9. Integration

1

a) x^5

$\rightarrow \frac{x^6}{6} + C$

b) x^9

$\rightarrow \frac{x^{10}}{10} + C$

c) x^{14}

$\rightarrow \frac{x^{15}}{15} + C$

d) x^{50}

$\rightarrow \frac{x^{51}}{51} + C$

e) x^{-3}

$\rightarrow \frac{x^{-2}}{-2} + C$

$= -\frac{1}{2x^2} + C$

f) x^{-6}

$\rightarrow \frac{x^{-5}}{-5} + C$

$= -\frac{1}{5x^5} + C$

g) x^{-10}

$\rightarrow \frac{x^{-9}}{-9} + C$

$= -\frac{1}{9x^9} + C$

h) x^{-27}

$\rightarrow \frac{x^{-26}}{-26} + C$

$= -\frac{1}{26x^{26}} + C$

2

a) $2x^3$

$\rightarrow \frac{2x^4}{4} + C$

$= \frac{x^4}{2} + C$

b) $5x^4$

$\rightarrow \frac{5x^5}{5} + C$

$= x^5 + C$

c) $\frac{1}{2}x^5$

$\rightarrow \frac{1}{2} \times \frac{x^6}{6} + C$

$= \frac{x^6}{12} + C$

d) $\frac{2}{5}x^3$

$\rightarrow \frac{\cancel{2}^{1}}{5} \times \frac{x^4}{\cancel{4}_{2}} + C$

$= \frac{x^4}{10} + C$

e) $\frac{2}{x^2} = 2x^{-2}$

$\rightarrow \frac{2x^{-1}}{-1} + C$

$= -\frac{2}{x} + C$

f) $\frac{3}{x^3} = 3x^{-3}$

$\rightarrow \frac{3x^{-2}}{-2} + C$

$= -\frac{3}{2x^2} + C$

g) $3x^{-5}$

$\rightarrow \frac{3x^{-4}}{-4} + C$

$= -\frac{3}{4x^4} + C$

h) $2x^{-2}$

$\rightarrow \frac{2x^{-1}}{-1} + C$

$= -\frac{2}{x} + C$

i) $\frac{14}{x^{-4}} = 14x^4$

$\rightarrow \frac{14x^5}{5} + C$

j) $\frac{2}{7x^2} = \frac{2}{7}x^{-2}$

$\rightarrow \frac{2}{7} \times \frac{x^{-1}}{-1} + C$

$= -\frac{2}{7x} + C$

k) $\frac{6}{5x^3} = \frac{6}{5}x^{-3}$

$\rightarrow \frac{\cancel{6}^{3}}{5} \times \frac{x^{-2}}{\cancel{-2}_{1}} + C$

$= \frac{3}{5x^2} + C$

l) $\frac{5}{2x^{-2}}$

$= \frac{5}{2}x^2$

$\rightarrow \frac{5}{2} \times \frac{x^3}{3} + C$

$= \frac{5x^3}{6} + C$

3 a) $x^{\frac{1}{2}}$

$\rightarrow \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \dfrac{2x^{\frac{3}{2}}}{3} + c$

