

2. Functions

1. a) $f(x) = 3x - 4$

$$f(2) = (3 \times 2) - 4 = 2$$

$$f(3) = (3 \times 3) - 4 = 5$$

$$f(4) = (3 \times 4) - 4 = 8$$

$$f(5) = (3 \times 5) - 4 = 11$$

Range 2, 5, 8, 11

b) $f(x) = x^2 - 3x + 4$

$$f(-2) = (-2)^2 - (3 \times -2) + 4 = 14$$

$$f(-1) = (-1)^2 - (3 \times -1) + 4 = 8$$

$$f(0) = (0)^2 - (3 \times 0) + 4 = 4$$

$$f(1) = (1)^2 - (3 \times 1) + 4 = 2$$

$$f(2) = (2)^2 - (3 \times 2) + 4 = 2$$

Range 2, 4, 8, 14

c) $f(x) = 3x^2 - 7$

$$f(-3) = 3(-3^2) - 7 = 20$$

$$f(-2) = 3(-2^2) - 7 = 5$$

$$f(0) = 3(0^2) - 7 = -7$$

$$f(2) = 3(2^2) - 7 = 5$$

$$f(3) = 3(3^2) - 7 = 20$$

Range -7, 5, 20

d) $f(x) = \frac{x^2 + 3}{2x - 1}$

$$f(1) = \frac{1^2 + 3}{(2 \times 1) - 1} = 4$$

$$f(3) = \frac{3^2 + 3}{(2 \times 3) - 1} = \frac{12}{5}$$

$$f(5) = \frac{5^2 + 3}{(2 \times 5) - 1} = \frac{28}{9}$$

$$f(7) = \frac{7^2 + 3}{(2 \times 7) - 1} = 4$$

Range 4, $\frac{12}{5}$, $\frac{28}{9}$

$$2. a) \{x: x \neq -1, x \in \mathbb{R}\}$$

$$b) \{x: x \neq \frac{3}{2}, x \in \mathbb{R}\}$$

$$c) \{x: x \neq -4, x \neq 1, x \in \mathbb{R}\}$$

$$d) \{x: x \neq 0, x \neq 3, x \in \mathbb{R}\}$$

$$e) \{x: x \neq -4, x \neq 4, x \in \mathbb{R}\}$$

$$f) \{x: x \neq -6, x \neq -2, x \in \mathbb{R}\}$$

$$3. a) f(g(x)) = f(x^2) \\ = (x^2) + 1 \\ = \underline{\underline{x^2 + 1}}$$

$$b) f(h(x)) = f(x^2 - 2) \\ = (x^2 - 2) + 1 \\ = \underline{\underline{x^2 - 1}}$$

$$c) f(f(x)) = f(x+1) \\ = (x+1) + 1 \\ = \underline{\underline{x+2}}$$

$$d) g(f(x)) = g(x+1) \\ = (x+1)^2 \\ = \underline{\underline{x^2 + 2x + 1}}$$

$$e) g(h(x)) = g(x^2 - 2) \\ = (x^2 - 2)^2 \\ = \underline{\underline{x^4 - 4x^2 + 4}}$$

$$f) g(g(x)) = g(x^2) \\ = (x^2)^2 \\ = \underline{\underline{x^4}}$$

$$g) h(f(x)) = h(x+1) \\ = (x+1)^2 - 2 \\ = x^2 + 2x + 1 - 2 \\ = \underline{\underline{x^2 + 2x - 1}}$$

$$h) h(g(x)) = h(x^2) \\ = (x^2)^2 - 2 \\ = \underline{\underline{x^4 - 2}}$$

$$i) h(h(x)) = h(x^2 - 2) \\ = (x^2 - 2)^2 - 2 \\ = x^4 - 4x^2 + 4 - 2 \\ = \underline{\underline{x^4 - 4x^2 + 2}}$$

$$\begin{aligned}
 4 \quad a) \quad f(g(x)) &= f(3x+1) \\
 &= (3x+1)^2 \\
 &= \underline{\underline{9x^2 + 6x + 1}}
 \end{aligned}$$

$$\begin{aligned}
 b) \quad f(h(x)) &= f(4-2x) \\
 &= (4-2x)^2 \\
 &= \underline{\underline{16 - 16x + 4x^2}}
 \end{aligned}$$

$$\begin{aligned}
 c) \quad f(f(x)) &= f(x^2) \\
 &= (x^2)^2 \\
 &= \underline{\underline{x^4}}
 \end{aligned}$$

