8. Quadratics

1 a) $98-2x^2$
$= 2(49-x^2)$
$= 2(7-x)(7+x)$

b) $8x^2-98$
$= 2(4x^2-49)$
$= 2(2x-7)(2x+7)$

c) $81-x^4$
$= (9-x^2)(9+x^2)$

d) $27w-12x^3$
$= 3w(9-4x^2)$
$= 3w(3-2x)(3+2x)$

e) $50x^3-2x$
$= 2x(25x^2-1)$
$= 2x(5x-1)(5x+1)$

f) $5r^3-20r$
$= 5r(r^2-4)$
$= 5r(r-2)(r-2)$

2. a) x^2+x-6
$= (x-2)(x+3)$

b) $2x^2-7x+3$
$= (2x-1)(x-3)$

c) $6x^2+7x+2$
$= (3x+2)(2x+1)$

d) $3x^2+10x+8$
$= (3x+4)(x+2)$

e) $12x^2-4x-1$
$= (6x+1)(2x-1)$

f) $8x^2+6x-9$
$= (4x-3)(2x+3)$

g) $6-x-x^2$
$= (3+x)(2-x)$

h) $15-7x-2x^2$
$= (5+x)(3-2x)$

i) $20+11x-3x^2$
$= (5-x)(4+3x)$

3 a) $x^2-11x+24=0$
$(x-8)(x-3)=0$
$x=8 \quad x=3$

b) $4x^2-9=0$
$(2x-3)(2x+3)=0$
$x=\frac{3}{2} \quad x=-\frac{3}{2}$

c) $n^2+3n-10=0$
$(n-2)(n+5)=0$
$n=2 \quad n=-5$

d) $2r^2-r-6=0$
$(2r+3)(r-2)=0$
$r=-\frac{3}{2} \quad r=2$

e) $7x^2-14x=-7$
$7x^2-14x+7=0$
$7(x^2-2x+1)=0$
$7(x-1)(x-1)=0$
$x=1$

f) $n^2+8n=-15$
$n^2+8n+15=0$
$(n+3)(n+5)=0$
$n=-3 \quad n=-5$

4 a) $f(x) = 5x^2 + 20x$

i) $5x^2 + 20x = 0$

$5x(x+4) = 0$

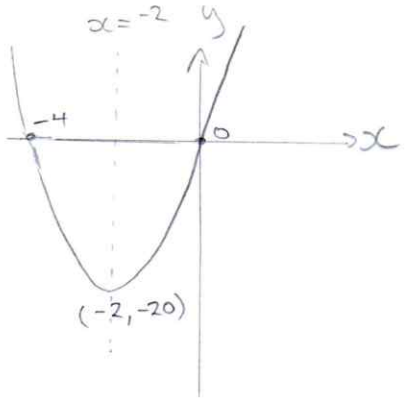
$x = 0 \quad x = -4$

y int $(x=0)$

$5 \times 0^2 + 20 \times 0$

$= 0$

so $(0,0)$



ii)

$x = -2$

iii) TP when $x = -2$

$5 \times (-2)^2 + (20 \times -2)$

$= -20 \qquad (-2,-20)$

b) $f(x) = x^2 + 6x + 8$

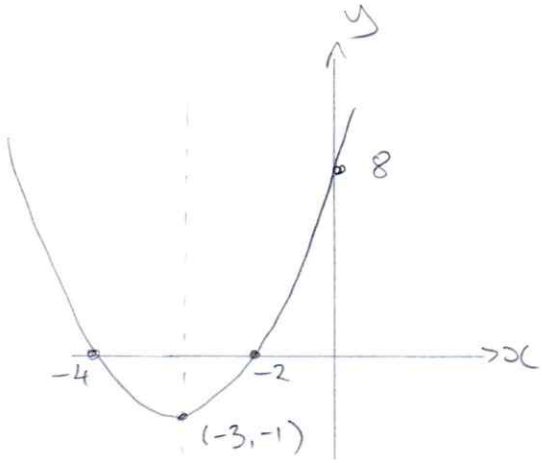
$0 = (x+4)(x+2)$

i) $\Rightarrow x = -4 \quad x = -2$

y int

$0^2 + 6 \times 0 + 8$

$(0,8)$



ii)