$= \dfrac{2\sqrt{x^3}}{3} + c$

b) $x^{2/3}$

$\rightarrow \dfrac{x^{\frac{5}{3}}}{\frac{5}{3}} + c$

$= \dfrac{3x^{\frac{5}{3}}}{5} + c$

$= \dfrac{3\sqrt[3]{x^5}}{5} + c$

c) $x^{\frac{5}{6}}$

$\rightarrow \dfrac{x^{\frac{11}{6}}}{\frac{11}{6}} + c$

$= \dfrac{6x^{\frac{11}{6}}}{11} + c$

$= \dfrac{6\sqrt[6]{x^{11}}}{11} + c$

d) $x^{\frac{1}{4}}$

$\rightarrow \dfrac{x^{\frac{5}{4}}}{\frac{5}{4}} + c$

$= \dfrac{4x^{\frac{5}{4}}}{5} + c$

$= \dfrac{4\sqrt[4]{x^5}}{5} + c$

e) $x^{3/7}$

$\rightarrow \dfrac{x^{\frac{10}{7}}}{\frac{10}{7}} + c$

$= \dfrac{7x^{\frac{10}{7}}}{10} + c$

$= \dfrac{7\sqrt[7]{x^{10}}}{10} + c$

f) $x^{2/9}$

$\rightarrow \dfrac{x^{\frac{11}{9}}}{\frac{11}{9}} + c$

$= \dfrac{9x^{\frac{11}{9}}}{11} + c$

$= \dfrac{9\sqrt[9]{x^{11}}}{11} + c$

g) $x^{\frac{1}{5}}$

$\rightarrow \dfrac{x^{\frac{6}{5}}}{\frac{6}{5}} + c$

$= \dfrac{5x^{\frac{6}{5}}}{6} + c$

$= \dfrac{5\sqrt[5]{x^6}}{6} + c$

h) $x^{3/11}$

$\rightarrow \dfrac{x^{\frac{14}{11}}}{\frac{14}{11}} + c$

$= \dfrac{11x^{\frac{14}{11}}}{14} + c$

$= \dfrac{11\sqrt[11]{x^{14}}}{14} + c$

4 a) $\sqrt{x} = x^{\frac{1}{2}}$

$\rightarrow \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} + c$

$= \dfrac{2x^{\frac{3}{2}}}{3} + c$

$= \dfrac{2\sqrt{x^3}}{3} + c$

b) $\sqrt{x^3} = x^{\frac{3}{2}}$

$\rightarrow \dfrac{x^{\frac{5}{2}}}{\frac{5}{2}} + c$

$= \dfrac{2\sqrt{x^5}}{5} + c$

c) $\sqrt[3]{x} = x^{\frac{1}{3}}$

$\rightarrow \dfrac{x^{\frac{4}{3}}}{\frac{4}{3}} + c$

$= \dfrac{3\sqrt[3]{x^4}}{4} + c$

d) $\sqrt[3]{x^2} = x^{\frac{2}{3}}$

$\rightarrow \dfrac{x^{\frac{5}{3}}}{\frac{5}{3}} + c$

$= \dfrac{3\sqrt[3]{x^5}}{5} + c$

e) $\sqrt[5]{x^4} = x^{\frac{4}{5}}$

$\rightarrow \dfrac{x^{\frac{9}{5}}}{\frac{9}{5}} + c$

$= \dfrac{5\sqrt[5]{x^9}}{9} + c$

f) $\dfrac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$\rightarrow \dfrac{x^{\frac{1}{2}}}{\frac{1}{2}} + c$

$= 2\sqrt{x} + c$

g) $\dfrac{1}{\sqrt[5]{x^3}} = x^{-\frac{3}{5}}$

$\rightarrow \dfrac{x^{\frac{2}{5}}}{\frac{2}{5}} + c$

$= \dfrac{5\sqrt[5]{x^2}}{2} + c$

h) $\dfrac{3}{\sqrt[4]{x^3}} = 3x^{-\frac{3}{4}}$

$\rightarrow \dfrac{3x^{\frac{1}{4}}}{\frac{1}{4}} + c$

$= 12\sqrt[4]{x} + c$

i) $\dfrac{1}{2\sqrt[3]{x^2}} = \frac{1}{2}x^{-\frac{2}{3}}$

$\rightarrow \frac{1}{2}\dfrac{x^{\frac{1}{3}}}{\frac{1}{3}} + c$

$= \frac{1}{2} \times 3x^{\frac{1}{3}} + c$

$= \frac{3}{2}\sqrt[3]{x} + c$

5 a) $$\int x^3 + 3x^2 + 5x \, dx$$
$$= \frac{x^4}{4} + \frac{3x^3}{3} + \frac{5x^2}{2} + c$$
$$= \frac{x^4}{4} + x^3 + \frac{5x^2}{2} + c$$

b) $$\int 3x^5 + 2x^4 - x \, dx$$
$$= \frac{3x^6}{6} + \frac{2x^5}{5} - \frac{x^2}{2} + c$$
$$= \frac{x^6}{2} + \frac{2x^5}{5} - \frac{x^2}{2} + c$$

c) $$\int (x^2 + 6x - 1) \, dx$$
$$= \frac{x^3}{3} + \frac{6x^2}{2} - x + c$$
$$= \frac{x^3}{3} + 3x^2 - x + c$$

d) $$\int x^{\frac{2}{3}} + 4x^2 \, dx$$
$$= \frac{x^{\frac{5}{3}}}{\frac{5}{3}} + \frac{4x^3}{3} + c$$
$$= \frac{3\sqrt[3]{x^5}}{5} + \frac{4x^3}{3} + c$$

