**(h)**	45·9m
2.	**a)**	2·5 cm	**(b)**	5·9 cm	**(c)**	6·1 cm	**(d)**	4·6 cm
	e)	19·9 cm	**(f)**	2·2 cm	**(g)**	9·1 cm	**(h)**	7 cm
	i)	2·9 cm	**(j)**	7·5 cm	**(k)**	29·9 cm		

Exercise 4

1. **a)** 34·9° **(b)** 26·9° **(c)** 40·8°
d) 96·4° **(e)** 119·9° **(f)** 77·4°

2. **a)** 22·3° **(b)** 15·3° **(c)** 66·4° **(d)** 39·6°
e) 22·2° **(f)** 42·0° **(g)** 98·4° **(h)** 67·3°

Exercise 5

1. **a)** 8·5 **(b)** 6·3 **(c)** 26° **(d)** 75·5°

2. 82·6cm^2

3. **a)** 11·8 **(b)** 8·2 **(c)** 110

4. **a)** 6·7cm **(b)** 48° **(c)** 100° **(d)** 8·9cm
e) 25·8cm^2

5. Golfer 1: 61·7m Golfer 2: 31·5m

6. 126·km

7. 19·7° ; 40·5m

8. 185·4m

9. 20·4cm

10. 214m

Mixed Exercise

1. £101 390

2. 70·7cm²

3. 13m

4. Proof

5. 3·8m

6. 9·46m²

7. **a)** 110° **(b)** 51·6m²

8. **a)** 24·1cm **(b)** 94° **(c)** 36cm

9. 108°

10. 11cm

11. 30·7°

12. **a)** 13·7cm **(b)** 54·7cm²

13. **a)** 104·5° **(b)** 11·6m²

14. £1313.50

15. **a)** – 13/35 [or equivalent] **(b)** cosine is negative