

Definition: the standard deviation of a list of numbers is a measure of how spread out (varied) the numbers are from the mean.

Formula. These formulae are given on the National 5 Mathematics exam paper. standard deviation = $\sqrt{\frac{\sum (x - \overline{x})^2}{n-1}}$ or $\sqrt{\frac{\sum x^2 - (\sum x)^2}{n}}$

Where n is how many numbers are in the list, \overline{x} is the mean and Σ means "add together"

You only need to use one of these formulae. In general, it is more helpful to just know the method rather than memorising the formula.

Example

- a) Find the mean of these five numbers:
 - 2, 3, 9, 6, 5
- b) Find the standard deviation of the same five numbers

Solution

- a) $\frac{2+3+9+6+5}{5} = \frac{25}{5} = 5$, so the <u>mean is 5</u>
- b) You have a choice of two methods:

Method 1 – using the formula $s = \sqrt{\frac{\sum (x - \overline{x})^2}{n - 1}}$

<u>Step 1</u> - Draw up a table showing $x, x - \overline{x}$ and $(x-\overline{x})^2$

x	$x - \overline{x}$	$(x-\overline{x})^2$
2		
3		
9		
6		
5		

<u>Step 2</u> – Complete the table,
remembering that
\overline{x} = the mean = 5.
Step 3 – find the total of the final column So $\sum (x - \overline{x})^2 = 30$

x	$x - \overline{x}$	$(x-\overline{x})^2$
2	-3	9
3	-2	4
9	4	16
6	1	1
5	0	0
TO	ΓAL	30

<u>Step 4</u> – use the formula, remembering that n = 5 as there were five numbers.

$$s = \sqrt{\frac{\sum (x - \overline{x})^2}{n - 1}}$$

= $\sqrt{\frac{30}{5 - 1}}$
= $\sqrt{\frac{30}{4}} = 2 \cdot 74 \ (2 \text{ d.p.})$

Method 2 – using the formula: $s =$	$\sqrt{\frac{\sum x^2 - (\sum x)}{n-1}}$
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<u>Step 1</u> - Draw up a table showing x and x^2

x	x^2
2	
3	
9	
6	
5	

Step	2 –	Compl	lete	the	tabl	e
_		-				

Ste	<u>p 3</u> – find t	he totals	
So	$\sum x = 25,$	$\sum x^2 = 13$	55

	x	x^2
	2	4
	3	9
	9	81
	6	36
	5	25
TOTAL	25	155

<u>Step 4</u> – use the formula, remembering that n = 5 as there are 5 numbers.

$$s = \sqrt{\frac{\sum x^2 - (\sum x)^2}{n}}$$

= $\sqrt{\frac{155 - \frac{25^2}{5}}{5 - 1}} = \sqrt{\frac{(155 - 125)}{4}}$
= $\sqrt{\frac{30}{4}}$
= $2 \cdot 74 \ (2 \text{ d.p.})$

Exercise 1

1. Calculate the mean and standard deviation for the following sets of data.

a)	20	21	19	22	21	20	19	20	21	20	
b)	303	299	306	298	304	307	299	302	305	299	300
c)	15.3	14.9	15.1	15.2	14.8	14.7	15.1	14.8	15.0	15.0	
d)	87	89	84	88	89	87	86	87	86	87	
e)	48	73	29	82	54	43	95	41	92	71	
f)	4.4	4.6	4.8	4·0	4.2	4.3	4.5	4.7	4.9	4.1	
g)	0.2	0.3	0.4	0.2	0.2	0.0	0.4	0.1	0.2	0.3	
h)	40	40	39	38	38	40	40	42	40	39	

2. A third year pupil conducting an experiment with a die got the following results

6	1	1	4	4	2	2	6	5	6
1	1	1	5	1	4	2	3	4	6
1	4	4	1	5	4	4	3	6	2
5	3	5	6	3	2	6	5	5	2
3	1	4	5	2	4	1	4	4	3

- a) Show these results in a frequency table
- **b**) Use your table to calculate the mean and standard deviation.
- **3**. A company that manufactures shoelaces spot checks the length (in cm) of the laces.

Here are the results for two different production lines.

Line A	26.8	27.2	26.5	27.0	27.3	27.5	26.1	26.4	27.9	27.3
Line B	26.8	26.7	27·1	27.0	26.9	27·0	27.3	26.9	27·0	27.3

Calculate the mean and standard deviation and comment on any differences between line A and line B.

4. The running times, in minutes, of films shown on television over a week are as follows.

110	95	135	70	100	125	140	105	95	105
95	95	110	90	110	100	125	105	90	120

Calculate the mean and standard deviation.

5. The temperatures, in °C, at a seaside resort were recorded at noon over a 10-day period.

19	20	19	17	21	18	19	24	25	28
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Calculate the mean and standard deviation.

6. John James plays golf with his brother Joe each month. They keep a note of their scores.

John	74	73	74	73	71	73	72	75	73	73	72	73
Joe	68	74	70	67	80	81	69	68	79	67	70	71

Calculate the mean and standard deviation and comment on John's and Joe's performance over the year.

7. The weekly takings in small store, to the nearest £, for a week in December and March are shown below

December	2131	2893	2429	3519	4096	4810
March	1727	2148	1825	2397	2901	3114

Calculate the mean and standard deviation and comment on any differences.

8. Two sixth year classes take part in a Sponsored Fast for Famine Relief. The number of hours each pupil lasted are shown below.

