

4. Trigonometry

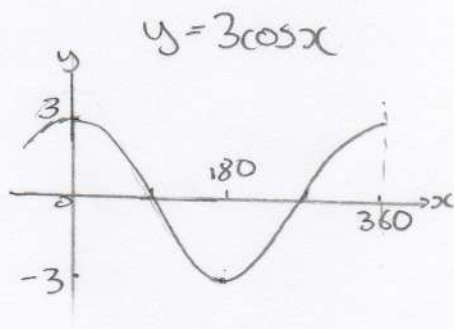
1. a) $y = 3 \cos x$ b) $y = 4 \sin x$ c) $y = -\sin x$ d) $y = -\cos x$

2. a) $y = -2 \sin x$ b) $y = \sin x + 1$ c) $y = \cos x - 3$ d) $y = \cos x + 4$

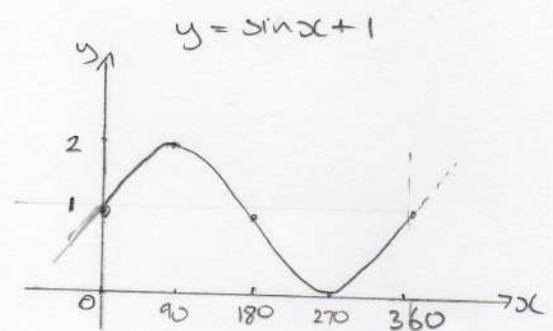
3. a) $y = -4 \sin x$ b) $y = -\sin x + 3$ c) $y = 3 \cos(x - 15^\circ)$
or
 $y = 2 \cos(x - 45^\circ)$

d) $y = 2 \sin(x + 45^\circ)$
or
 $y = 2 \cos(x - 45^\circ)$

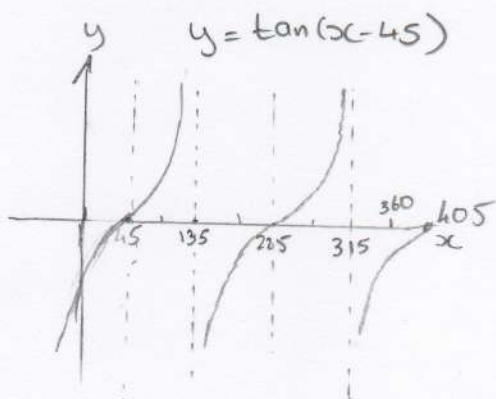
4. a)



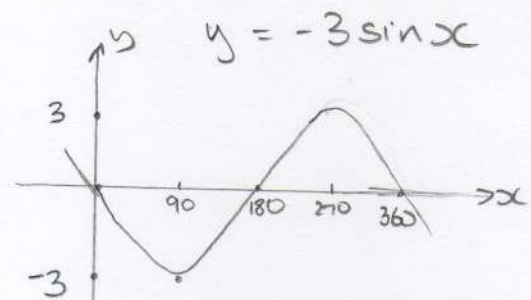
b)



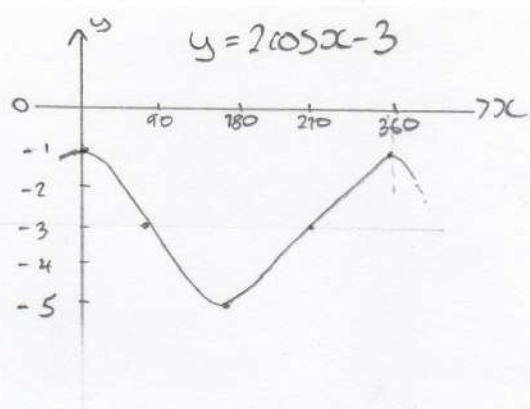
c)



