

## Topheavy Fractions and Mixed Numbers

In an exam, you would be expected to add, take away, multiply or divide fractions, including topheavy fractions (e.g.  $\frac{7}{2}$ ) or those which may be expressed as a mixed number (e.g.  $3\frac{1}{2}$ ).

You need to be able to change a fraction from a topheavy fraction or vice versa. To do this we can think about the link between fractions and dividing.

$$\frac{a}{b} = a \div b = p\frac{q}{b}, \text{ where } p \text{ is the quotient and } q \text{ is the remainder when doing } a \div b.$$

### Example 1

Change the topheavy fractions to mixed numbers: (a)  $\frac{13}{3}$  (b)  $\frac{46}{7}$

**Solutions**

$$(a) \ 13 \div 3 = 4 \text{ r } 1, \quad \text{so } \frac{13}{3} = 4\frac{1}{3} \qquad (b) \ 46 \div 7 = 6 \text{ r } 4, \quad \text{so } \frac{46}{7} = 6\frac{4}{7}$$

To go the other way, we use the fact that  $a\frac{b}{c} = \frac{ac+b}{c}$

### Example 2

Change the mixed numbers to topheavy fractions: (a)  $6\frac{2}{5}$  (b)  $10\frac{4}{9}$

**Solutions**

$$(a) \ 5 \times 6 + 2 = 32, \quad \text{so } 6\frac{2}{5} = \frac{32}{5} \qquad (b) \ 9 \times 10 + 4 = 94, \quad \text{so } 10\frac{4}{9} = \frac{94}{9}$$

### Exercise 1

1. Change each of these top heavy fractions to mixed numbers :-

(a)  $\frac{15}{2}$

(b)  $\frac{16}{3}$

(c)  $\frac{42}{5}$

(d)  $\frac{91}{20}$

(e)  $\frac{25}{4}$

(f)  $\frac{63}{8}$

(g)  $\frac{122}{11}$

(h)  $\frac{629}{25}$

2. Change each of the following to a mixed number and simplify where possible :-

(a)  $\frac{30}{4}$

(b)  $\frac{25}{10}$

(c)  $\frac{131}{5}$

(d)  $\frac{100}{15}$

(e)  $\frac{305}{25}$

(f)  $\frac{78}{8}$

(g)  $\frac{1005}{25}$

(h)  $\frac{100005}{100}$

3. Change each of the following mixed numbers to a top heavy fraction :-

(a)  $3\frac{1}{2}$

(b)  $4\frac{1}{3}$

(c)  $7\frac{3}{5}$

(d)  $10\frac{5}{6}$

(e)  $7\frac{8}{9}$

(f)  $5\frac{11}{12}$

(g)  $10\frac{1}{50}$

(h)  $15\frac{8}{15}$

4. How many  $\frac{1}{4}$  litre glasses of juice can I get from :-

(a) 2 litres

(b) 10 litres

(c)  $\frac{1}{2}$  litre

(d)  $3\frac{3}{4}$  litres ?

## Adding and Subtracting Fractions

You can only add and subtract fractions when the denominators are the same. When they are not the same, we have to change the fractions into another fraction that *is* the same.

A quick method for doing this, and the one used in these notes, is known as the ‘**kiss and smile**’ method because of the shape formed when you draw lines between the terms you are combining. The method is outlined and explained on page 24.

Adding works in exactly the same way as taking away. The only difference is that step three involves an add sum rather than a take away sum.

If either or both the numbers are mixed numbers, then the ‘whole number’ part and the ‘fraction’ part can be dealt with separately.