$x = -3$

iii) TP when $x = -3$

$(-3)^2 + (6 \times -3) + 8$

$= -1 \qquad (-3,-1)$

c) $f(x) = 15 - 2x - x^2$

i) $0 = (5+x)(3-x)$

$x = -5 \quad x = 3$

y int

$15 - 2 \times 0 - 0^2$

$= 15$

$(0,15)$

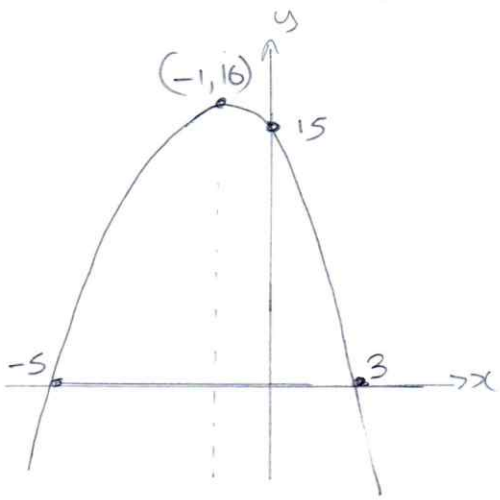
ii)

$x = -1$

iii) TP when $x = -1$

$15 - (2 \times -1) - (-1)^2$

$= 16 \qquad (-1,16)$



5. a) $x^2 - 8x + 12$

$x^2 - 8x + 16 - 16 + 12$

$= (x-4)^2 - 4$

b) $x^2 + 10x + 21$

$x^2 + 10x + 25 - 25 + 21$

$= (x+5)^2 - 4$

c) $x^2 - 6x - 16$

$x^2 - 6x + 9 - 9 - 16$

$= (x-3)^2 - 25$

d) $2x^2 - 4x + 1$

$2(x^2 - 2x + \frac{1}{2})$

$2(x^2 - 2x + 1 - 1 + \frac{1}{2})$

$= 2((x-1)^2 - \frac{1}{2})$

$= 2(x-1)^2 - 1$

e) $3x^2 - 18x + 10$

$3(x^2 - 6x + \frac{10}{3})$

$3(x^2 - 6x + 9 - 9 + \frac{10}{3})$

$= 3((x-3)^2 - \frac{17}{3})$

$= 3(x-3)^2 - 17$

f) $2x^2 - 11x + 10$

$2(x^2 - \frac{11}{2}x + 5)$

$2(x^2 - \frac{11}{2}x + \frac{121}{16} - \frac{121}{16} + 5)$

$= 2((x - \frac{11}{4})^2 - \frac{41}{16})$

$= 2(x - \frac{11}{4})^2 - \frac{41}{8}$

g) $10 + 4x - x^2$

$-x^2 + 4x + 10$

$-1(x^2 - 4x - 10)$

$-1(x^2 - 4x + 4 - 4 - 10)$

$= -1((x-2)^2 - 14)$

$= -(x-2)^2 + 14$

or

$14 - (x-2)^2$

h) $3 - 6x - x^2$

$-x^2 - 6x + 3$

$-1(x^2 + 6x - 3)$

$-1(x^2 + 6x + 9 - 9 - 3)$

$= -1((x+3)^2 - 12)$

$= -(x+3)^2 + 12$

i) $15 + 8x - 2x^2$

$-2x^2 + 8x + 15$

$-2(x^2 - 4x - \frac{15}{2})$

$-2(x^2 - 4x + 4 - 4 - \frac{15}{2})$

$= -2((x-2)^2 - \frac{23}{2})$

$= -2(x-2)^2 + 23$

6. $x^2 + 2x + 7$

$x^2 + 2x + 1 - 1 + 7$

$= (x+1)^2 + 6$

For $\frac{1}{(x+1)^2 + 6}$ max value is when denominator is at its smallest.

This will be when $(x+1)^2 = 0$

so $x = -1$ will give 0.

