St Peter the Apostle High School Maths Department

Higher Practice Questions

9. Integration

1 Integrate the following:

(a) x^5

(b) x^9

(c) x^{14}

(d) x^{50}

(e) x^{-3}

(f) x^{-6}

(g) x^{-10}

(h) x^{-27}

2 Integrate the following:

(a) $2x^3$

(b) $5x^4$

(c) $\frac{1}{2}x^5$

(d) $\frac{2}{5}x^3$

(e) $\frac{2}{x^2}$

(f) $\frac{3}{x^3}$

(g) $3x^{-5}$

(h) $2x^{-2}$

(i) $\frac{14}{x^{-4}}$

(j) $\frac{2}{7x^2}$

(k) $\frac{6}{5x^3}$

(l) $\frac{5}{2x^{-2}}$

3 Integrate the following:

(a) $x^{\frac{1}{2}}$

(b) $x^{\frac{2}{3}}$

(c) $x^{\frac{5}{6}}$

(d) $x^{\frac{1}{4}}$

(e) $x^{\frac{3}{7}}$

(f) $x^{\frac{2}{9}}$

(g) $x^{\frac{1}{5}}$

(h) $x^{\frac{3}{11}}$

4 Integrate the following:

(a) $\sqrt{x}$

(b) $\sqrt{x^3}$

(c) $\sqrt[3]{x}$

(d) $\sqrt[3]{x^2}$

(e) $\sqrt[5]{x^4}$

(f) $\dfrac{1}{\sqrt{x}}$

(g) $\dfrac{1}{\sqrt[5]{x^3}}$

(h) $\dfrac{3}{\sqrt[4]{x^3}}$

(i) $\dfrac{1}{2\sqrt[3]{x^2}}$

5 Find :

(a) $\int (x^3 + 3x^2 + 5x)dx$

(b) $\int (3x^5 + 2x^4 - x)dx$

(c) $\int (x^2 + 6x - 1)dx$

(d) $\int \left(x^{\frac{2}{3}} + 4x^2\right)dx$

(e) $\int \left(3x^{\frac{1}{2}} - 2x^{-5}\right)dx$

(f) $\int \left(5x^{-2} - 3x^{\frac{1}{2}}\right)dx$

(g) $\int \left(\frac{1}{2\sqrt[3]{x}} + x^2\right) dx$