e) $$\int 3x^{\frac{1}{2}} - 2x^{-5} \, dx$$
$$= \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2x^{-4}}{-4} + c$$
$$= 2\sqrt{x^3} + \frac{1}{2x^4} + c$$

f) $$\int 5x^{-2} - 3x^{\frac{1}{2}} \, dx$$
$$= \frac{5x^{-1}}{-1} - \frac{3x^{\frac{3}{2}}}{\frac{3}{2}} + c$$
$$= -\frac{5}{x} - 2\sqrt{x^3} + c$$

g) $$\int \frac{1}{2}x^{-\frac{1}{3}} + x^2 \, dx$$
$$= \frac{1}{2 \times \frac{2}{3}} x^{2/3} + \frac{x^3}{3} + c$$
$$= \frac{3\sqrt[3]{x^2}}{4} + \frac{x^3}{3} + c$$

h) $$\int (3x^7 - \frac{1}{5}x^{-\frac{3}{4}} \, dx$$
$$= \frac{3x^8}{8} - \frac{1}{5 \times \frac{1}{4}} x^{\frac{1}{4}} + c$$
$$= \frac{3x^8}{8} - \frac{4\sqrt[4]{x}}{5} + c$$

i) $$\int \frac{3}{5}x^{-\frac{5}{2}} + 5 \, dx$$
$$= \frac{3}{5 \times \frac{-3}{2}} x^{\frac{-3}{2}} + 5x + c$$
$$= \frac{6}{-15} x^{-\frac{3}{2}} + 5x + C$$
$$= -\frac{2}{5\sqrt{x^3}} + 5x + c$$

6. a) $\int (x+1)(x+2)\,dx$

$\int x^2 + 3x + 2 \; dx$

$= \frac{x^3}{3} + \frac{3x^2}{2} + 2x + c$

b) $\int (x-3)(x+4)\,dx$

$\int x^2 + x - 12 \; dx$

$= \frac{x^3}{3} + \frac{x^2}{2} - 12x + c$

c) $\int x(x+4)\,dx$

$\int x^2 + 4x \; dx$

$= \frac{x^3}{3} + \frac{4x^2}{2} + c$

$= \frac{x^3}{3} + 2x^2 + c$

d) $\int x^{-2}(x^3 - 2x^2)\,dx$

$\int x - 2x^0 \; dx$

$\int x - 2 \; dx$

$= \frac{x^2}{2} - 2x + c$

e) $\int x^{-1}(x^2 + x)\,dx$

$\int x + x^0 \; dx$

$\int x + 1 \; dx$

$= \frac{x^2}{2} + x + c$

f) $\int (x^{-1} + x)^2\,dx$

$\int x^{-2} + x^0 + x^0 + x^2 \; dx$

$\int x^{-2} + 2 + x^2 \; dx$

$= \frac{x^{-1}}{-1} + 2x + \frac{x^3}{3} + c$

$= -\frac{1}{x} + 2x + \frac{x^3}{3} + c$

7 a) $\int x^{-1}(x^2+3x)\,dx$

$\int x + 3 \; dx$

$= \frac{x^2}{2} + 3x + C$

b) $\int x^{-1}(2x^3+x^2+x)\,dx$

$\int 2x^2 + x + 1 \; dx$

$= \frac{2x^3}{3} + \frac{x^2}{2} + x + C$

c) $\int x^{-2}(x^4+x^3-6)\,dx$

$\int x^2 + x - 6x^{-2} \; dx$

$= \frac{x^3}{3} + \frac{x^2}{2} - \frac{6x^{-1}}{-1} + C$

$= \frac{x^3}{3} + \frac{x^2}{2} + \frac{6}{x} + C$

d) $\int x^{-\frac{1}{2}}(x^2-1)\,dx$

$\int x^{3/2} - x^{-\frac{1}{2}} \; dx$

$= \frac{x^{\frac{5}{2}}}{\frac{5}{2}} - \frac{x^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= \frac{2\sqrt{x^5}}{5} - 2\sqrt{x} + C$