Max value $\Rightarrow \frac{1}{0+6} = \frac{1}{6}$ at $x = -1$

7. a) $9 - 8x - x^2$

$-x^2 - 8x + 9$

$-1(x^2 + 8x - 9)$

$-1(x^2 + 8x + 16 - 16 - 9)$

$= -1((x+4)^2 - 25)$

$= -(x+4)^2 + 25$

$p = -1 \quad q = 4 \quad r = 25$

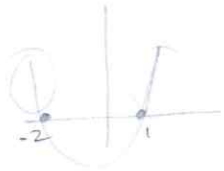
b) $\frac{1}{-(x+4)^2 + 25}$

$\Rightarrow$ max value is $\frac{1}{25}$

8.a) $x^2 + x - 2 > 0$

$(x+2)(x-1) > 0$

$x = -2 \quad x = 1$

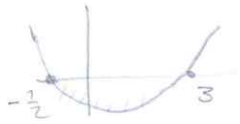


$x > 1$ or $x < -2$

b) $2x^2 - 5x - 3 < 0$

$(2x+1)(x-3) < 0$

$x = -\frac{1}{2} \quad x = 3$

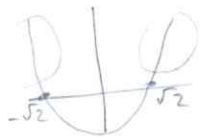


$-\frac{1}{2} < x < 3$

c) $x^2 - 2 > 0$

$x^2 > 2$

$x > \pm\sqrt{2}$

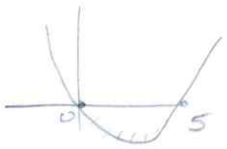


$x > \sqrt{2}$ or $x < -\sqrt{2}$

d) $x^2 - 5x < 0$

$x(x-5) < 0$

$x = 0 \quad x = 5$

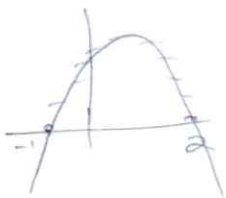


$0 < x < 5$

e) $2 + x - x^2 > 0$

$(2-x)(1+x) > 0$

$x = 2 \quad x = -1$

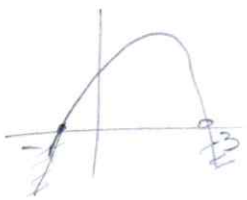


$-1 < x < 2$

f) $3 + 2x - x^2 < 0$

$(3-x)(1+x) < 0$

$x = 3 \quad x = -1$



$x < -1$ or $x > 3$

9 a) $x^2 + 5x + 4 = 0$

$a = 1 \quad b = 5 \quad c = 4$

$b^2 - 4ac$

$(5^2) - (4 \times 1 \times 4)$

$= 25 - 16$

$= 9$

Real + distinct

b) $4x^2 + 12x + 9 = 0$

$a = 4 \quad b = 12 \quad c = 9$

$b^2 - 4ac$

$(12)^2 - (4 \times 4 \times 9)$

$= 144 - 144$

$= 0$

Real and equal

c) $2x^2 + x - 3 = 0$

$a = 2 \quad b = 1 \quad c = -3$

$b^2 - 4ac$

$(1)^2 - (4 \times 2 \times -3)$

$= 1 - (-24)$

$= 25$

Real and distinct

d) $x^2 + x + 2 = 0$

$a = 1 \quad b = 1 \quad c = 2$

$b^2 - 4ac$

$(1)^2 - (4 \times 1 \times 2)$

$= 1 - 8$

$= -7$

Roots are non-real

e) $2x^2 - 5x + 3 = 0$

$a = 2 \quad b = -5 \quad c = 3$

$b^2 - 4ac$

$(-5)^2 - (4 \times 2 \times 3)$

$= 25 - 24$

$= 1$

Real and distinct

f) $3x^2 - 6x + 3 = 0$

$a = 3 \quad b = -6 \quad c = 3$

$b^2 - 4ac$

$(-6)^2 - (4 \times 3 \times 3)$

$= 36 - 36$

$= 0$

Real and equal

g) $9x^2 + 1 = -6x$

$9x^2 + 6x + 1 = 0$

$a = 9 \quad b = 6 \quad c = 1$

$b^2 - 4ac$

$(6)^2 - (4 \times 9 \times 1)$

$= 36 - 36$

$= 0$

Real and equal

h) $(x-1)^2 + 3x^2 = 6x - 11$

$x^2 - 2x + 1 + 3x^2 - 6x + 11 = 0$

$4x^2 - 8x + 12 = 0$

$a = 4 \quad b = -8 \quad c = 12$

$b^2 - 4ac$

$(-8)^2 - (4 \times 4 \times 12)$

$= 64 - 192$

$= -128$

Roots are non-real

10. a) $2x^2+4x-p=0$

$a=2 \quad b=4 \quad c=-p$

$b^2-4ac=0$

$(4)^2-(4\times 2\times -p)=0$

$16+8p=0$

$8p=-16$

$p=-2$

b) $x^2+(p+1)x+9=0$

$a=1 \quad b=p+1 \quad c=9$

$b^2-4ac=0$

$(p+1)^2-(4\times 1\times 9)=0$

$p^2+2p+1-36=0$

$p^2+2p-35=0$

$(p+7)(p-5)=0$

$p=-7$ or $p=5$

c) $(p+1)x^2+2px+(p-2)=0$

$a=p+1 \quad b=2p \quad c=p-2$

$b^2-4ac=0$