(h) $\int \left(3x^7 - \frac{1}{5\sqrt[4]{x^3}}\right) dx$

(i) $\int \left(\frac{3}{5\sqrt{x^5}} + 5\right) dx$

6 Find:

(a) $\int (x + 1)(x + 2)dx$

(b) $\int (x - 3)(x + 4)dx$

(c) $\int x(x + 4)dx$

(d) $\int \frac{1}{x^2}(x^3 - 2x^2)dx$

(e) $\int \frac{1}{x}(x^2 + x)dx$

(f) $\int \left(\frac{1}{x} + x\right)^2 dx$

7 Find:

(a) $\int \dfrac{x^2+3x}{x} \, dx$

(b) $\int \dfrac{2x^3 + x^2 + x}{x} \, dx$

(c) $\int \dfrac{x^4+x^3-6}{x^2} \, dx$

(d) $\int \dfrac{(x+1)(x-1)}{\sqrt{x}} \, dx$

(e) $\int \dfrac{1+\sqrt{x}}{x^3} \, dx$

(f) $\int \dfrac{3x^4 + 5x^2 + 1}{2x^2} \, dx$

8 Evaluate each of the following definite integrals

(a) $\int_{-1}^{1}(5x^3 - 2x) \, dx$

(b) $\int_{-1}^{1}(3x + 2)(x - 2) \, dx$

(c) $\int_{-2}^{0}(x - 3)^2 \, dx$

(d) $\int_{0}^{2}(3x^2 + 8x - 5) \, dx$

(e) $\int_{-1}^{2}(1 + 3x)(1 - x) \, dx$

(f) $\int_{1}^{3}\left(x^2 - \dfrac{1}{x^2}\right) \, dx$

9

(a) Find a, when $a > 0$

    (i) $\int_0^a (2x + 2)dx = 8$ **(ii)** $\int_0^a x^2\, dx = \dfrac{64}{3}$
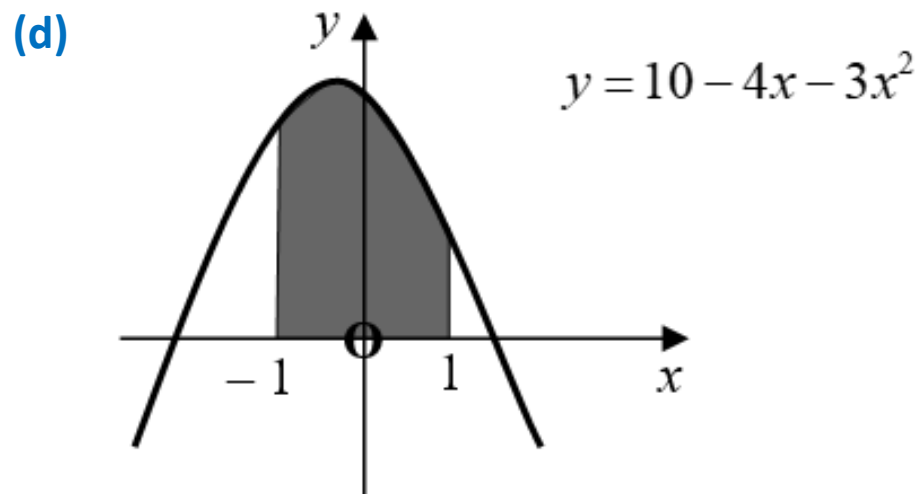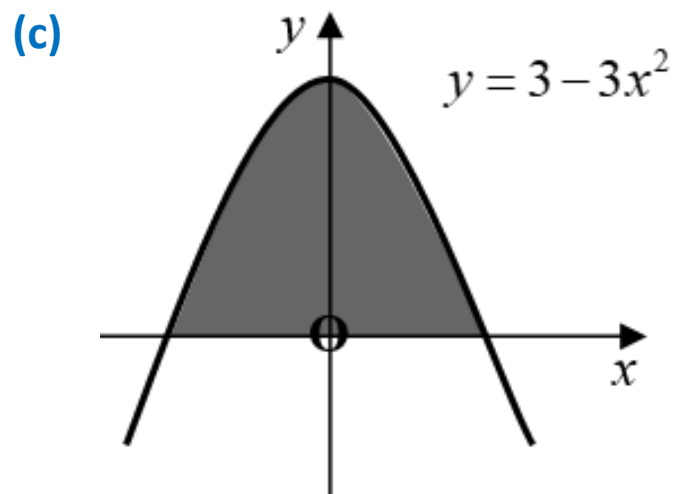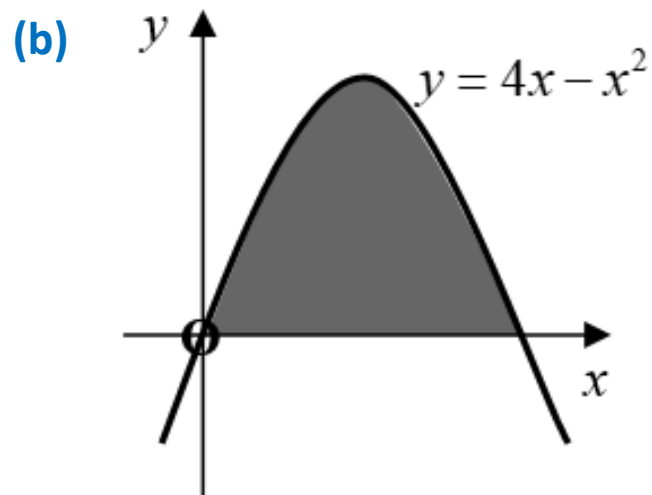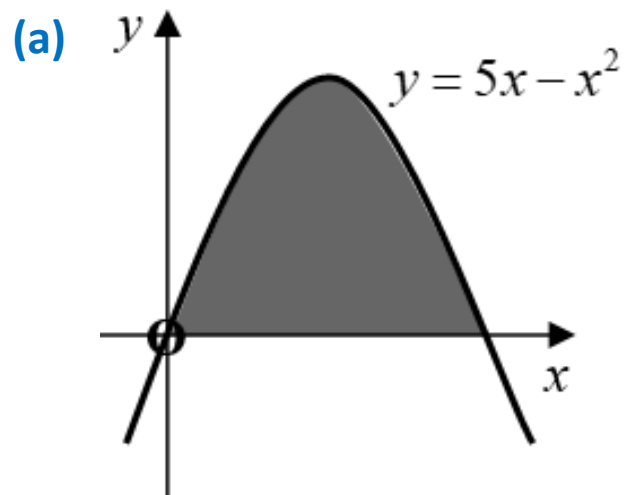
(b) Given that $\int_0^a 3\sqrt{x}\, dx = 16$, calculate the value of a

