6. Differentiation

1 a) $f(x) = x^5$

$f'(x) = 5x^4$

b) $y = x^9$

$\frac{dy}{dx} = 9x^8$

c) $f(x) = x^{14}$

$f'(x) = 14x^{13}$

d) $y = x^{-2}$

$\frac{dy}{dx} = -2x^{-3}$

$= \frac{-2}{x^3}$

e) $f(x) = x^{-8}$

$f'(x) = -8x^{-9}$

$= \frac{-8}{x^9}$

f) $y = x^{-20}$

$\frac{dy}{dx} = -20x^{-21}$

$= \frac{-20}{x^{21}}$

2 a) $y = 2x^3$

$\frac{dy}{dx} = 6x^2$

b) $y = 5x^4$

$\frac{dy}{dx} = 20x^3$

c) $f(x) = \frac{1}{2}x^6$

$f'(x) = 3x^5$

d) $f(x) = \frac{2}{3}x^9$

$f'(x) = 6x^8$

e) $y = \frac{3}{x^3}$

$y = 3x^{-3}$

$\frac{dy}{dx} = -9x^{-4}$

$= \frac{-9}{x^4}$

f) $y = \frac{25}{x^4}$

$y = 25x^{-4}$

$\frac{dy}{dx} = -100x^{-5}$

$= \frac{-100}{x^5}$

g) $y = 9x^4$

$\frac{dy}{dx} = 36x^3$

h) $f(x) = x^{2/3}$

$f'(x) = \frac{2}{3}x^{-\frac{1}{3}}$

$= \frac{2}{3x^{1/3}}$

$= \frac{2}{3\sqrt[3]{x}}$

i) $f(x) = 10x^{2/5}$

$f'(x) = 4x^{-3/5}$

$= \frac{4}{x^{3/5}}$

$= \frac{4}{\sqrt[5]{x^3}}$

j) $f(x) = 4x^{3/4}$

$f'(x) = 3x^{-\frac{1}{4}}$

$= \frac{3}{x^{1/4}}$

$= \frac{3}{\sqrt[4]{x}}$

k) $y = \frac{1}{2x^4} = \frac{1}{2}x^{-4}$

$\frac{dy}{dx} = -2x^{-5}$

$= \frac{-2}{x^5}$

l) $y = \frac{3}{4x^5} = \frac{3}{4}x^{-5}$

$\frac{dy}{dx} = -\frac{15}{4}x^{-6}$

$= -\frac{15}{4x^6}$

3 a) $y = (x+1)(x+2)$

$y = x^2 + 3x + 2$

$\frac{dy}{dx} = \underline{\underline{2x+3}}$

b) $f(x) = (x+2)(x-3)$

$f(x) = x^2 - x - 6$

$f'(x) = \underline{\underline{2x-1}}$

c) $y = (x+2)^2$

$y = x^2 + 4x + 4$

$\frac{dy}{dx} = \underline{\underline{2x+4}}$

d) $f(x) = (x-3)(x+4)$

$f(x) = x^2 + x - 12$

$f'(x) = \underline{\underline{2x+1}}$

e) $y = (x-5)(2x-2)$

$y = 2x^2 - 12x + 10$

$\frac{dy}{dx} = \underline{\underline{4x-12}}$

f) $y = x(x-4)$

$y = x^2 - 4x$

$\frac{dy}{dx} = \underline{\underline{2x-4}}$

g) $y = (2x-3)(x+4)$

$y = 2x^2 + 5x - 12$

$\frac{dy}{dx} = \underline{\underline{4x+5}}$

h) $f(x) = x^2(x-2)$

$f(x) = x^3 - 2x^2$

$f'(x) = \underline{\underline{3x^2 - 4x}}$

i) $y = \frac{1}{x^2}(x^3 + 2x)$

$y = x^{-2}(x^3 + 2x)$

$y = x + 2x^{-1}$

$\frac{dy}{dx} = \underline{\underline{1 - \frac{2}{x^2}}}$

j) $f(x) = \frac{1}{x^2}(x^2 + x)$