6C1	20	22	21	20	22	20	22	20	20	24	21	22	23	22	22	23
6C2	15	20	24	23	22	24	18	24	22	23	24	17	20	24	24	20

Calculate the mean and standard deviation for each class and comment on how well each class did.

Comparing Statistics

The mean, median and mode are <u>averages</u>. They say whether a list of numbers is higher or lower on average.

The range, semi interquartile range and standard deviation are <u>measures of spread</u>. They say whether a list of numbers is more or less varied/consistent.

- A lower range, IQR or standard deviation means the numbers are more consistent.
- A higher range, IQR or standard deviation means the numbers are more varied.

Example

The temperature in Aberdeen has a mean of 3°C and a standard deviation of 5.

In London it has a mean of 9°C and a standard deviation of 3.

Compare the temperatures in London and Aberdeen.

You would get NO MARKS (as you are stating the obvious) for:

- "Aberdeen has a lower mean"
- "London has a higher mean"
- "Aberdeen has a higher standard deviation"
- "London has a lower standard deviation".

You would get NO MARKS (as your sentence makes no sense) for:

- "Aberdeen is lower" (no mention of temperature)
- "The first one has a lower temperature" (no mention of Aberdeen or London)
- "In London it is more consistent" (no mention of what 'it' is)

You WOULD get marks for:

- The temperature in Aberdeen is lower than London and the temperature is less consistent
- The temperature in London is higher and more consistent than Aberdeen" or similar

Exercise 2

1. The weights of 6 plums are

40·5g	37·8g	42·1g	35·9g	46·3g	41·6g					
a)	a) Calculate the mean and standard deviation.									
The we	eights of 6 apple	es are								
140·5g	137·8g	142·1g	135·9g	146·3g	141·6g					

b) **Write down** the mean and standard deviation.

2. During a recent rowing competition the times, in minutes, recorded for a 2000 metre race were

7.2 7.3 7.3 7.5 7.6 8.4

a) Calculate the mean and standard deviation of these times. Give both answers correct to 2 decimal places.

b) In the next race the mean time was 7.76 and the standard deviation was 0.49.Make two valid comments about this race compared to the one in part (a).

3. 6 friends joined "Super Slimmers", a weight loss class. Their weights were recorded and the results are shown below.

65kg 72kg 74kg 81kg 90kg 98kg

a) Calculate the mean and standard deviation of the weights.

After 6 weeks the mean weight was 74kg and the standard deviation was 8.6

b) Compare the mean and standard deviation of the friend's weights.

4. Stewart and Jenni complete a crossword puzzle every day. Here are the times (in minutes) that Stewart took to complete it each day for a week.

63 71 68 59 69 75 57

a) Calculate the mean and standard deviation for Stewart's times.

Every day Jenni took exactly 5 minutes longer than Stewart to complete the puzzle.

b) Write down Jenni's mean and standard deviation.

5. The number of hours spent studying by a group of 6 student nurses over a week were

20 23 14 21 27 24

- **a**) Calculate the mean and standard deviation of this data.
- b) A group of student teachers had a mean of 21.5 and a standard deviation of 6.Make two valid comments to compare the study times of the 2 groups of students.
- 6. Barbara is looking for a new 'A-Pod' and searches for the best deal.

The costs of the 'A-Pod' are shown below.

- £175 £185 £115 £87 £150 £230
- **a**) Calculate the mean and standard deviation of the above data.
- b) A leading competitor, the 'E-Pod', has a mean price of £170 and a standard deviation of 26.7. Make two valid comparisons between the 2 products.
- 7. In Bramley's Toy Shop there are 6 styles of teddy bear. The price of each is sh

£19 £25 £17 £32 £20 £22

a) Calculate the mean and standard deviation of these prices.

In the same shop the prices of the dolls have a mean of £22.50 and a standard deviation of $2 \cdot 3$.

b) Compare the two sets of data making particular reference to the spread of the prices.



Answers

Exercise 1

1.		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)			
	mean	20.3	302	14.99	87	62.8	4.45	0.23	39.6			
	SD	0.95	3.19	0.19	1.49	22.9	0.30	0.13	1.17			
2.	3.44, 1.72											
3.	line A 27, 0.55; line B 27,0.19; line B more consistent											
4.	106, 16.7											
5.	21, 3.6											
6.	John 73, 1·04	4;Joe 72,	5.20	Joe has perforr	Joe has lower mean score but John has better overall performance (lower standard deviation)							
7.	Dec 3313, 10	025; Mar 2	2352, 565	Decem variatio	December has higher mean takings but March has less variation in takings							
8.	6C1 21·5,1·2	6;6C2 21	l·5, 2·88	Same a	Same average but 6C1 has lower SD so less spread out							
Exerc	ise 2											

1.	a)	40·7g, 3·6	(b)	140·7g, 3·6
2.	a)	7.55; 0.44	(b)	slightly higher mean so slower times on average in 2nd race higher SD so times are less consistent than 1st race
3.	a)	80kg, 12·2	(b)	on average weight is less and less spread out
4.	a)	66; 6.56	(b)	71; 6.56
5.	a)	21.5; 4.42	(b)	On average study times same but teachers are more varied
6.	a)	£157, 51·3	(b)	on average E-Pod more expensive and less spread out
7.	a)	£22·50, 5·4	(b)	prices of dolls are less spread out than teddies