(c) Determine p, given that $\int_1^p x^{\frac{1}{2}}\, dx = 42$

(d) If $\int_0^k x^{\frac{1}{3}}\, dx = 12$, calculate the value of k

10 Find the shaded area in the following diagrams:

(a)

$y = 5x - x^2$

(b)

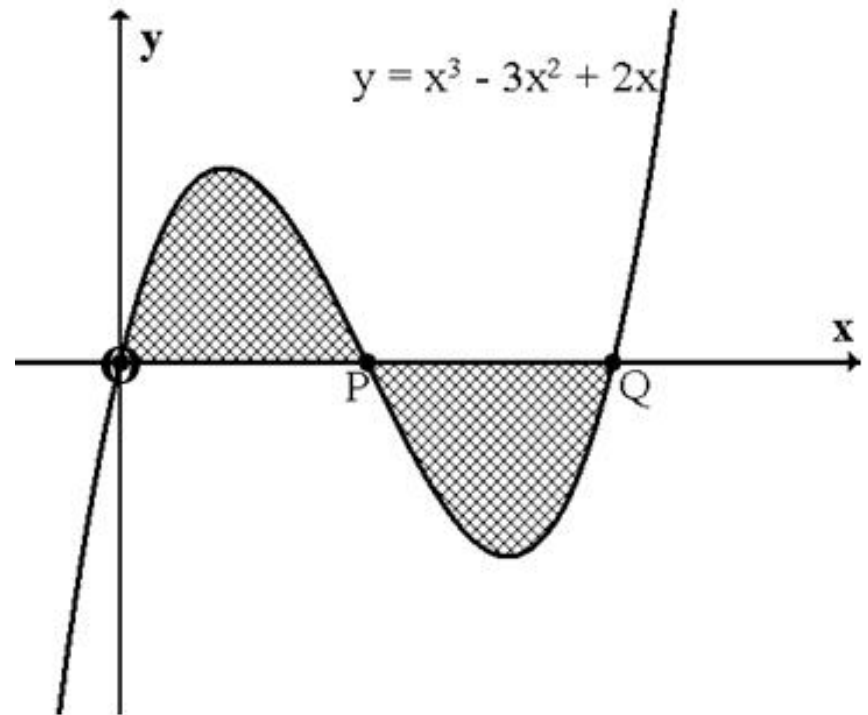$y = 4x - x^2$

(c)

$y = 3 - 3x^2$

(d)