$f(x) = x^{-2}(x^2 + x)$

$f(x) = 1 + x^{-1}$

$f'(x) = -x^{-2}$

$= \underline{\underline{-\frac{1}{x^2}}}$

k) $y = \left(\frac{1}{x} + 1\right)^2$

$y = (x^{-1}+1)(x^{-1}+1)$

$y = x^{-2} + 2x^{-1} + 1$

$\frac{dy}{dx} = -2x^{-3} - 2x^{-2}$

$= \underline{\underline{\frac{-2}{x^3} - \frac{2}{x^2}}}$

l) $y = \frac{1}{x^2}(x-5)^2$

$y = x^{-2}(x^2 - 10x + 25)$

$y = 1 - 10x^{-1} + 25x^{-2}$

$\frac{dy}{dx} = 10x^{-2} - 50x^{-3}$

$= \underline{\underline{\frac{10}{x^2} - \frac{50}{x^3}}}$

4 a) $y = \dfrac{x^2+3x+5}{x}$

$y = x^{-1}(x^2+3x+5)$

$y = x + 3 + 5x^{-1}$

$\dfrac{dy}{dx} = 1 - 5x^{-2}$

$= 1 - \dfrac{5}{x^2}$

b) $y = \dfrac{2x^3+x^2+x}{x}$

$y = x^{-1}(2x^3+x^2+x)$

$y = 2x^2 + x + 1$

$\dfrac{dy}{dx} = 4x + 1$

c) $f(x) = \dfrac{x^4+x^3-6x}{x^2}$

$f(x) = x^{-2}(x^4+x^3-6x)$

$f(x) = x^2 + x - 6x^{-1}$

$f'(x) = 2x + 1 + 6x^{-2}$

$= 2x + 1 + \dfrac{6}{x^2}$

d) $y = \dfrac{3+x^2}{x^2}$

$y = x^{-2}(3+x^2)$

$y = 3x^{-2} + 1$

$\dfrac{dy}{dx} = -6x^{-3}$

$= -\dfrac{6}{x^3}$

e) $y = \dfrac{x+2}{\sqrt{x}}$ $\quad [\sqrt{x} = x^{\frac{1}{2}}]$

$y = x^{-\frac{1}{2}}(x+2)$

$y = x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$

$\dfrac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - x^{-\frac{3}{2}}$

$= \dfrac{1}{2x^{\frac{1}{2}}} - \dfrac{1}{x^{3/2}}$

$= \dfrac{1}{2\sqrt{x}} - \dfrac{1}{\sqrt{x^3}}$

f) $f(x) = \dfrac{3x^2+5x+1}{2x^2}$

$f(x) = x^{-2}\left(\frac{3}{2}x^2 + \frac{5}{2}x + \frac{1}{2}\right)$

$f(x) = \frac{3}{2} + \frac{5}{2}x^{-1} + \frac{1}{2}x^{-2}$

$f'(x) = -\frac{5}{2}x^{-2} - x^{-3}$

$= \dfrac{-5}{2x^2} - \dfrac{1}{x^3}$

5

a) $f(x) = x^3 + 3x^2 + 5x$

$f'(x) = 3x^2 + 6x + 5$

$f'(2) = 3\times(2)^2 + 6\times(2) + 5$

$= \underline{\underline{29}}$

b) $f(x) = 3x^5 + 2x^4 - x$

$f'(x) = 15x^4 + 8x^3 - 1$

$f'(1) = 15\times(1)^4 + 8\times(1)^3 - 1$

$= \underline{\underline{22}}$

c) $f(x) = x^2 + 6x - 1$

$f'(x) = 2x + 6$

$f'(-4) = 2\times(4) + 6$

$= \underline{\underline{-2}}$

d) $f(x) = x^{2/3} + 4x^2$

$f'(x) = \frac{2}{3}x^{-\frac{1}{3}} + 8x$

$= \frac{2}{3\sqrt[3]{x}} + 8x$

$f'(8) = \frac{2}{3\sqrt[3]{8}} + 8\times(8)$