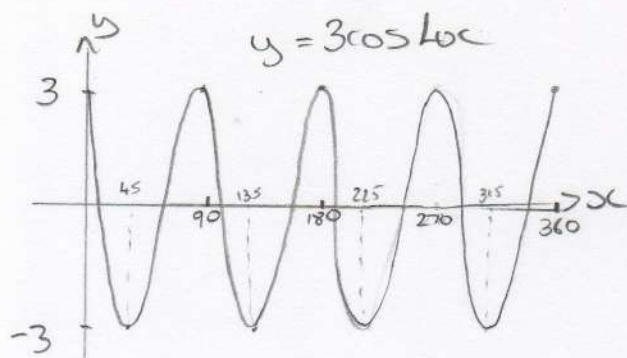
d)



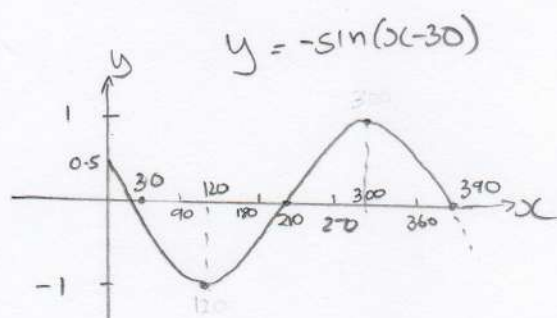
e)



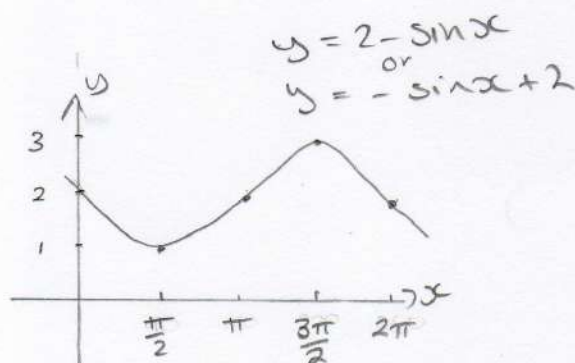
f)



g)



h)