### Example 1

Take away:  $\frac{7}{8} - \frac{2}{3}$

**Solution**

Step one (smile)

$$\frac{7}{8} - \frac{2}{3}$$

Step two (kiss)

$$\frac{7}{8} \times \frac{2}{3}$$

Step three – take away (or add) the top line

$\begin{aligned}\frac{7}{8} - \frac{2}{3} &= \frac{\quad}{24} \\ &= \frac{21-16}{24} \\ &= \frac{5}{24}\end{aligned}$
---

### Example 2

Add:  $4\frac{2}{3} + 3\frac{4}{5}$

**Solution**

To add the whole numbers we just do  $4 + 3 = 7$

Now we use 'kiss and smile' to add  $\frac{2}{3}$  and  $\frac{4}{5}$ .

$$\begin{aligned}\frac{2}{3} + \frac{4}{5} &= \frac{10+12}{15} \\ &= \frac{22}{15} \\ &= 1\frac{7}{15}\end{aligned}$$

$$\frac{2}{3} \times \frac{4}{5}$$

Finally add 7 and  $1\frac{7}{15}$ . The final answer is  $8\frac{7}{15}$ .

## Exercise 2

1. Express each sum as a fraction in its simplest form:

a) $\frac{1}{5} + \frac{3}{5}$	(b) $\frac{2}{5} + \frac{1}{10}$	(c) $\frac{3}{4} + \frac{1}{8}$	(d) $\frac{1}{6} + \frac{2}{3}$
e) $\frac{1}{9} + \frac{2}{3}$	(f) $\frac{1}{3} + \frac{1}{4}$	(g) $\frac{3}{5} + \frac{1}{4}$	(h) $\frac{1}{4} + \frac{1}{6}$
i) $\frac{1}{3} + \frac{5}{8}$	(j) $\frac{1}{2} + \frac{2}{5}$	(k) $\frac{3}{4} + \frac{1}{6}$	(l) $\frac{1}{2} + \frac{3}{7}$
m) $\frac{2}{7} + \frac{1}{8}$	(n) $\frac{1}{5} + \frac{3}{8}$	(o) $\frac{2}{9} + \frac{3}{7}$	(p) $\frac{1}{3} + \frac{4}{7}$

2. Express each difference as a fraction in its simplest form:

a) $\frac{3}{4} - \frac{1}{4}$	(b) $\frac{1}{2} - \frac{1}{6}$	(c) $\frac{5}{6} - \frac{2}{3}$	(d) $\frac{11}{12} - \frac{5}{6}$
e) $\frac{11}{12} - \frac{2}{3}$	(f) $\frac{1}{2} - \frac{1}{16}$	(g) $\frac{2}{3} - \frac{1}{4}$	(h) $\frac{1}{2} - \frac{2}{5}$
i) $\frac{4}{5} - \frac{1}{2}$	(j) $\frac{7}{8} - \frac{3}{16}$	(k) $\frac{11}{12} - \frac{1}{3}$	(l) $\frac{7}{12} - \frac{1}{3}$
m) $\frac{5}{8} - \frac{2}{5}$	(n) $\frac{5}{6} - \frac{3}{5}$	(o) $\frac{7}{9} - \frac{3}{7}$	(p) $\frac{5}{8} - \frac{7}{16}$

## Multiplying and Dividing Fractions

We multiply and divide fractions in the same way as we do for algebraic fractions (shown on page 23).

Multiplying fractions is a straightforward procedure – you **multiply the tops and multiply the bottoms**.

$$\text{e.g. } \frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5 \times 7} = \frac{6}{35} \qquad \frac{a}{c} \times \frac{b}{c} = \frac{ab}{c^2}$$

It is easiest to cancel before you multiply. You are allowed to cancel *anything* from the top row with *anything* from the bottom row.

### Example 1 – multiplying with cancelling

Write a single fraction in its simplest form:  $\frac{35}{34} \times \frac{17}{42}$

**Solution**

$$\begin{aligned}\text{Cancelling gives: } \frac{35}{34} \times \frac{17}{42} &= \frac{\overset{5}{\cancel{35}}}{34} \times \frac{17}{\underset{6}{\cancel{42}}} && \text{(cancelling factors of 7)} \\ &= \frac{5}{\cancel{34}_2} \times \frac{\overset{1}{\cancel{17}}}{6} && \text{(cancelling factors of 17)} \\ &= \frac{5}{2} \times \frac{1}{6} \\ &= \frac{5}{12} && \text{(multiplying tops and bottoms)}\end{aligned}$$

To multiply mixed numbers, we have to change them to topheavy fractions first.

