St Peter the Apostle High School

Maths Department

Higher Practice Questions

5. Recurrence Relations

1 A family take out a loan of £3000. The interest charged on this works out as 1·2% per calendar month. They set up a payment plan of £500 per month.

(a) Write down a recurrence relation for the amount they owe.
(b) How much will the family owe after 3 months?
(c) How many payments will it take for the loan to be repaid?

2 An investor saves £50 000 in an account, gaining 4·5% interest per year. They withdraw £1800 every year.

(a) Write down a recurrence relation for the amount of money in the account.
(b) Find how much they would have in this savings account after 5 years.

3 The air pressure in a used car tyre was 35 psi. This is above its recommended minimum pressure of 30 psi. The tyre loses 11% of its air pressure during each month. The owner has been refilling the tyre with 3 psi of air at the end of every month.

(a) Find a recurrence relation showing the air pressure of the tyre.
(b) What is the air pressure of the tyre at the end of the 2^{nd} month?
(c) After how many months would the tyre end up below the recommended minimum pressure?

4 For each recurrence relation find, rounding your answer to 2 decimal places where applicable:

(a) u_2 : $u_{n+1} = 0.2\,u_n + 4, \quad u_0 = 3$
(b) u_3 : $u_{n+1} = 0.1\,u_n + 5, \quad u_0 = 7$
(c) u_4 : $u_{n+1} = -0.5\,u_n + 20, \quad u_0 = 16$
(d) u_3 : $u_{n+1} = -u_n - 7, \quad u_0 = 1$
(e) u_2 : $u_n = 0.9\,u_{n-1} + 450, \quad u_0 = 2$

5 A sequence in defined by the recurrence relation $v_{n+1} = 1.2\,v_n - 8, \quad v_0 = 150$

(a) Calculate the value of v_3.
(b) Find the smallest value of n for which $v_n > 400$.

6 For each recurrence relation below find u_1 and u_2, and the limit, where it exists.

(a) $u_{n+1} = 2\,u_n + 3, \quad u_0 = 3$

(b) $u_{n+1} = 0.7\,u_n + 12, \quad u_0 = 30$

7 A recurrence relation is defined as $u_{n+1} = a\,u_n + b$

(a) Find u_1 and u_2 of the sequence when $u_0 = 30,\ a = 0.5$ and $b = 7$

(b) Describe why this particular sequence has a limit.

(c) Calculate the limit of this sequence.

8 A sequence is defined by the recurrence relation

$$u_{n+1} = k\,u_n - 5, \quad u_0 = 0$$

(a) Given that $u_2 = -7$, find the value of k

(b) **(i)** Why does this sequence tend to a limit as $n \to \infty$

(ii) Find the value of this limit.

9 Two sequences are generated by the recurrence relations

$$u_{n+1} = 0.2\,u_n + 4.8$$
$$v_{n+1} = k\,v_n + 4$$

The two sequences approach the same limit as $n \to \infty$

(a) Evaluate this limit

(b) Hence determine the value of k.

10 Two sequences are defined by the recurrence relations

$$u_{n+1} = 0.2\,u_n + p, \quad u_0 = 1 \quad \text{and} \quad v_{n+1} = 0.6\,v_n + q, \quad v_0 = 1$$

If both sequences have the same limit, express p in terms of q .

11 A recurrence relation is defined by

$$u_{n+1} = a\,u_n + b \quad \text{where } -1 < a < 1 \text{ and } u_0 = 25$$

(a) If $u_1 = 30$, and $u_2 = 31$ find the values of a and b
(b) Find the limit of this recurrence relation as $n \to \infty$

12 For the recurrence relation $u_{n+1} = m\,u_n + c$

it is known that $u_0 = 2$, $u_1 = 4$ and $u_2 = 7$

Find the values of m and c

13 A recurrence relation is defined by $u_{n+1} = a\,u_n + b$ for some constants a and b

(a) If $u_2 = 190$, $u_3 = 430$ and $u_4 = 910$ calculate the values of a and b
(b) What is the initial value u_0, of this sequence?

14 A new ’24 hour antibiotic’ is being tested on a patient in hospital.
It is known that over a 24 hour period the amount of antibiotic remaining in the bloodstream is reduced by 70%. On the first day of the trial, an initial 220 mg dose is given to a patient at 7 am.

(a) After 24 hours and just prior to the second dose being given, how much antibiotic remains in the patient’s bloodstream?

The patient is then given a further 220 mg dose at 7 am and at this time each subsequent morning.

(b) A recurrence relation of the form $u_{n+1} = a\,u_n + b$ can be used to model this course of treatment. Write down the values of a and b.

It is also known that more than 350 mg of the drug in the bloodstream results in unpleasant side effects.

(c) Is it safe to administer this antibiotic over an extended period of time?