N5 Percentages

Expressions & Formulae

SPTA Mathematics – Topic Questions with Notes

Revision of Basic Percentages

Exercise 1

1. Calculate:

   (a) 50% of £25·50 (b) 75% of £28 (c) 25% of £4·40

   (d) 10% of £6·80 (e) 20% of £45 (f) 30% of £160

   (g) 40% of £18 (h) 60% of £8 (i) 70% of £5

   (j) 80% of £9·50 (k) 90% of £2200 (l) 15% of £3

   (m) 17·5% of £400 (n) 22·5% of £200 (o) 8·2% of £600

   (p) $17\frac{1}{2}$% of £20 (q) $8\frac{1}{2}$% of £40 (r) $12\frac{1}{2}$% of £4

2. What is:

   (a) $33\frac{1}{3}$% of £90? (b) $66\frac{2}{3}$% of £120?

3. At a dance, only 28% of the 150 people were female.
   How many were: (i) female? (ii) male?

4. A bottle holds 500 millilitres of diluted juice. 96·5% of this is water.
   How many millilitres of water is this?

5. Mavis bought a 750 gram box of chocolates on Saturday afternoon.
   By evening only 15% of them were left.
   What weight of chocolates remained?

6. The village of Elderslie has 3800 residents. Only 2% of them attended a local meeting.

   (a) How many villagers attended the meeting?

   (b) How many did not bother to go?

7. A jet was flying at 32 000 feet when one of its engines failed.
The jet dropped by 42% in height. By how many feet did it drop?

8. When David was 14 he was 140 cm tall. Over the next year he grew by 2·5%.
What was his height when he reached 15 years?

9. At Stanford City Football Club, 95% of its home support are season ticket holders.
The stadium has room for 44 200 home supporters.
How many home supporters do not have a season ticket?

10. Mrs. Nicolson borrows £1200. She must pay back the loan plus interest at a rate of 9% per year.
Calculate the amount she has to pay if she wishes to pay back the loan (plus interest) in:
(a) 1 year (b) 6 months (c) 9 months (d) 4 months (e) 5 months.

11. Of the 40 guests at a party, 32 of them were women.
What percentage were women?

12. Of the 180 cars which took part in a rally, 45 of them were green.
What percentage of them were not green?

13. From my weekly pay of £280, I spend £84 in rent.
What percentage of my pay do I spend on rent?

14. 2000 people were stuck at the airport, due to flight delays.
The first flight to leave was to Orkney. It left carrying 72 of the people.
What percentage of the people already at the airport remained there?

Percentages

In National 5 exam percentage questions, you will always be asked to increase or decrease an amount by a percentage – this will usually be either **compound interest**, or **appreciation** or **depreciation**.

For every question, there is a longer way and a quicker way to do it. Use the one you are happiest with. In the examples below, the quicker method will be preferred.

Percentage	Longer method	Quicker method
3% increase	Multiply by **0·03**, then add answer on	*[100% + 3% = 103%]* Multiply by **1·03**
3% decrease	Multiply by **0·03**, then take answer away	*[100% – 3% = 97%]* Multiply by **0·97**
2·4% increase	Multiply by **0·024**, then add answer on	*[100% + 2·4% = 102·4%]* Multiply by **1·024**
15% decrease	Multiply by **0·15**, then take answer away	*[100% – 15% = 85%]* Multiply by **0·85**
4·5% decrease	Multiply by **0·045**, then take answer away	*[100% – 4·5% = 95·5%]* Multiply by **0·955**

Compound Interest

Compound interest is always an example of **appreciation** (because the amount in the account is always going up), so you always add the amount on each time.

There are two types of questions you may be asked about compound interest:

- If the question asks **how much money** is in the account (the **balance**) you do the appreciation calculation as normal. See Example 1.
- If the question asks what the **interest** was, you do the appreciation calculation and then *subtract* the original amount. See Example 2.

Example 1

A bank account pays 7% interest per annum. £3000 is invested in the account. How much money is in the account after 15 years?

Solution

The longer method is not a good method here unless you want to do 15 lines of calculations. We use the quicker method.

Interest is appreciation. For 7% interest, the multiplier is 1·07.