$$\begin{aligned}
 d) \quad g(f(x)) &= g(x^2) \\
 &= 3(x^2) + 1 \\
 &= \underline{\underline{3x^2 + 1}}
 \end{aligned}$$

$$\begin{aligned}
 e) \quad g(h(x)) &= g(4-2x) \\
 &= 3(4-2x) + 1 \\
 &= 12 - 6x + 1 \\
 &= \underline{\underline{13 - 6x}}
 \end{aligned}$$

$$\begin{aligned}
 f) \quad g(g(x)) &= g(3x+1) \\
 &= 3(3x+1) + 1 \\
 &= 9x + 3 + 1 \\
 &= \underline{\underline{9x + 4}}
 \end{aligned}$$

$$\begin{aligned}
 g) \quad h(f(x)) &= h(x^2) \\
 &= 4 - 2(x^2) \\
 &= \underline{\underline{4 - 2x^2}}
 \end{aligned}$$

$$\begin{aligned}
 h) \quad h(g(x)) &= h(3x+1) \\
 &= 4 - 2(3x+1) \\
 &= 4 - 6x - 2 \\
 &= \underline{\underline{2 - 6x}}
 \end{aligned}$$

$$\begin{aligned}
 i) \quad h(h(x)) &= h(4-2x) \\
 &= 4 - 2(4-2x) \\
 &= 4 - 8 + 4x \\
 &= \underline{\underline{-4 + 4x}}
 \end{aligned}$$

$$5. \quad a) \quad f(g(x)) = f(\cos x) \\ = 2(\cos x) \\ = \underline{\underline{2\cos x}} \quad b) \quad g(f(x)) = g(2x) \\ = \cos(2x) \\ = \underline{\underline{\cos 2x}}$$

$$c) \quad f(f(x)) = f(2x) \\ = 2(2x) \\ = \underline{\underline{4x}}$$

$$6. \quad a) \quad h(x-1) \\ = 3(x-1)^2 + 6(x-1) + 2 \\ = 3(x^2 - 2x + 1) + 6x - 6 + 2 \\ = 3x^2 - 6x + 3 + 6x - 6 + 2 \\ = \underline{\underline{3x^2 - 1}} \quad \text{as required}$$

$$b) \quad h(x+1) \\ = 3(x+1)^2 + 6(x+1) + 2 \\ = 3(x^2 + 2x + 1) + 6x + 6 + 2 \\ = 3x^2 + 6x + 3 + 6x + 6 + 2 \\ = \underline{\underline{3x^2 + 12x + 11}}$$

$$c) \quad \frac{h(x+1) - h(x-1)}{2} \\ = \frac{(3x^2 + 12x + 11) - (3x^2 - 1)}{2} \\ = \frac{12x + 12}{2} \\ = \underline{\underline{6x + 6}}$$

$$\begin{aligned}
 7. a) f(x-1) &= \\
 &= \frac{(x-1)}{1-(x-1)} \\
 &= \frac{x-1}{\underline{\underline{2-x}}}
 \end{aligned}$$

$$\begin{aligned}
 b) f\left(\frac{1}{x}\right) &= \frac{1}{x} \div \frac{x-1}{x} \\
 &= \frac{\left(\frac{1}{x}\right)}{1-\left(\frac{1}{x}\right)} \Rightarrow \frac{1}{\cancel{x}} \times \frac{\cancel{x}}{x-1} \\
 &= \frac{1}{\underline{\underline{x-1}}} \\
 &= \frac{\frac{1}{x}}{\frac{x}{x}-\frac{1}{x}}
 \end{aligned}$$

$$c) f\left(\frac{1}{x+1}\right)$$

$$\begin{aligned}
 &= \frac{\left(\frac{1}{x+1}\right)}{1-\left(\frac{1}{x+1}\right)} = \frac{1}{x+1} \div \frac{x}{x+1} \\
 &\Rightarrow \frac{1}{x+1} \times \frac{x+1}{x} \\
 &= \frac{\frac{1}{x+1}}{\frac{x+1}{x+1} - \frac{1}{x+1}} = \frac{1}{\underline{\underline{x}}}
 \end{aligned}$$

$$d) f\left(\frac{1}{1-x}\right)$$

$$\begin{aligned}
 &= \frac{\left(\frac{1}{1-x}\right)}{1-\left(\frac{1}{1-x}\right)} = \frac{1}{1-x} \div \frac{-x}{1-x} \\
 &\Rightarrow \frac{1}{1-x} \times \frac{1-x}{-x} \\
 &= \frac{\frac{1}{1-x}}{\frac{1-x}{1-x} - \frac{1}{1-x}} = -\frac{1}{\underline{\underline{x}}}
 \end{aligned}$$