5. a) $30^\circ = \frac{\pi}{6}$

b) $45^\circ = \frac{\pi}{4}$

c) $60^\circ = \frac{\pi}{3}$

d) $90^\circ = \frac{\pi}{2}$

e) $180^\circ = \pi$

f) $360^\circ = 2\pi$

g) $120^\circ = \frac{2\pi}{3}$

h) $135^\circ = \frac{3\pi}{4}$

i) $150^\circ = \frac{5\pi}{6}$

j) $210^\circ = \frac{7\pi}{6}$

k) $225^\circ = \frac{5\pi}{4}$

l) $240^\circ = \frac{4\pi}{3}$

m) $270^\circ = \frac{3\pi}{2}$

n) $300^\circ = \frac{5\pi}{3}$

o) $315^\circ = \frac{7\pi}{4}$

p) $330^\circ = \frac{11\pi}{6}$

q) $540^\circ = 3\pi$

r) $720^\circ = 4\pi$

6. a) $\frac{37}{180} \times \pi$
 $= \underline{\underline{0.646}}$

b) $\frac{142}{180} \times \pi$
 $= \underline{\underline{2.48}}$

c) $\frac{226}{180} \times \pi$
 $= \underline{\underline{3.94}}$

d) $\frac{281}{180} \times \pi$
 $= \underline{\underline{4.90}}$

e) $\frac{307}{180} \times \pi$
 $= \underline{\underline{5.36}}$

f) $\frac{453}{180} \times \pi$
 $= \underline{\underline{7.91}}$

$$7a) \pi = 180^\circ$$

$$b) 2\pi = 360^\circ$$

$$c) 3\pi = 540^\circ$$

$$d) \frac{\pi}{2} = 90^\circ$$

$$e) \frac{3\pi}{2} = \cancel{135^\circ} 270^\circ$$

$$f) \frac{5\pi}{2} = 450^\circ$$

$$g) \frac{\pi}{3} = 60^\circ$$

$$h) \frac{2\pi}{3} = 120^\circ$$

$$i) \frac{4\pi}{3} = 240^\circ$$

$$j) \frac{5\pi}{3} = 300^\circ$$

$$k) \frac{7\pi}{3} = 420^\circ$$

$$l) \frac{\pi}{4} = 45^\circ$$

$$m) \frac{3\pi}{4} = 135^\circ$$

$$n) \frac{5\pi}{4} = 225^\circ$$

$$o) \frac{7\pi}{4} = 315^\circ$$

$$p) \frac{\pi}{6} = 30^\circ$$

$$q) \frac{5\pi}{6} = 150^\circ$$

$$r) \frac{7\pi}{6} = 210^\circ$$

$$s) \frac{11\pi}{6} = 330^\circ$$

$$t) \frac{\pi}{12} = 15^\circ$$

$$u) \frac{5\pi}{12} = 75^\circ$$

$$8. a) 180 = \pi \text{ radians}$$

$$\Rightarrow 1 \text{ radian} = \frac{180}{\pi} \\ = \underline{\underline{57.3^\circ}}$$