$y = 10 - 4x - 3x^2$

11 The diagram shows part of the graph of

$$y = x^3 - 3x^2 + 2x$$

(a) Find the coordinates of P and Q
(b) Calculate the shaded area

$y = x^3 - 3x^2 + 2x$

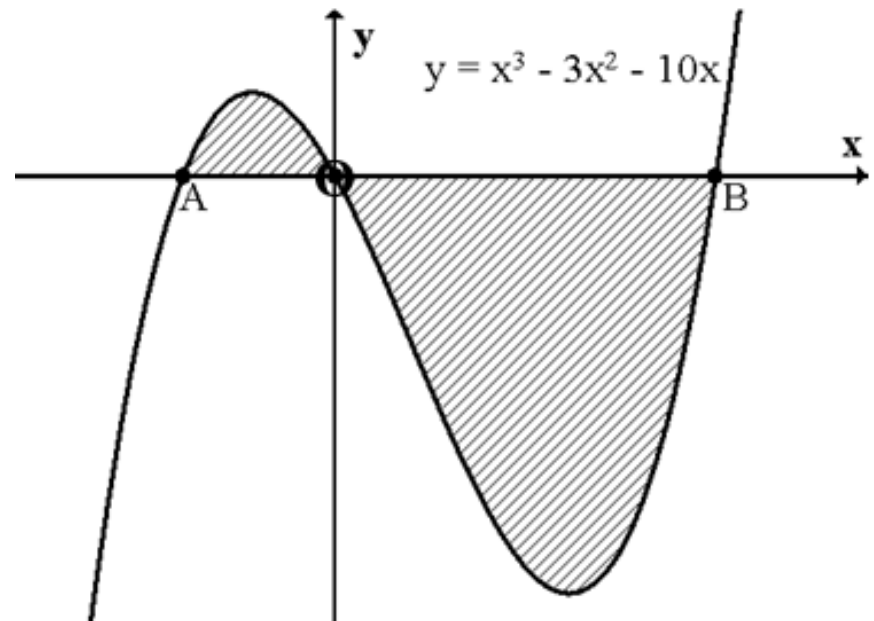**12** The diagram shows part of the graph of

$$y = x^3 - 3x^2 - 10x$$

(a) Find the coordinates of A and B
(b) Calculate the shaded area
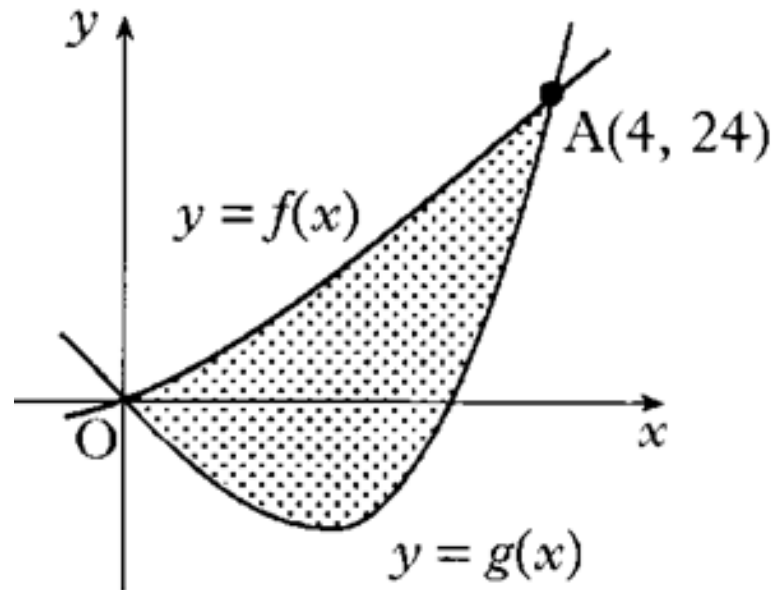