e) $\int x^{-3}(1+x^{\frac{1}{2}})\,dx$

$\int x^{-3} + x^{-\frac{5}{2}} \; dx$

$= \frac{x^{-2}}{-2} + \frac{x^{-\frac{3}{2}}}{-\frac{3}{2}} + C$

$= -\frac{1}{2x^2} - \frac{2}{3\sqrt{x^3}} + C$

f) $\int \frac{1}{2}x^{-2}(3x^4+5x^2+1)\,dx$

$\int \frac{3}{2}x^2 + \frac{5}{2}x^0 + \frac{1}{2}x^{-2} \; dx$

$\int \frac{3}{2}x^2 + \frac{5}{2} + \frac{1}{2}x^{-2} \; dx$

$= \frac{3x^3}{2\times3} + \frac{5}{2}x + \frac{1x^{-1}}{2\times-1} + C$

$= \frac{x^3}{2} + \frac{5}{2}x - \frac{1}{2x} + C$

8 a) $\int_{-1}^{1} (5x^3 - 2x)\,dx$

$= \left[\frac{5x^4}{4} - x^2\right]_{-1}^{1}$

$= \left(\frac{5\times1^4}{4} - 1^2\right) - \left(\frac{5\times(-1)^4}{4} - (-1)^2\right)$

$= \left(\frac{1}{4}\right) - \left(\frac{1}{4}\right)$

$= \underline{\underline{0}}$

b) $\int_{-1}^{1} (3x^2 - 4x - 4)\,dx$

$= \left[x^3 - 2x^2 - 4x\right]_{-1}^{1}$

$= (1^3 - 2\times1^2 - 4\times1) - ((-1)^3 - 2(-1)^2 - 4(-1))$

$= (-5) - (1)$

$= \underline{\underline{-6}}$

c) $\int_{-2}^{0} (x^2 - 6x + 9)\,dx$

$= \left[\frac{x^3}{3} - 3x^2 + 9x\right]_{-2}^{0}$

$= \left(\frac{0^3}{3} - 3\times0^2 + 9\times0\right) - \left(\frac{(-2)^3}{3} - 3\times(-2)^2 + 9(-2)\right)$

$= (0) - \left(-\frac{98}{3}\right)$

$= \underline{\underline{\frac{98}{3}}}$ or $\underline{\underline{32^2/3}}$

d) $\int_{0}^{2} (3x^2 + 8x - 5)\,dx$

$= \left[x^3 + 4x^2 - 5x\right]_{0}^{2}$

$= (2^3 + 4(2)^2 - 5(2)) - (0^3 + 4(0)^2 - 5(0))$

$= (14) - (0)$

$= \underline{\underline{14}}$

e) $\int_{-1}^{2} 1 + 2x - 3x^2\,dx$

$= \left[x + x^2 - x^3\right]_{-1}^{2}$

$= (2 + (2)^2 - (2)^3) - ((-1) + (-1)^2 - (-1)^3)$

$= (-2) - (1)$

$= \underline{\underline{-3}}$

f) $\int_{1}^{3} (x^2 - x^{-2})\,dx$

$= \left[\frac{x^3}{3} - \frac{x^{-1}}{-1}\right]_{1}^{3}$

$= \left[\frac{x^3}{3} + \frac{1}{x}\right]_{1}^{3}$

$= \left(\frac{3^3}{3} + \frac{1}{3}\right) - \left(\frac{1^3}{3} + \frac{1}{1}\right)$

$= (9\tfrac{1}{3}) - (1\tfrac{1}{3})$

$= \underline{\underline{8}}$

9. a) i) $\int_0^a (2x+2)\,dx = 8$

$\left[x^2+2x\right]_0^a = 8$

$(a^2+2a)-(0) = 8$

$a^2+2a-8=0$

$(a-2)(a+4)=0$

$\underline{\underline{a=2}}$ $\cancel{a=-4}$

ii) $\int_0^a x^2\,dx = \frac{64}{3}$

$\left[\frac{x^3}{3}\right]_0^a = \frac{64}{3}$

$\left(\frac{a^3}{3}\right)-(0) = \frac{64}{3}$

$a^3 = 64$

$\underline{\underline{a=4}}$

b) $\int_0^a 3x^{\frac{1}{2}}\,dx = 16$

$\frac{3x^{\frac{3}{2}}}{\frac{3}{2}} \Rightarrow \left[2\sqrt{x^3}\right]_0^a = 16.$

$\Rightarrow (2\sqrt{a^3}) = 16$

$\sqrt{a^3} = 8$

$a^3 = 64.$

$\underline{\underline{a=4}}$

c) $\int_1^p x^{\frac{1}{2}}\,dx = 42$

$\frac{x^{\frac{3}{2}}}{\frac{3}{2}} \Rightarrow \left[\frac{2\sqrt{x^3}}{3}\right]_1^p = 42$