$(2p)^2-(4\times(p+1)(p-2))=0$

$4p^2-(4(p^2-p-2))=0$

$4p^2-(4p^2-4p-8)=0$

$4p^2-4p^2+4p+8=0$

$4p+8=0$

$p=-2$

d) $(p+1)x^2-2(p+3)x+3p=0$

$a=p+1 \quad b=-2(p+3) \quad c=3p$

$=(-2p-6)$

$b^2-4ac=0$

$(-2p-6)^2-(4\times(p+1)\times 3p)=0$

$(4p^2+24p+36)-(12p^2+12p)=0$

$4p^2+24p+36-12p^2-12p=0$

$-8p^2+12p+36=0$

$-4(2p^2-3p-9)=0$

$-4(2p+3)(p-3)=0$

$p=-\frac{3}{2}$ or $p=3$

e) $p^2x^2+2(p+1)x+4=0$

$a=p^2 \quad b=2(p+1) \quad c=4$

$=(2p+2)$

$b^2-4ac=0$

$(2p+2)^2-(4\times p^2\times 4)=0$

$(4p^2+8p+4)-(16p^2)=0$

$-12p^2+8p+4=0$

$-4(3p^2-2p-1)=0$

$-4(3p+1)(p-1)=0$

$p=-\frac{1}{3}$ or $p=1$

11. $kx^2 + 3x + 3 = k$

$kx^2 + 3x + 3 - k = 0$

$a = k \quad b = 3 \quad c = 3 - k$

$b^2 - 4ac$

$= (3)^2 - (4 \times k \times (3-k))$

$= 9 - (12k - 4k^2)$

$= 9 - 12k + 4k^2$

$= (3-2k)(3-2k)$

$= (3-2k)^2$

Any value of k will give an answer ≥ 0 as $(3-2k)^2$

So roots always real.

12. $(x-2)(x-3) = k^2$

$x^2 - 5x + 6 - k^2 = 0$

$a = 1 \quad b = -5 \quad c = 6 - k^2$

$b^2 - 4ac$

$= (-5)^2 - (4 \times 1 \times (6-k^2))$

$= 25 - (24 - 4k^2)$

$= 1 + 4k^2$

As k is squared it will always be +ve so $1 + 4k^2 > 0$

So roots always real

13 a) $y = 6 + 2x \quad y = 5 - x^2$

$6 + 2x = 5 - x^2$

$x^2 + 2x + 1 = 0$

$(x+1)^2 = 0$

$x = -1$

Only one point of contact so tangent

$y = 6 + (2 \times -1) = 4$

Point of contact $(-1, 4)$

b) $y = 17 - 7x \quad y = -x^2 - x + 8$

$17 - 7x = -x^2 - x + 8$

$x^2 - 6x + 9 = 0$

use either method →

$b^2 - 4ac$

$(-6)^2 - (4 \times 1 \times 9)$

$= 36 - 36$

$= 0$ As $b^2 - 4ac = 0$ line is a tangent

$x^2 - 6x + 9 = 0$

$(x-3)^2 = 0$

$x = 3$

$y = 17 - (7 \times 3) = -4$

Point of contact $(3, -4)$

14. $y = -8x + k \quad y = 6x - x^2$

$-8x + k = 6x - x^2$

$x^2 - 14x + k = 0$

$a = 1 \quad b = -14 \quad c = k$

$b^2 - 4ac = 0$

$(-14)^2 - (4 \times 1 \times k) = 0$

$196 - 4k = 0$

$4k = 196$

$k = 49$

$\Rightarrow x^2 - 14x + 49 = 0$

$(x-7)(x-7) = 0$

$x = 7$

$y = (6 \times 7) - (7)^2$

$= -7$

$(7, -7)$

Eq of tangent $y = -8x + k$

$y = -8x + 49$

15. $y = mx + 8 \quad y = x^2 + 9$

$mx + 8 = x^2 + 9$

$x^2 - mx + 1 = 0$

$a = 1 \quad b = -m \quad c = 1$

$b^2 - 4ac = 0$

$(-m)^2 - (4 \times 1 \times 1) = 0$

$m^2 - 4 = 0$

$m^2 = 4$

$m = \pm 2$

16. $f(x) = x^2 + 3 \quad g(x) = x + 4$

a) i) $f(g(x)) = f(x+4)$

$= (x+4)^2 + 3$

$= x^2 + 8x + 16 + 3$

$= x^2 + 8x + 19$

ii) $g(f(x)) = g(x^2 + 3)$

$= (x^2 + 3) + 4$

$= x^2 + 7$

b) $f(g(x)) + g(f(x)) = 0$

$x^2 + 8x + 19 + x^2 + 7 = 0$

$2x^2 + 8x + 26 = 0$

$a = 2 \quad b = 8 \quad c = 26$

$b^2 - 4ac$

$(8)^2 - (4 \times 2 \times 26)$

$= 64 - 208$

$= -144$

As $b^2 - 4ac < 0$ no real roots.