$y = x^3 - 3x^2 - 10x$

13 Parts of the graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram.
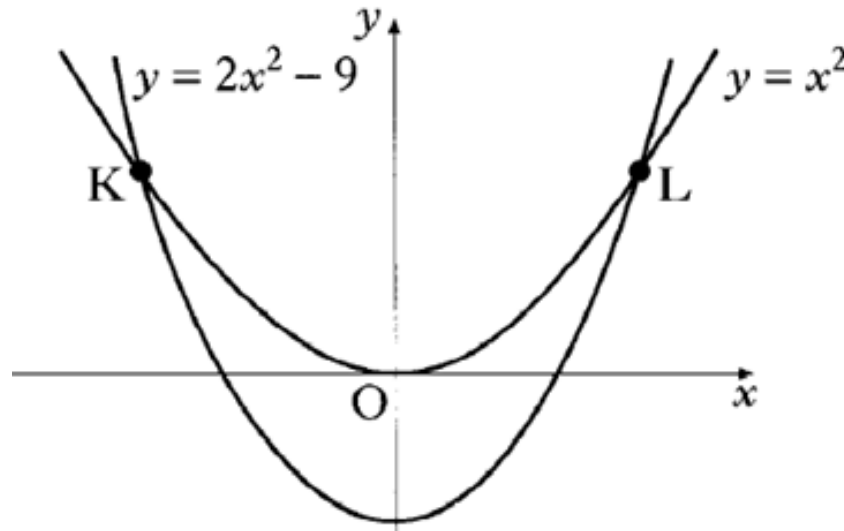
The graphs intersect at $A\,(4, 24)$ and the origin.

Find the shaded are enclosed between the curves

$y = f(x)$

$A(4, 24)$

$y = g(x)$

14 The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown
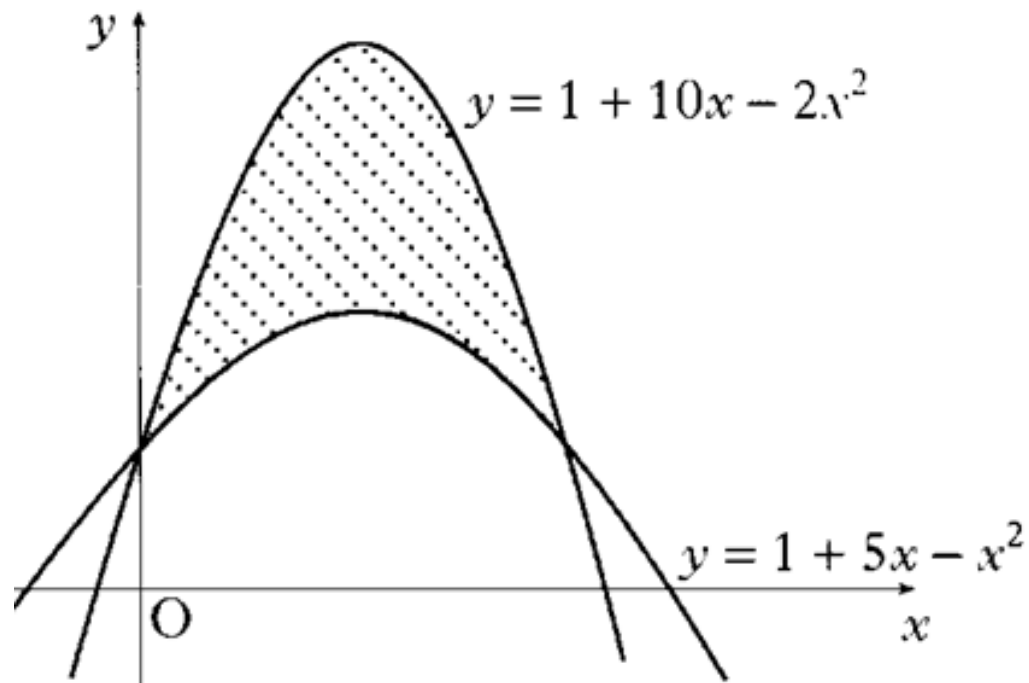
(a) Find the coordinates of the points K and L

(b) Calculate the area enclosed between the curves

15 Calculate the shaded area enclosed between the parabolas with the equations

$$y = 1 + 10x - 2x^2 \quad \text{and} \quad y = 1 + 5x - x^2$$

16 Given the gradient $\frac{dy}{dx}$ of the curve at the point (x, y), and a point on the curve, find the equation of each curve