$\Rightarrow \left(\frac{2\sqrt{p^3}}{3}\right) - \left(\frac{2\sqrt{1^3}}{3}\right) = 42$

$\frac{2\sqrt{p^3}}{3} - \frac{2}{3} = 42$

$\frac{2\sqrt{p^3}}{3} = \frac{128}{3}.$ $\left(42\tfrac{2}{3}\right)$

$\sqrt{p^3} = \frac{128\times 3}{3\times 2}$

$\sqrt{p^3} = 64$

$p = \sqrt[3]{64^2}$

$p = \underline{\underline{16}}$

d) $\int_0^k x^{\frac{1}{3}}\,dx = 12$

$\frac{x^{\frac{4}{3}}}{\frac{4}{3}} \Rightarrow \left[\frac{3\sqrt[3]{x^4}}{4}\right]_0^k = 12$

$\left(\frac{3\sqrt[3]{k^4}}{4}\right) - (0) = 12$

$k^{\frac{4}{3}} = \frac{12\times 4}{3}$

$k^{\frac{4}{3}} = 16$

$k = 16^{3/4}$

$k = \sqrt[4]{16^3}$

$k = \underline{\underline{8}}$

10. a) $y = 5x - x^2$

$y = x(5-x)$

so $\underline{\underline{x=0}}$ $\underline{\underline{x=5}}$

$$\int_0^5 5x - x^2 \, dx$$

$$= \left[\frac{5x^2}{2} - \frac{x^3}{3}\right]_0^5$$

$$= \left(\frac{5\times5^2}{2} - \frac{5^3}{3}\right) - \left(\frac{5\times0^2}{2} - \frac{0^3}{3}\right)$$

$$= \left(\frac{125}{2} - \frac{125}{3}\right) - (0)$$

$$= \underline{\underline{\frac{125}{6} \text{ units}^2}}$$

b) $y = 4x - x^2$

$y = x(4-x)$

so $\underline{\underline{x=0}}$ $\underline{\underline{x=4}}$

$$\int_0^4 4x - x^2 \, dx$$

$$= \left[2x^2 - \frac{x^3}{3}\right]_0^4$$

$$= \left((2\times4^2) - \frac{4^3}{3}\right) - \left(2\times0^2 - \frac{0^3}{3}\right)$$

$$= \left(32 - \frac{64}{3}\right) - (0)$$

$$= \underline{\underline{\frac{32}{3} \text{ units}^2}}$$

c) $y = 3 - 3x^2$

$y = 3(1 - x^2)$

$y = 3(1-x)(1+x)$

$\underline{\underline{x=1}}$, $\underline{\underline{x=-1}}$

$$\int_{-1}^1 3 - 3x^2 \, dx$$

$$= \left[3x - x^3\right]_{-1}^1$$

$$= \left((3\times1) - (1)^3\right) - \left(3\times(-1) - (-1)^3\right)$$

$$= (3 - 1) - (-3 - (-1))$$

$$= (2) - (-2)$$

$$= \underline{\underline{4 \text{ units}^2}}$$

d) $y = 10 - 4x - 3x^2$

(limits given this time)

$$\int_{-1}^1 10 - 4x - 3x^2 \, dx$$

$$= \left[10x - 2x^2 - x^3\right]_{-1}^1$$

$$= \left((10\times1) - 2\times(1)^2 - (1)^3\right) - \left(10\times(-1) - 2\times(-1)^2 - (-1)^3\right)$$

$$= (10 - 2 - 1) - (-10 - 2 + 1)$$

$$= (7) - (-11)$$

$$= \underline{\underline{18 \text{ units}^2}}$$

11. a) $y = x^3 - 3x^2 + 2x$

$y = x(x^2 - 3x + 2)$

$y = x(x-1)(x-2)$ $\Rightarrow P(1,0)$ $Q(2,0)$

so $x = 0$, $x = 1$, $x = 2$.

b) $\int_0^1 x^3 - 3x^2 + 2x \, dx$

$= \left[\frac{x^4}{4} - x^3 + x^2 \right]_0^1$

$= \left(\frac{1^4}{4} - 1^3 + 1^2 \right) - (0)$

$= \left(\frac{1}{4} - 1 + 1 \right)$

$= \frac{1}{4}$

$\int_1^2 x^3 - 3x^2 + 2x \, dx$

$= \left[\frac{x^4}{4} - x^3 + x^2 \right]_1^2$

$= \left(\frac{2^4}{4} - 2^3 + 2^2 \right) - \left(\frac{1}{4} \right)$

$= (4 - 8 + 4) - \left(\frac{1}{4} \right)$

$= -\frac{1}{4}$.