$3000 \times 1 \cdot 07^{15} = 8277 \cdot 094622$

$= \underline{\underline{£8277 \cdot 09}}$ (must show units (£) and round to 2 d.p. for money)

Example 2

A savings account pays 2·4% interest per annum. If you put £2500 in the account, how much compound interest will you receive after 3 years?

Longer method	Quicker method
Year 1: 0·024 × 2500 = £60 2500 + 60 = £2560	
Year 2: 0·024 × 2560 = £61·44 2560 + 61·44 = £2621·44	2500 × 1·024³ = 2684·35
Year 3: 0·024 × 2621·44 = £62·91 2621·44 + 62·91 = £2684·35	
Interest: 2684·35 − 2500 = £184·35	**Interest:** 2684·35 − 2500 = £184·35

Exercise 2

1. At the beginning of the year, Mr. Bradford borrows £5000 from the bank. The rate of compound interest is 8%. He agrees to pay back £108 per month.

Calculate how much he still owes at the end of the second year.

2. For each of the investments below, calculate

(i) the amount due at the end of the term

(ii) the total interest

	Bank/ Building Society	**Amount Invested (£)**	**Rate of interest (per year)**	**Number of Years**
a)	Hamilton Bank	2000	8 %	2
b)	Allied Friendly	5000	6 %	3
c)	Northern Hill	4800	7 %	2
d)	Highland Bank	3500	7·5 %	3
e)	Church National	1600	5·5 %	4
f)	Southern Rock	1750	11 %	3
g)	London Savings Bank	20 000	6%	3
h)	Bath & Eastern	18 000	8·5%	2
i)	Royal Bank of Britain	50 000	9%	3
j)	Bingford & Bradley	400	4·8%	2

Appreciation and Depreciation

Definition: Appreciation means an increase in value.
Definition: Depreciation means a decrease in value.

Example 1

Peterhead has a population of 30 000. Its population depreciates by 15% per year. What is its population after two years? Round your answer to 3 significant figures.

Solution

Depreciation means decrease, so we will be taking away.
100% − 15% = 85%, so we use **0·85** in the quicker method.
The question is for two years so need to repeat 2 times (a power of 2).

Longer method	Quicker method
Year 1: 0·15×30000 = 4500 30000 − 4500 = 25500	
Year 2: 0·15×25500 = 3825 25500 − 3825 = 21675	$30000 \times 0{\cdot}85^2 = 21675$
Answer: 21700 (3 s.f.)	**Answer:** 21700 (3 s.f.)

(remember we must state the unrounded answer (21675) first before rounding).

Formula. This formula is **not** given on the National 5 Mathematics exam paper.

The percentage increase or decrease is found using $\frac{\text{increase (or decrease)}}{\text{original amount}} \times 100$

Example 2 – find the percentage first

A house cost £240 000 when first bought. One year later its value has appreciated to £250 800.

a) **Find the rate of appreciation** (in an exam, this part would have 1 mark)
b) **If the house continues to appreciate at this rate, what will its value be after a further 4 years?**

Solution

a) The increase is 250 800 – 240 000=£10 800

Using the formula, the percentage increase is given by:

$$\frac{10800}{240000}\times 100 = \underline{\underline{4\cdot 5\%}}$$

b) Using the quicker method:

Appreciation means increase, so we will be adding.
100% + 4·5% = 104·5%, so we use **1·045** in the quicker method.
The question is for four years so need to repeat 4 times (a power of 4).
(Note the question says a further *four years – so we start with £250 800 not £240 000).*

$$250800\times 1\cdot 045^4 = 299083\cdot 665$$

Answer: after a further 4 years, the house will be worth £299 000 (3 s.f.)

Example 3 – no start value

A vintage car depreciated in value by 5% and then appreciated in value by 12%. How much had its value changed overall?

Solution

It does not matter that we do not have a start value. We can just do the multiplication calculation with the multipliers alone.

The multiplier for a 5% depreciation is 0·95 (since 100 – 5 = 95)
The multiplier for a 12% appreciation is 1·12 (since 100 + 12 = 112)

$$0\cdot 95\times 1\cdot 12 = 1\cdot 064$$

1·064 is the multiplier for a 6·4% appreciation.

