St Peter the Apostle High School

Maths Department

Higher Practice Questions

8. Quadratics

1 Factorise fully

(a) $98 - 2x^2$ **(b)** $8x^2 - 98$ **(c)** $81 - x^4$

(d) $27w - 12x^3$ **(e)** $50x^3 - 2x$ **(f)** $5r^3 - 20r$

2 Factorise fully

(a) $x^2 + x - 6$ **(b)** $2x^2 - 7x + 3$ **(c)** $6x^2 + 7x + 2$

(d) $3x^2 + 10x + 8$ **(e)** $12x^2 - 4x - 1$ **(f)** $8x^2 + 6x - 9$

(g) $6 - x - x^2$ **(h)** $15 - 7x - 2x^2$ **(i)** $20 + 11x - 3x^2$

3 Solve each of these quadratic equations

(a) $x^2 - 11x + 24 = 0$ **(b)** $4x^2 - 9 = 0$ **(c)** $n^2 + 3n - 10 = 0$

(d) $2r^2 - r - 6 = 0$ **(e)** $7x^2 - 14x = -7$ **(f)** $n^2 + 8n = -15$

4 For each of the quadratic functions given below

(i) Find the coordinates of the points where the graph of $y = f(x)$ cuts the axes.

(ii) State the equation of the axis of symmetry of $y = f(x)$

(iii) Write down the coordinates of the turning point of $y = f(x)$

(iv) Sketch the graph of $y = f(x)$

(a) $f(x) = 5x^2 + 20x$

(b) $f(x) = x^2 + 6x + 8$

(c) $f(x) = 15 - 2x - x^2$

5 Express each of the following in the form $a(x-p)^2+q$

(a) $x^2-8x+12$ **(b)** $x^2+10x+21$ **(c)** $x^2-6x-16$

(d) $2x^2-4x+1$ **(e)** $3x^2-18x+10$ **(f)** $2x^2-11x+10$
(* tricky – fractions)

(g) $10+4x-x^2$ **(h)** $3-6x-x^2$ **(i)** $15+8x-2x^2$

6 Express x^2+2x+7 in the form $(x+p)^2+q$, and hence state the maximum value of $\dfrac{1}{x^2+2x+7}$ and the corresponding value of x.

7 **(a)** Show that the function $f(x)=9-8x-x^2$ can be written in the form $f(x)=p(x+q)^2+r$, where p, q and r are constants

(b) Hence, or otherwise, find the minimum value of $g(x)=\dfrac{1}{f(x)}$

8 Solve the following quadratic inequalities

(a) $x^2 + x - 2 > 0$ **(b)** $2x^2 - 5x - 3 < 0$ **(c)** $x^2 - 2 > 0$

(d) $x^2 - 5x < 0$ **(e)** $2 + x - x^2 > 0$ **(f)** $3 + 2x - x^2 < 0$

9 By calculating the value of the discriminant, determine the nature of the roots of each equation.

(a) $x^2 + 5x + 4 = 0$ **(b)** $4x^2 + 12x + 9 = 0$ **(c)** $2x^2 + x - 3 = 0$

(d) $x^2 + x + 2 = 0$ **(e)** $2x^2 - 5x + 3 = 0$ **(f)** $3x^2 - 6x + 3 = 0$

(g) $9x^2 + 1 = -6x$ **(h)** $(x - 1)^2 + 3x^2 = 6x - 11$

10 For each equation find the value(s) for p so that the roots are equal

(a) $2x^2 + 4x - p = 0$

(b) $x^2 + (p+1)x + 9 = 0$

(c) $(p+1)x^2 + 2px + (p-2) = 0$

(d) $(p+1)x^2 - 2(p+3)x + 3p = 0$

(e) $p^2x^2 + 2(p+1)x + 4 = 0$

11 Given that k is a real number, show that the roots of the equation $kx^2 + 3x + 3 = k$ are always real numbers

12 Show that the roots of the equation $(x-2)(x-3) = k^2$ are always real when k is real

13 Prove that

(a) $y = 6 + 2x$ is a tangent to $y = 5 - x^2$ and find the point of contact

(b) $y = 17 - 7x$ is a tangent to the parabola $y = -x^2 - x + 8$ and find the point of contact

14 The line $y = -8x + k$ is a tangent to the parabola $y = 6x - x^2$
Find the equation of the tangent

15 The line $y = mx + 8$ is a tangent to $y = x^2 + 9$. Find two values for m

16 Functions f and g are defined on the set of real numbers by

- $f(x) = x^2 + 3$
- $g(x) = x + 4$

(a) Find expressions for

(i) $f(g(x))$
(ii) $g(f(x))$

(b) Show that $f(g(x)) + g(f(x)) = 0$ has no real roots