Total area $\frac{1}{4} + \frac{1}{4} = \frac{1}{2}$ units^2

12. a) $y = x^3 - 3x^2 - 10x$

$y = x(x^2 - 3x - 10)$

$y = x(x+2)(x-5)$ $\Rightarrow A(-2,0)$ $B(5,0)$

$x = 0$, $x = -2$ $x = 5$

b) $\int_{-2}^0 x^3 - 3x^2 - 10x \, dx$

$= \left[\frac{x^4}{4} - x^3 - 5x^2 \right]_{-2}^0$

$= (0) - \left(\frac{(-2)^4}{4} - (-2)^3 - 5 \times (-2)^2 \right)$

$= (0) - (4 - (-8) - 20)$

$= (0) - (-8)$

$= 8$

$\int_0^5 x^3 - 3x^2 - 10x \, dx$

$= \left[\frac{x^4}{4} - x^3 - 5x^2 \right]_0^5$

$= \left(\frac{5^4}{4} - 5^3 - 5 \times 5^2 \right) - (0)$

$= \left(\frac{625}{4} - 125 - 125 \right)$

$= -\frac{375}{4}$

Total area $\underset{(8)}{\frac{32}{4}} + \frac{375}{4} = \frac{407}{4}$ units^2 $\left(101\frac{3}{4} \text{ units}^2 \right)$

13. Top - bottom

$$\int_0^4 (x^2+2x)-(x^3-x^2-6x)\,dx$$

$$\int_0^4 -x^3+2x^2+8x \quad dx$$

$$= \left[-\frac{x^4}{4}+\frac{2x^3}{3}+4x^2\right]_0^4$$

$$= \left(-\frac{(4)^4}{4}+\frac{2\times(4^3)}{3}+4\times 4^2\right)-(0)$$

$$= \left(-64+\frac{128}{3}+64\right)$$

$$= \underline{\underline{\frac{128}{3}\text{units}^2}}$$

14. a) $y=x^2 \qquad y=2x^2-9$

$$x^2 = 2x^2-9$$

$$2x^2-x^2-9=0$$

$$x^2-9=0$$

$$(x-3)(x+3)=0$$

$$\underline{\underline{x=3}} \qquad \underline{\underline{x=-3}}$$

$y=3^2 = 9 \qquad \underline{\underline{L(3,9)}}$

$y=(-3)^2 = 9 \qquad \underline{\underline{K(-3,9)}}$

b) Top - bottom

$$\int_{-3}^{3} (x^2)-(2x^2-9)\,dx$$

$$\int_{-3}^{3} -x^2+9 \quad dx$$

$$= \left[-\frac{x^3}{3}+9x\right]_{-3}^{3}$$

$$= \left(-\frac{(3^3)}{3}+9\times3\right)-\left(-\frac{(-3)^3}{3}+9\times(-3)\right)$$

$$= (-9+27)-(9-27)$$

$$= (18)-(-18)$$

$$= 36\text{units}^2$$

15. $y = 1 + 10x - 2x^2 \qquad y = 1 + 5x - x^2$

$$1 + 10x - 2x^2 = 1 + 5x - x^2$$

$$x^2 - 5x = 0$$

$$x(x-5) = 0$$

$$\underline{\underline{x = 0}} \qquad \underline{\underline{x = 5}}$$

(top) − (bottom)