---

### Example 2 – multiplying mixed numbers

Multiply  $2\frac{1}{4} \times 1\frac{3}{5}$

**Solution**

First change to topheavy fractions.  $2\frac{1}{4} \times 1\frac{3}{5}$  becomes  $\frac{9}{4} \times \frac{8}{5}$ . Next:

$$\begin{aligned}\frac{9}{4} \times \frac{8}{5} &= \frac{9}{\cancel{4}_1} \times \frac{\overset{2}{\cancel{8}}}{5} && \text{(cancelling a factor of 4)} \\ &= \frac{9}{1} \times \frac{2}{5} \\ &= \frac{18}{5} \quad \left( \text{or } 3\frac{3}{5} \right)\end{aligned}$$

To divide two fractions, you:

1. flip the second fraction upside down
2. and change the sum to be a multiply sum:

$$\begin{array}{l} \text{e.g.} \quad \frac{3}{5} \div \frac{2}{7} = \frac{3}{5} \times \frac{7}{2} = \frac{21}{10} \qquad \frac{x}{y} \div \frac{a}{x} = \frac{x}{y} \times \frac{x}{a} = \frac{x^2}{ay} \end{array}$$

### Example 3 – dividing mixed numbers

Divide:  $4\frac{1}{3} \div \frac{4}{5}$

#### **Solution**

First change to topheavy fractions.  $4\frac{1}{3} \div \frac{4}{5}$  becomes  $\frac{13}{3} \div \frac{4}{5}$ . Next:

$$\begin{aligned}\frac{13}{3} \div \frac{4}{5} &= \frac{13}{3} \times \frac{5}{4} && \text{(flipping and multiplying)} \\ &= \frac{13 \times 5}{3 \times 4} \\ &= \frac{65}{12} \quad \left( \text{or } 5\frac{5}{12} \right)\end{aligned}$$

### **Exercise 3**

1. Express each product as a fraction in its simplest form:

<b>a)</b> $\frac{1}{4} \times \frac{4}{7}$	<b>(b)</b> $\frac{1}{3} \times \frac{3}{10}$	<b>(c)</b> $\frac{1}{2} \times \frac{4}{7}$	<b>(d)</b> $\frac{2}{3} \times \frac{1}{8}$
<b>e)</b> $\frac{4}{5} \times \frac{1}{16}$	<b>(f)</b> $\frac{6}{7} \times \frac{2}{3}$	<b>(g)</b> $\frac{3}{5} \times \frac{10}{21}$	<b>(h)</b> $\frac{3}{8} \times \frac{4}{21}$
<b>i)</b> $\frac{21}{32} \times \frac{4}{7}$	<b>(j)</b> $\frac{1}{9} \times \frac{12}{13}$	<b>(k)</b> $\frac{5}{16} \times \frac{6}{25}$	<b>(l)</b> $\frac{5}{7} \times \frac{14}{15}$
<b>m)</b> $\frac{7}{9} \times \frac{12}{35}$	<b>(n)</b> $\frac{12}{13} \times \frac{39}{48}$	<b>(o)</b> $\frac{2}{3} \times \frac{5}{9}$	<b>(p)</b> $\frac{5}{8} \times \frac{11}{15}$