$$b) 2 \text{ radians}$$

$$= 2 \times 57.3 \\ = \underline{\underline{115^\circ}}$$

$$c) 3 \text{ radians}$$

$$= 3 \times 57.3 \\ = \underline{\underline{172^\circ}}$$

$$d) 4 \text{ radians}$$

$$= 4 \times 57.3 \\ = \underline{\underline{229^\circ}}$$

$$e) 1.4 \text{ radians}$$

$$= 1.4 \times 57.3 \\ = \underline{\underline{80.2^\circ}}$$

$$f) 2.7 \text{ radians}$$

$$= 2.7 \times 57.3 \\ = \underline{\underline{155^\circ}}$$

$$9. a) \sin 30$$

$$= \frac{1}{2}$$



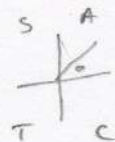
$$b) \cos 60$$

$$= \frac{1}{2}$$



$$c) \tan 45$$

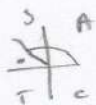
$$= 1$$



$$d) \sin 120$$

$$= \sin 60$$

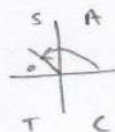
$$= \frac{\sqrt{3}}{2}$$



$$e) \cos 150$$

$$= -\cos 30$$

$$= -\frac{\sqrt{3}}{2}$$



$$f) \tan 135$$

$$= -\tan 45$$

$$= -1$$



$$g) \sin 90$$

$$= 1$$

$$h) \cos 180$$

$$= -1$$

$$i) \tan 270$$

$$= \text{undefined}$$

$$j) \sin 210$$

$$= -\sin 30$$

$$= -\frac{1}{2}$$



$$k) \cos 225$$

$$= -\cos 45$$

$$= -\frac{1}{\sqrt{2}}$$



$$l) \tan 240$$

$$= \tan 60$$

$$= \sqrt{3}$$



$$m) \sin 300$$

$$= -\sin 60$$

$$= -\frac{\sqrt{3}}{2}$$



$$n) \cos 330$$

$$= \cos 30$$

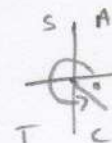
$$= \frac{\sqrt{3}}{2}$$



$$o) \tan 315$$

$$= -\tan 45$$

$$= -1$$



$$p) \sin 540$$

$$= \sin 180 \quad (540 - 360)$$

$$= 0$$

$$q) \cos 405$$

$$= \cos 45$$

$$= \frac{1}{\sqrt{2}}$$



$$r) \tan 390$$

$$= \tan 30$$

$$= \frac{1}{\sqrt{3}}$$



$$10. a) \sin \frac{\pi}{6} (30)$$

$$= \underline{\underline{\frac{1}{2}}}$$

$$b) \cos \frac{\pi}{4} (45)$$

$$= \underline{\underline{\frac{1}{\sqrt{2}}}}$$

$$c) \tan \frac{\pi}{3} (60)$$

$$= \underline{\underline{\sqrt{3}}}$$

$$d) \sin \pi (180)$$

$$= \underline{\underline{0}}$$

$$e) \cos 2\pi (360)$$

$$= \underline{\underline{1}}$$

$$f) \tan \frac{3\pi}{2} (270)$$

$$= \underline{\underline{\text{undefined}}}$$

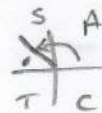
$$g) \sin \frac{5\pi}{6} (150)$$



$$= \sin 30$$

$$= \underline{\underline{\frac{1}{2}}}$$

$$h) \cos \frac{3\pi}{4} (135)$$



$$= -\cos 45$$

$$= -\underline{\underline{\frac{1}{\sqrt{2}}}}$$

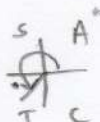
$$i) \tan \frac{2\pi}{3} (120)$$



$$= -\tan 60$$

$$= -\underline{\underline{\sqrt{3}}}$$

$$j) \sin \frac{7\pi}{6} (210)$$



$$= -\sin 30$$

$$= -\underline{\underline{\frac{1}{2}}}$$

$$k) \cos \frac{5\pi}{4} (225)$$



$$= -\cos 45$$

$$= -\underline{\underline{\frac{1}{\sqrt{2}}}}$$

$$l) \tan \frac{4\pi}{3} (240)$$



$$= \tan 60$$

$$= \underline{\underline{\sqrt{3}}}$$

$$m) \sin \frac{11\pi}{6} (330)$$



$$= -\sin 30$$

$$= -\underline{\underline{\frac{1}{2}}}$$

$$n) \cos \frac{7\pi}{4} (315)$$



$$= \cos 45$$

$$= \underline{\underline{\frac{1}{\sqrt{2}}}}$$

$$o) \tan \frac{5\pi}{3} (300)$$



$$= -\tan 60$$

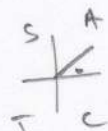
$$= -\underline{\underline{\sqrt{3}}}$$

$$p) \sin \frac{13\pi}{6} (390)$$

$$\sin 390$$

$$= \sin 30$$

$$= \underline{\underline{\frac{1}{2}}}$$



$$q) \cos \frac{10\pi}{3} (600)$$

$$\cos 600$$

$$= \cos 240$$

$$= -\cos 60$$

$$= -\underline{\underline{\frac{1}{2}}}$$



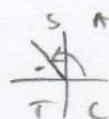
$$r) \tan \frac{11\pi}{4} (495)$$

$$\tan 495$$

$$= \tan 135$$

$$= -\tan 45$$

$$= -\underline{\underline{1}}$$



$$\begin{aligned}
 11. a) \quad & \sin \frac{\pi}{6} + \cos \frac{\pi}{6} \\
 & \quad (30) \quad \quad (30) \\
 & = \frac{1}{2} + \frac{\sqrt{3}}{2} \\
 & = \frac{1+\sqrt{3}}{2}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad & \cos \frac{\pi}{4} - \sin \frac{\pi}{4} \\
 & \quad (45) \quad \quad (45) \\
 & = \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \\
 & = \underline{\underline{0}}
 \end{aligned}$$

$$\begin{aligned}
 e) \quad & \tan^2 \frac{\pi}{6} + \tan^2 \frac{\pi}{3} \\
 & \quad (30) \quad \quad (60) \\
 & = (\tan 30)^2 + (\tan 60)^2 \\
 & = \left(\frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{3}}\right) + (\sqrt{3} \times \sqrt{3}) \\
 & = \frac{1}{3} + 3 \\
 & = \underline{\underline{3\frac{1}{3}}} \quad \left(\frac{10}{3}\right)
 \end{aligned}$$

$$\begin{aligned}
 g) \quad & \sin^2 \frac{4\pi}{3} \\
 & \quad (240) \\
 & = (\sin 240)^2 \\
 & = \left(-\frac{\sqrt{3}}{2} \times -\frac{\sqrt{3}}{2}\right) \\
 & = \underline{\underline{\frac{3}{4}}}
 \end{aligned}$$