$$\int_0^5 (1 + 10x - 2x^2) - (1 + 5x - x^2)\, dx$$

$$\int_0^5 -x^2 + 5x \quad dx$$

$$= \left[-\frac{x^3}{3} + \frac{5x^2}{2} \right]_0^5$$

$$= \left(-\frac{5^3}{3} + \frac{5 \times 5^2}{2} \right) - (0)$$

$$= \left(-\frac{125}{3} + \frac{125}{2} \right)$$

$$= \underline{\underline{\frac{125}{6} \text{units}^2}}$$

16 a) $\frac{dy}{dx} = 3x^2 - 6x + 1$

$\int 3x^2 - 6x + 1 \; dx$

(3,4) $y = x^3 - 3x^2 + x + C$

$4 = (3)^3 - (3 \times 3^2) + 3 + C$

$4 = 27 - 27 + 3 + C$

$C = 1$

$\Rightarrow y = \underline{\underline{x^3 - 3x^2 + x + 1}}$

b) $\frac{dy}{dx} = 4x^3 - 6x^2$

$\int 4x^3 - 6x^2 \; dx$

(1,9) $y = x^4 - 2x^3 + C$

$9 = (1)^4 - 2 \times 1^3 + C$

$9 = 1 - 2 + C$

$C = 10$

$\Rightarrow y = \underline{\underline{x^4 - 2x^3 + 10}}$

17 a) $\frac{dy}{dx} = 4x$

$\int 4x \; dx$

(1,3) $y = 2x^2 + C$

$3 = 2 \times 1^2 + C$

$C = 1$

$\Rightarrow y = \underline{\underline{2x^2 + 1}}$

b) $\frac{dy}{dx} = \frac{1}{\sqrt{x}} \Rightarrow x^{-\frac{1}{2}}$

$\int x^{-\frac{1}{2}} \; dx$

(9,10) $y = 2\sqrt{x} + C$

$10 = 2\sqrt{9} + C$

$10 = 6 + C$

$C = 4$

$\Rightarrow y = \underline{\underline{2\sqrt{x} + 4}}$

18 a) $\frac{dy}{dx} = 4x^3 + \frac{2}{x^2}$

$\int 4x^3 + 2x^{-2} \; dx$

$x=1$
$y=0$

$y = x^4 - \frac{2}{x} + C$

$0 = 1^4 - \frac{2}{1} + C$

$C = 1$

$\Rightarrow y = \underline{\underline{x^4 + \frac{2}{x} + 1}}$

b) $\frac{dy}{dx} = \frac{x^2+1}{x^2} \Rightarrow x^{-2}(x^2+1)$

$1 + x^{-2}$

$\int 1 + x^{-2} \; dx$

$x=2$
$y=4$

$y = x - \frac{1}{x} + C$

$4 = 2 - \frac{1}{2} + C$

$C = \frac{5}{2}$

$\Rightarrow y = \underline{\underline{x - \frac{1}{x} + \frac{5}{2}}}$

19. $\frac{dy}{dx} = 4x - 6x^2$

$\int 4x - 6x^2 \, dx$

$(-1, 9)$ $\quad y = 2x^2 - 2x^3 + c$

$9 = 2(-1)^2 - (2\times(-1)^3) + c$

$9 = 2 + 2 + c$

$c = 5$

$\Rightarrow \quad \underline{\underline{y = 2x^2 - 2x^3 + 5}}$

20. a) $y = x^3 - 4x^2 + x + 6$

$(-1, 0) \Rightarrow y = (-1)^3 - 4(-1)^2 + (-1) + 6$

$= (-1) - (4) - 1 + 6$

$= \underline{\underline{0}}$

Hence $(-1,0)$ lies on eq^n

$\underline{\text{or}}$ use synth division (best for part (b))

$$\begin{array}{r|rrrr} -1 & 1 & -4 & 1 & 6 \\ & & -1 & 5 & -6 \\ \hline & 1 & -5 & 6 & \boxed{0} \end{array}$$

Hence $(x+1)$ is a factor

b) From synt division

$(x+1)(x^2 - 5x + 6) = 0$

$(x+1)(x-3)(x-2) = 0$

$x = 1 \quad x = 3 \quad x = 2$

so Point Q is $\underline{\underline{(2,0)}}$

c) $\int_{-1}^{2} x^3 - 4x^2 + x + 6 \, dx$

$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x \right]_{-1}^{2}$

$= \left(\frac{2^4}{4} - \frac{4\times 2^3}{3} + \frac{2^2}{2} + 6\times 2 \right) - \left(\frac{-1^4}{4} - \frac{4(-1)^3}{3} + \frac{(-1)^2}{2} + (6\times -1) \right)$

$= \left(4 - \frac{32}{3} + 2 + 12 \right) - \left(\frac{1}{4} + \frac{4}{3} + \frac{1}{2} - 6 \right)$

$= \left(\frac{22}{3} \right) - \left(-\frac{47}{12} \right)$

$= \underline{\underline{\frac{45}{4} \text{ units}^2}}$