$= \underline{\underline{64\tfrac{1}{3}}}$

e) $f(x) = 3x^{\frac{1}{2}} - 2x^{-5}$

$f'(x) = \frac{3}{2}x^{-\frac{1}{2}} + 10x^{-6}$

$= \frac{3}{2\sqrt{x}} + \frac{10}{x^6}$

$f'(1) = \frac{3}{2\sqrt{1}} + \frac{10}{1^6}$

$= \underline{\underline{11\tfrac{1}{2}}}$

f) $f(x) = 5x^{-2} - 3x^{\frac{1}{2}}$

$f'(x) = -10x^{-3} - \frac{3}{2}x^{-\frac{1}{2}}$

$= \frac{-10}{x^3} - \frac{3}{2\sqrt{x}}$

$f'(4) = \frac{-10}{(4)^3} - \frac{3}{2\sqrt{4}}$

$= -\frac{5}{32} - \frac{3}{4}$

$= \underline{\underline{-\frac{29}{32}}}$

g) $f(x) = \frac{1}{2\sqrt[3]{x}} + x^2$

$f(x) = \frac{1}{2}x^{-\frac{1}{3}} + x^2$

$f'(x) = -\frac{1}{6}x^{-\frac{4}{3}} + 2x$

$= -\frac{1}{6\sqrt[3]{x^4}} + 2x$

$f'(-1) = -\frac{1}{6\sqrt[3]{(-1)^4}} + 2(-1)$

$= -\frac{1}{6} - 2 = \underline{\underline{-2\tfrac{1}{6}}}$

h) $f(x) = 3x^7 - \frac{1}{5\sqrt[4]{x^3}}$

$f(x) = 3x^7 - \frac{1}{5}x^{-\frac{3}{4}}$

$f'(x) = 21x^6 + \frac{3}{20}x^{-\frac{7}{4}}$

$= 21x^6 + \frac{3}{20\sqrt[4]{x^7}}$

$f'(1) = 21(1)^6 + \frac{3}{20\sqrt[4]{1^7}}$

$= 21\tfrac{3}{20}$

6a) $y = x^2 \quad (1,4)$

$\frac{dy}{dx} = 2x$

$m = 2 \times 1$
$= \underline{\underline{2}}$

$y - b = m(x-a)$ || $m = 2$, $(1,4)$

$y - 4 = 2(x-1)$

$y - 4 = 2x - 2$

$\underline{\underline{2x - y + 2 = 0}}$

b) $y = x^3 + 3 \quad (2,11)$

$\frac{dy}{dx} = 3x^2$

$m = 3 \times (2)^2$
$= 12$

$y - b = m(x-a)$ || $m = 12$, $(2,11)$

$y - 11 = 12(x-2)$

$y - 11 = 12x - 24$

$\underline{\underline{12x - y - 13 = 0}}$

c) $y = x^2 - 6x + 5 \quad x = 2$

$\frac{dy}{dx} = 2x - 6$

$m = (2 \times 2) - 6$
$= \underline{\underline{-2}}$

$y - b = m(x-a)$ || $m = -2$, $(2, ?)$, $y = (2)^2 - (6\times 2) + 5$, $= -3$, $\underline{\underline{(2,-3)}}$

$y + 3 = -2(x-2)$

$y + 3 = -2x + 4$

$\underline{\underline{2x + y - 1 = 0}}$

d) $y = \sqrt{x} + 1 \quad x = 1$

$y = x^{\frac{1}{2}} + 1$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$m = \frac{1}{2\sqrt{x}} \quad = \underline{\underline{\frac{1}{2}}}$

$y - b = m(x-a)$ || $m = \frac{1}{2}$, $(1, ?)$, $y = \sqrt{1} + 1$, $= 2$, $\underline{\underline{(1,2)}}$