(a) $\frac{dy}{dx} = 3x^2 - 6x + 1$ $\quad$ $(3, 4)$

(b) $\frac{dy}{dx} = 4x^3 - 6x^2$ $\quad$ $(1, 9)$

17 Find the equations of the curves which satisfy the following conditions:

(a) $\frac{dy}{dx} = 4x$ $\quad$ and the curve passes through the point $(1, 3)$

(b) $\frac{dy}{dx} = \frac{1}{\sqrt{x}}$ $\quad$ and the curve passes through the point $(9, 10)$

18 Find the solution to the following differential equations

(a) $\dfrac{dy}{dx} = 4x^3 + \dfrac{2}{x^2}$ and $y = 0$ when $x = 1$

(b) $\dfrac{dy}{dx} = \dfrac{u^2+1}{u^2}$ and $y = 4$ when $u = 2$
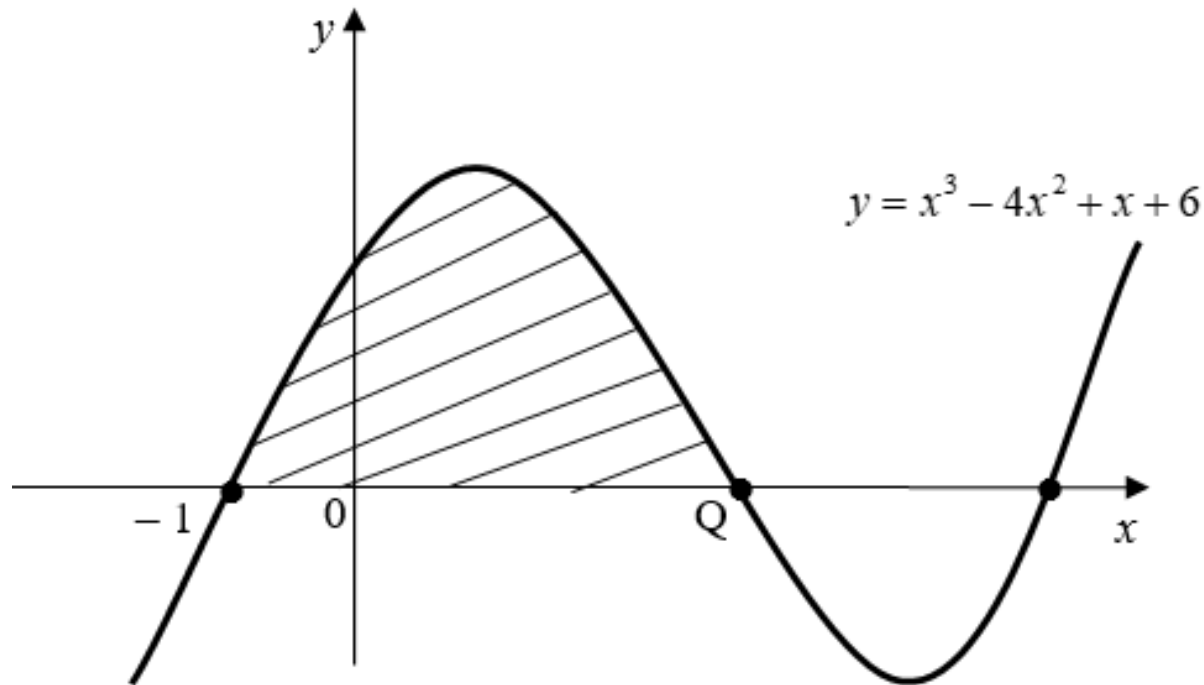
11 A curve $y = f(x)$ is such that $\dfrac{dy}{dx} = 4x - 6x^2$

The curve passes through the point $(-1, 9)$

Express y in terms of x

20 The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$

$$y = x^3 - 4x^2 + x + 6$$

(a) Show that the graph cuts the x – axis at $(-1, 0)$.

(b) Hence, or otherwise, find the coordinates of Q

(c) Find the shaded area