Answer: a 5% depreciation, followed by a 12% depreciation is equivalent to a 6·4% appreciation overall.

Tip: Always use multipliers and powers in any National 5 percentages exam question.

Exercise 3

1. The Smiths buy a house for £60,000. If it appreciates in value at the rate of 9% per year, how much will it be worth in 5 years time?

2. Amanda wins some money and decides to spend £200 on some jewellery. If it appreciates at the rate of 2% per year, how much will the jewellery be worth 3 years from now?

3. In 1990 the world population was estimated to be 5300 million, and was increasing at the rate of 1·7% per annum.

What will the population be in the year 2000? (answer to 2 significant figures)

4. Peter buys a car for £3000. If it depreciates at the rate of 20% per annum, how much will he be able to sell it for in 3 years time?

5. Brian buys a new car costing £12600. It depreciates in value by 30% in the first year and by 20% each year after that.

How much will he be able to trade it in for in 3 years time

6. Each year a factory's machinery depreciates by 25% of its value at the beginning of the year. The initial value of the machinery was £360 000.

a) What was the value of the machinery after 1 year

b) The machinery was to be scrapped at the end of the year when its value fell below half its original value. After how many years should the machinery be scrapped?

7. Joseph invests £4500 in a bank that pays 6·4% interest per annum.

If Joseph does not touch the money in the bank, how much interest will he have gained after 3 years? Give your answer to the nearest penny.

8. Jane bought a painting in an auction for £32 250.

Unfortunately the painting depreciated in value by 7% each year.

Calculate how much the painting was worth after 2 years.

Give your answer to 3 significant figures.

9. Non calculator

Last year (2008) a company made a profit of £1 000 000. This year (2009) it expects to increase its profit by 20% and by 2010 to have increased it by a further 25%.

Calculate the profit the company expects to make in 2010.

10. A patient in hospital is given 200mg of a drug at 0900. 12% of the amount of the drug at the beginning of each hour is lost, through natural body processes, by the end of that hour.

How many mg of the drug will be **lost** by 1200?

11. Holly buys an antique watch costing £1200. The watch appreciates in value by 3·7% per annum.

How much will the watch be worth in 4 years time?

Give your answer to the nearest pound.

12. A local council recycles 28 000 tonnes of glass each year. After a publicity campaign they expect to increase the amount of glass recycled by 12% each year.

a) How much glass do they expect to recycle in 3 years time?

Give your answer correct to **3 significant figures**.

b) The council aim to double the amount of glass recycled in 6 years.

If this rate is maintained, will the council meet their target?

Give a reason for your answer.

13. Non calculator

Arthur's new car cost him £15 000. The value of it will depreciate by 20% each year.

How much will Arthur's car be worth when he trades it in for a new one in 2 years time?

14. Barry bought a house last year costing £115 000. This year it is valued at £110 400.

a) Calculate the percentage decrease in the value of the house.

b) If the value of the house continues to decrease at this rate what will the house be worth in a further 3 years time?

Give your answer to 3 significant figures.

15. Marcus invested £3000 in a bank which paid 2·5% interest per year.

a) Calculate how much money Marcus would have in his account after 3 years.

b) How long would it take for Marcus' money to increase by 12%?

16. In 2007 a company made a profit of £45 000. Over the next three years its profit dropped by 3% each year due to increased manufacturing costs.

Calculate, correct to 3 significant figures, the company's profit in 2010.

17. The value of an industrial machine is expected to decrease each year by 14·2% of its value at the beginning of the year.

If it was valued at £15500 at the **beginning** of 2011, what will its expected value be at the **end** of 2013? **Give your answer correct to the nearest pound**.

18. The membership of the 'Watch your Weight' slimming club is 40 000 and is increasing at the rate of 4% per month.

The membership of 'World of Slimming' is 70 000 but is decreasing at the rate of 9% per month.

a) Calculate the membership of the 'Watch your Weight' club after 3 months, giving your answer correct to 4 significant figures.

b) How many months will it take for the membership of the 'Watch your Weight' club to be more than the 'World of Slimming'?