$\sin 240$ 
 $= -\sin 60$
 $= -\frac{\sqrt{3}}{2}$

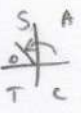
$$\begin{aligned}
 b) \quad & 2 \sin \frac{\pi}{6} - 2 \cos \frac{\pi}{6} \\
 & \quad (180) \quad \quad (180) \\
 & = 2 \times 0 - (2 \times -1) \\
 & = \underline{\underline{2}}
 \end{aligned}$$

$$\begin{aligned}
 d) \quad & \sin^2 \frac{\pi}{3} + \cos^2 \frac{\pi}{3} \\
 & \quad (60) \\
 & = (\sin 60)^2 + (\cos 60)^2 \\
 & = \left(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}\right) + \left(\frac{1}{2} \times \frac{1}{2}\right) \\
 & = \frac{3}{4} + \frac{1}{4} \\
 & = \underline{\underline{1}}
 \end{aligned}$$

$$\begin{aligned}
 f) \quad & \cos^2 \left(\frac{5\pi}{4}\right) \\
 & \quad 225 \\
 & = (\cos 225)^2 \\
 & = \left(-\frac{1}{\sqrt{2}} \times -\frac{1}{\sqrt{2}}\right) \\
 & = \underline{\underline{\frac{1}{2}}}
 \end{aligned}$$

$\cos 225$ 
 $= -\cos 45$
 $= -\frac{1}{\sqrt{2}}$

$$\begin{aligned}
 h) \quad & 2 \sin \frac{11\pi}{6} \cos \frac{5\pi}{6} \\
 & \quad (330) \quad \quad (150) \\
 & = 2 \times \left(-\frac{1}{2}\right) \times \left(-\frac{\sqrt{3}}{2}\right) \\
 & = \frac{2\sqrt{3}}{4} \\
 & = \underline{\underline{\frac{\sqrt{3}}{2}}}
 \end{aligned}$$

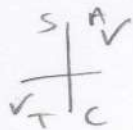
$\sin 330$ 
 $= -\sin 30$
 $= -\frac{1}{2}$
 $\cos 150$ 
 $= -\cos 30$
 $= -\frac{\sqrt{3}}{2}$

12. a) $5 \tan x - 6 = 2$

$$\tan x = \frac{2+6}{5}$$

$$\tan x = \frac{8}{5}$$

$$(\tan^{-1}) = 58^\circ$$



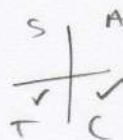
$$180 + 58 = 238$$

$$x = \underline{\underline{58^\circ, 238^\circ}}$$

b) $4 \sin x - 2 = -3$

$$\sin x = \frac{-3+2}{4}$$

$$\sin x = -\frac{1}{4}$$



$$(\sin^{-1}) = (14.5^\circ)$$

$$180 + 14.5 = 194.5$$

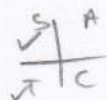
$$360 - 14.5 = 345.5$$

$$x = 194.5^\circ, 345.5^\circ$$

$$x = \underline{\underline{3.39 \text{ rads}, 6.03 \text{ rads}}}$$

c) $3 \cos 2x + 2 = 0$

$$\cos 2x = -\frac{2}{3}$$



$$(x = 48.2^\circ)$$

$$180 - 48.2 = 131.8$$

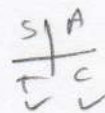
$$180 + 48.2 = 228.2$$

$$2x = 131.8, 228.2, 491.8, 588.2$$

$$x = \underline{\underline{65.9^\circ, 114.1^\circ, 245.9^\circ, 294.1^\circ}}$$