$$\begin{aligned}
 8. a) f(g(x)) &= f\left(\frac{x+2}{3}\right) \\
 &= 3\left(\frac{x+2}{3}\right) - 2 \\
 &= \underline{\underline{x}}
 \end{aligned}$$

$$\begin{aligned}
 g(f(x)) &= g(3x-2) \\
 &= \frac{(3x-2)+2}{3} \\
 &= \underline{\underline{x}}
 \end{aligned}$$

b) As solution is x for both $f(g(x))$ and $g(f(x))$ functions are the inverse of each other.

$$\begin{aligned} 9. \text{ a) } f(g(x)) &= f\left(\frac{x-5}{2}\right) \\ &= 2\left(\frac{x-5}{2}\right) + 5 \\ &= \underline{\underline{x}} \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(2x+5) \\ &= \frac{(2x+5)-5}{2} \\ &= \underline{\underline{x}} \end{aligned}$$

b) As solution is x for both $f(g(x))$ and $g(f(x))$ functions are the inverse of each other

$$\begin{aligned} 10. \text{ a) } f(g(x)) &= f\left(\frac{x+3}{6}\right) \\ &= 6\left(\frac{x+3}{6}\right) - 3 \\ &= \underline{\underline{x}} \end{aligned}$$

$$\begin{aligned} g(f(x)) &= g(6x-3) \\ &= \frac{(6x-3)+3}{6} \\ &= \underline{\underline{x}} \end{aligned}$$

b) As solution is x for both $f(g(x))$ and $g(f(x))$ functions are the inverse of each other

$$11. a) f(x) = 3x - 4$$

$$y = 3x - 4$$

$$y + 4 = 3x$$

$$x = \frac{y+4}{3}$$

$$\Rightarrow f^{-1}(x) = \frac{x+4}{3}$$

$$b) g(x) = 5x + 2$$

$$y = 5x + 2$$

$$y - 2 = 5x$$

$$x = \frac{y-2}{5}$$

$$\Rightarrow g^{-1}(x) = \frac{x-2}{5}$$

$$c) h(x) = 2x - 6$$

$$y = 2x - 6$$

$$y + 6 = 2x$$

$$x = \frac{y+6}{2}$$

$$\Rightarrow h^{-1}(x) = \frac{x+6}{2}$$

$$d) f(x) = \frac{1}{2}x + 5$$

$$y = \frac{1}{2}x + 5$$

$$y - 5 = \frac{1}{2}x$$

$$x = 2y - 10$$

$$\Rightarrow f^{-1}(x) = 2x - 10$$

$$e) g(x) = \frac{1}{4}x - 3$$

$$y = \frac{1}{4}x - 3$$

$$y + 3 = \frac{1}{4}x$$

$$x = 4y + 12$$

$$\Rightarrow g^{-1}(x) = 4x + 12$$

$$f) h(x) = 7 - 3x$$

$$y = 7 - 3x$$

$$y - 7 = -3x$$

$$x = \frac{7-y}{3}$$

$$h^{-1}(x) = \frac{7-x}{3}$$

$$g) f(x) = 2 - 4x$$

$$y = 2 - 4x$$

$$y - 2 = -4x$$

$$x = \frac{2-y}{4}$$

$$\Rightarrow f^{-1}(x) = \frac{2-x}{4}$$

$$h) g(x) = \frac{2x-4}{5}$$

$$y = \frac{2x-4}{5}$$

$$5y = 2x - 4$$

$$5y + 4 = 2x$$

$$x = \frac{5y+4}{2}$$

$$\Rightarrow g^{-1}(x) = \frac{5x+4}{2}$$

$$i) h(x) = \frac{3x+2}{4}$$

$$y = \frac{3x+2}{4}$$

$$4y = 3x + 2$$

$$4y - 2 = 3x$$

$$x = \frac{4y-2}{3}$$

$$\Rightarrow h^{-1}(x) = \frac{4x-2}{3}$$

$$j) f(x) = \frac{6-3x}{2}$$

$$y = \frac{6-3x}{2}$$

$$2y - 6 = -3x$$

$$x = \frac{6-2y}{3}$$

$$2y = 6 - 3x$$

$$\Rightarrow f^{-1}(x) = \frac{6-2x}{3}$$