19. A woman had a Body Mass Index (BMI) of 30. After following a healthy eating plan she managed to reduce her BMI to 27·6 in 1 month.

a) Calculate the percentage reduction in her BMI.

b) If she managed to continue to reduce her BMI by the same percentage in each of the next 3 months, what was her BMI then? Give your answer correct to 3 significant figures.

20. The value of an antique chair increased in value by 12½ % each year.

The chair was bought for £4800. What was its value at the end of 3 years?

21. Non Calculator

Charlene's house is valued at £120 000 and is expected to appreciate at the rate of 10% per annum for the next three years.

If this happens, what will the house be valued at in three years time?

22. Three years ago I bought a new car which cost £10 500. An offer from the garage at the time stated:

"Keep the car for 3 years, return it to us and we will refund half the original cost"

The car depreciated in value by 20% during the first year and by 15% in subsequent years.

By calculating the value of the car after 3 years decide whether the garage's offer, in this case, was a good one or not. Give a reason for your answer.

23. A piece of jewellery was bought for £2580 two years ago. Its present value is 65% of its original price.

a) What is its present day value?

An expert estimates that it will increase in value at a rate of 12% per annum over the next few years.

b) How many years will it take for the jewellery to regain its original value?

24. Bill invested £10 000 in the Dodgy Building Society but his money lost 5% per annum over the first 2 years.

At the end of this time he decided to move his money to the Goody Building Society which guaranteed that his money would gain 6% per annum over the next 2 years.

How much did Bill gain or lose over the four years?

25. Chocolate fountains have become very popular at parties.

It takes a minimum of 900g of melted chocolate to operate a fountain properly.

On one occasion 2kg of melted chocolate was added to the fountain.

23% of the remaining chocolate was used every 20 minutes.

Was there still enough chocolate left to operate the fountain properly one hour later?

You must show all working and give a reason for your answer.

26. In 2008 the Portable Phone Company announced that their profits were £850 000. In the next 3 years their profits increased by 4·2% each year. How much profit did the company make in 2011? **Give your answer to the nearest thousand.**

Reversing a Percentage Change

To go backwards with a percentage change, we divide by the multiplier instead multiplying.

Example 1

Andy gets a 3% pay rise. After the pay rise he is earning £40 685 a year. What was he earning each year before the pay rise?

Solution

For a 3% increase, the multiplier is 1·03.
40 685 ÷ 1·03 = £39 500

Example 2

A ship's value depreciates by 2·5% a year. After two years, it is worth £240 000. How much was it worth originally? Round your answer to two significant figures.

Solution

For a 2·5% depreciation, the multiplier is 0·975. For two years, we use a power of 2.
240 000 ÷ $0·975^2$ = £252 465 = £250 000 (2 s.f.)

Exercise 4

1. These amounts have been reduced by 15%. What was the original amount?

a)	£85	**(b)**	212·5 mm	**(c)**	£63.75
d)	25·5 litres	**(e)**	357 miles	**(f)**	435·2 m
g)	1 275 km	**(h)**	£4 462.50	**(i)**	10 200 m
j)	605·2 cm	**(k)**	£658.75	**(l)**	76·5 kg

2. These amounts have been increased by 22%. What was the original amount?

a)	£26.84	**(b)**	£54.90	**(c)**	£87.84
d)	103·7 ml	**(e)**	£21.35	**(f)**	122 cm
g)	3 111 m	**(h)**	10 370 km	**(i)**	68·32 m
j)	£13 664	**(k)**	118·95 litres	**(l)**	£7 564

3. A shop is having a sale. There is '20% OFF'. Calculate the original cost of these items.

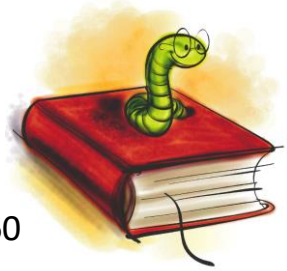
a) £32

(b) £52

(c) £20

d) £340

(e) £5.60



(f)

£239.20

g) £1 640

(h) £20.80

(i)

£21.20

4. A company gave their workers a 7% wage rise. Calculate how much each of these people were earning **each year before** the increase.

a)	Irene	£13 375 per year	**(b)**	Billy	£19 324.20 per year
c)	Peter	£26 322 per year	**(d)**	Isobel	£40 060.80 per year
e)	Stewart	£481.50 per week	**(f)**	Jackie	£1 820 per month
g)	Alan	£75 per week	**(h)**	Anne	£1 200 per month

5. A gym's membership has increased by 17% over the past year. It now has 585 members.

How many members did it have a year ago?