d) $4 \sin 3x + 1 = -2$

$$\sin 3x = -\frac{3}{4}$$



$$(3x = 48.6^\circ)$$

$$180 + 48.6 = 228.6$$

$$360 - 48.6 = 311.4$$

$$3x = 228.6, 311.4, 588.6, 671.4$$

$$x = 76.2, 103.8, 196.2, 223.8,$$

$$\underline{\underline{316.2, 343.8}}$$

e) $4 \cos(x+26) + 3 = 0$

$$\cos(x+26) = -\frac{3}{4}$$



$$(x = 41.4^\circ)$$

$$180 - 41.4 = 138.6$$

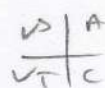
$$180 + 41.4 = 221.4$$

$$(x+26) = 138.6, 221.4$$

$$x = \underline{\underline{112.6^\circ, 195.4^\circ}}$$

f) $\sqrt{2} \cos(x + \frac{\pi}{8}) + 1 = 0$

$$\cos(x + \frac{\pi}{8}) = -\frac{1}{\sqrt{2}}$$



$$(x = 45)$$

$$180 - 45 = 135 = \frac{3\pi}{4}$$

$$180 + 45 = 225 = \frac{5\pi}{4}$$

$$(x + \frac{\pi}{8}) = \frac{3\pi}{4}, \frac{5\pi}{4}$$

$$x = \frac{3\pi}{4} - \frac{\pi}{8}, \frac{5\pi}{4} - \frac{\pi}{8}$$

$$= \frac{9\pi}{12} - \frac{2\pi}{12}, \frac{15\pi}{12} - \frac{2\pi}{12}$$

$$= \underline{\underline{\frac{7\pi}{12}, \frac{13\pi}{12}}}$$

13 a) $4\sin^2 x = 3$
 $\sin^2 x = \frac{3}{4}$
 $\sin x = \pm \frac{\sqrt{3}}{2}$ 

$x = 60, 120, 240, 300$

c) $\sin^2 x - 4\sin x + 3 = 0$
 $(x^2 - 4x + 3)$
 $(x-3)(x-1)$

$(\sin x - 3)(\sin x - 1) = 0$

$\sin x - 3 = 0$ $\sin x - 1 = 0$

$\sin x = 3$

no solution

$\sin x = 1$

$x = 90$

b) $4\cos^2 x = 2$
 $\cos^2 x = \frac{1}{2}$
 $\cos x = \pm \frac{1}{\sqrt{2}}$ 

$x = 45, 135, 225, 315$

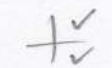
$x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

d) $2\cos^2 x + \cos x - 1 = 0$
 $(2x^2 + x - 1)$
 $(2x-1)(x+1)$

$(2\cos x - 1)(\cos x + 1) = 0$

$2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$



$x = 60, 300$

$\cos x + 1 = 0$

$\cos x = -1$

$x = 180$

$x = 60, 180, 300$

e) $6\sin^2 x + 5\sin x + 1 = 0$
 $(6x^2 + 5x + 1)$
 $(3x+1)(2x+1)$

$(3\sin x + 1)(2\sin x + 1) = 0$

$\sin x = -\frac{1}{3}$ $\sin x = -\frac{1}{2}$



$(x = 19.5)$

$180 + 19.5 = 199.5$

$360 - 19.5 = 340.5$



$(x = 30)$

$180 + 30 = 210$

$360 - 30 = 330$

$x = 199.5, 210, 330, 340.5$

↓

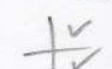
$x = 3.48 \text{ rads}, 3.67 \text{ rads}, 5.76 \text{ rads}, \underline{\underline{5.94 \text{ rads}}}$
 $(\frac{7\pi}{6}) \quad (\frac{11\pi}{6})$

f) $15\cos^2 x + 7\cos x - 2 = 0$
 $(15x^2 + 7x - 2)$
 $(5x-1)(3x+2)$

$(5\cos x - 1)(3\cos x + 2) = 0$

$5\cos x - 1 = 0$

$\cos x = \frac{1}{5}$



$x = 60, 300$

$3\cos x + 2 = 0$

$\cos x = -\frac{2}{3}$



$(x = 180.2)$

$180 - 48.2 = 131.8$

$180 + 48.2 = 228.2$

$x = 60, 131.8, 228.2, 300$

14. a) $y = p \sin x + q$

p amplitude $\frac{4 - (-2)}{2} = \underline{\underline{3}}$

q therefore is $+1$ as graph is between 4 and -2 instead of 3 and -3 .

$\underline{\underline{p = 3}} \quad \underline{\underline{q = 1}}$

b) $y = 3 \sin x + 1$.

cuts x -axis at $y = 0$

$3 \sin x + 1 = 0$

$\sin x = -\frac{1}{3}$

s	A
$\sqrt{1}$	$c \checkmark$

$(x = 19.5^\circ)$

$180 + 19.5 = 199.5$

$360 - 19.5 = 340.5$

$\underline{\underline{A(199.5, 0)}}$

$\underline{\underline{B(340.5, 0)}}$