2. Express as a single fraction:

<b>a)</b> $\frac{1}{4} \div \frac{1}{3}$	<b>(b)</b> $\frac{2}{5} \div \frac{2}{7}$	<b>(c)</b> $\frac{4}{5} \div \frac{3}{4}$	<b>(d)</b> $\frac{3}{7} \div \frac{2}{5}$
<b>e)</b> $\frac{5}{12} \div \frac{5}{3}$	<b>(f)</b> $\frac{5}{9} \div \frac{1}{3}$	<b>(g)</b> $\frac{2}{5} \div \frac{9}{10}$	<b>(h)</b> $\frac{3}{7} \div \frac{11}{14}$
<b>i)</b> $\frac{4}{9} \div \frac{2}{3}$	<b>(j)</b> $\frac{2}{5} \div \frac{4}{5}$	<b>(k)</b> $\frac{24}{35} \div \frac{20}{21}$	<b>(l)</b> $\frac{6}{25} \div \frac{9}{20}$
<b>m)</b> $\frac{8}{21} \div \frac{9}{14}$	<b>(n)</b> $\frac{10}{21} \div \frac{8}{9}$	<b>(o)</b> $\frac{20}{33} \div \frac{15}{44}$	<b>(p)</b> $\frac{7}{30} \div \frac{5}{20}$

## *Answers*

### *Exercise 1*

## Sill to be Done!!

### *Exercise 2*

1.    **a)**     $\frac{4}{5}$     **(b)**     $\frac{1}{2}$     **(c)**     $\frac{7}{8}$     **(d)**     $\frac{5}{6}$     **(e)**     $\frac{7}{9}$     **(f)**     $\frac{7}{12}$
- g)**     $\frac{17}{20}$     **(h)**     $\frac{5}{12}$     **(i)**     $\frac{23}{24}$     **(j)**     $\frac{9}{10}$     **(k)**     $\frac{11}{12}$     **(l)**     $\frac{13}{14}$
- m)**     $\frac{23}{56}$     **(n)**     $\frac{23}{40}$     **(o)**     $\frac{41}{63}$     **(p)**     $\frac{19}{21}$
- 
2.    **a)**     $\frac{1}{2}$     **(b)**     $\frac{1}{3}$     **(c)**     $\frac{1}{6}$     **(d)**     $\frac{1}{12}$     **(e)**     $\frac{1}{4}$     **(f)**     $\frac{7}{16}$
- g)**     $\frac{5}{12}$     **(h)**     $\frac{1}{10}$     **(i)**     $\frac{3}{10}$     **(j)**     $\frac{11}{16}$     **(k)**     $\frac{7}{12}$     **(l)**     $\frac{1}{4}$
- m)**     $\frac{9}{40}$     **(n)**     $\frac{7}{30}$     **(o)**     $\frac{22}{63}$     **(p)**     $\frac{3}{16}$

### *Exercise 3*

1.    **a)**     $\frac{1}{7}$     **(b)**     $\frac{1}{10}$     **(c)**     $\frac{2}{7}$     **(d)**     $\frac{1}{12}$     **(e)**     $\frac{1}{20}$     **(f)**     $\frac{4}{7}$
- g)**     $\frac{2}{7}$     **(h)**     $\frac{1}{14}$     **(i)**     $\frac{3}{8}$     **(j)**     $\frac{4}{39}$     **(k)**     $\frac{3}{40}$     **(l)**     $\frac{2}{3}$
- m)**     $\frac{4}{15}$     **(n)**     $\frac{3}{4}$     **(o)**     $\frac{10}{27}$     **(p)**     $\frac{11}{24}$

2.      (a)       $\frac{3}{4}$       (b)       $\frac{7}{5}$       (c)       $\frac{16}{15}$       (d)       $\frac{15}{14}$       (e)       $\frac{1}{4}$       (f)       $\frac{5}{3}$
- (g)       $\frac{4}{9}$       (h)       $\frac{6}{11}$       (i)       $\frac{2}{3}$       (j)       $\frac{1}{2}$       (k)       $\frac{18}{25}$       (l)       $\frac{8}{15}$
- (m)       $\frac{16}{27}$       (n)       $\frac{15}{28}$       (o)       $\frac{16}{9}$       (p)       $\frac{14}{15}$