6. The number of school pupils not wearing school uniform has decreased by 72% since the start of last year. There are now 42 pupils not wearing school uniform.

How many pupils were not wearing school uniform at the start of last year?

7. My house has increased in value by 15% in the last two years. It is now worth £230 000.

How much was it worth 2 years ago?

8. I bought a new car in September of last year. By this September the car had

depreciated by 20% and was now worth £9600.

How much did I pay for the car last September?

9. Jane bought a painting in an auction. Unfortunately the painting depreciated in value by 7% and is now worth £4185.

How much was the painting worth when it was bought?

10. An antique chair has increased in value by 34% since it was bought. It is now worth £3 484.

What was it worth when it was bought?

Answers

Exercise 1

1. (a) £12·75 (b) £21 (c) £1·10 (d) 68p (e) £9
   (f) £48 (g) £7·20 (h) £4·80 (i) £3·50 (j) £7·60
   (k) £1980 (l) 45p (m) £70 (n) £45 (o) £49·20
   (p) £3·50 (q) £3·40 (r) 50p
2. (a) £30 (b) £80 3. (i) 42 (ii) 108 4. 482·5mm
5. 112·5g 6. (a) 76 (ii) 3724 7. 13440ft 8. 143·5cm 9. 2210
10. (a) £1308 (b) £1254 (c) £1281 (d) £1236 (e) £1245
11. 80% 12. 75% 13. 30% 14. 96·4%

Exercise 2

1. £3240

2. **a)** £2332.80, £332.80 **(b)** £5955.08, £955.08 **(c)** £5495.52, £695.52

d) £4348.04, £848.04 **(e)** £1982.12, £382.12 **(f)** 2393.35, 643.35

g) £23820.32, 3820.32 **(h)** £21190.05, 3190.05

i) £64751.45, £14751.45 **(j)** £439.32, £39.32

Exercise 3

1. £92317 **2.** £212.24

3. 6300 million **4.** £1536 **5.** £5644.80

6. **a)** £270 000 **(b)** after 3 years

7. £920.48 **8.** £27 900 **9.** £1 500 000

10. 136·3mg **11.** £1 388

12. **a)** 39 300 tonnes **(b)** just falls short of doubling

13. £9 600 **14.** **(a)** 4% **(b)** £97 700

15. **a)** £3 230.67 **(b)** 5 years **16.** £41 100

17. £9 790 **18.** **(a)** 44 990 **(b)** 5 months

19. **a)** 8% **(b)** 21·5 **20.** £6 834.38 **21.** £159 720

22. Car is valued at more than half the original value.

23. **a)** £1 677 **(b)** 4 years

24. Gained £140.49 **25.** Yes, since 913g > 900g **26.** £962 000

Exercise 4

1. **(a)** £100 **(b)** 250mm **(c)** £75

(d) 30 litres **(e)** 420 miles **(f)** 512 m

(g) 1 500 km **(h)** £5 250 **(i)** 12 000m

(j) 712 cm **(k)** £775 **(l)** 90 kg

2. **(a)** £22 **(b)** £45 **(c)** £72

(d) 85 ml **(e)** £17.50 **(f)** 100 cm

(g) 2 550 m **(h)** 8 500 km **(i)** 56 m

(j) £11 200 **(k)** 97·5 litres **(l)** £6 200

3. **(a)** £40 **(b)** £65 **(c)** £25

(d) £425 **(e)** £7 **(f)** £299

(g) £2 050 **(h)** £26 **(i)** £26.50

4. **(a)** £12 500 **(b)** £18 060 **(c)** £24 600

(d) £37 440 **(e)** £23 400 **(f)** £20 411.22

(g) £3 644.86 **(h)** £13 457.94

5. 500 **6.** 150 **7.** £200 000

8. £12 000 **9.** £4 500 **10.** £2 600