$y - 2 = \frac{1}{2}(x-1)$

$2y - 4 = x - 1$

$\underline{\underline{x - 2y + 3 = 0}}$

e) $y = x^2 + 2x - 3 \quad (1,0)$

$\frac{dy}{dx} = 2x + 2$

$m = (2 \times 1) + 2$
$= \underline{\underline{4}}$

$y - b = m(x-a)$ || $m = 4$, $(1,0)$

$y - 0 = 4(x-1)$

$y = 4x - 4$

$\underline{\underline{4x - y - 4 = 0}}$

f) $f(x) = 5x^2 - 6x^{\frac{1}{2}} \quad x = 1$

$f'(x) = 10x - 3x^{-\frac{1}{2}}$

$m = (10 \times 1) - \frac{3}{\sqrt{1}}$
$= \underline{\underline{7}}$

$y - b = m(x-a)$ || $m =$, $(1, ?)$, $y = 5\times(1)^2 - 6\times\sqrt{1}$, $= \underline{-1}$, $(1,-1)$

$y + 1 = 7(x-1)$

$y + 1 = 7x - 7$

$\underline{\underline{7x - y - 8 = 0}}$

7. $y = 5x^2 - 2x + 7$

$\frac{dy}{dx} = 10x - 2$

$m = 8$ so

$10x - 2 = 8$

$10x = 10$

$\underline{\underline{x = 1}}$

$y = 5\times(1)^2 - 2(1) + 7$

$= \underline{10}$

$\underline{\underline{(1,10)}}$

$y - b = m(x - a)$ || $m = 8$, $(1,10)$

$y - 10 = 8(x - 1)$

$y - 10 = 8x - 8$

$\underline{\underline{8x - y + 2 = 0}}$

8. $f(x) = 2x^2 + 8x - 3$

$f'(x) = 4x + 8$

$m = -4$ so

$4x + 8 = -4$

$4x = -12$

$x = -3$

$y = 2\times(-3)^2 + 8\times(-3) - 3$

$= -\underline{9}$

$\underline{\underline{(-3,-9)}}$

$y - b = m(x - a)$ || $m = -4$, $(-3,-9)$

$y + 9 = -4(x + 3)$

$y + 9 = -4x - 12$

$\underline{\underline{4x + y + 21 = 0}}$

9. $y = ax^2 + b$

$m = 30$ at $(3,1)$

$\frac{dy}{dx} = 2ax$

so $2ax = 30$

$2a \times 3 = 30$

$6a = 30$

$\underline{\underline{a = 5}}$

sub $a = 5$

$y = 5x^2 + b$. $(3,1)$

$1 = 5\times(3)^2 + b$

$\underline{\underline{b = -44}}$

10. a) $y = x^3 - 6x^2 + 9x$

$\frac{dy}{dx} = 3x^2 - 12x + 9$

st pts occur when $\frac{dy}{dx} = 0$

$3x^2 - 12x + 9 = 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1$ and $x = 3$

$y = 1^3 - 6\times1^2 + 9\times1 = 4$ $\quad \underline{(1,4)}$

$y = 3^3 - 6\times3^2 + 9\times3 = 0$ $\quad \underline{(3,0)}$

	$\rightarrow$	1	$\rightarrow$	3	$\rightarrow$
$\frac{dy}{dx}$	+	0	−	0	+
slope	/	—	\	—	/

Max TP $\underline{(1,4)}$ $\qquad$ Min TP $\underline{(3,0)}$

b) $y = x^4 - 4x^3$

$\frac{dy}{dx} = 4x^3 - 12x^2$

st pts occur when $\frac{dy}{dx} = 0$

$4x^3 - 12x^2 = 0$

$4x^2(x-3) = 0$

$4x^2 = 0$ or $x - 3 = 0$

$\underline{x = 0}$ $\qquad$ $\underline{x = 3}$

$y = 0^4 - 4\times0^3 = 0$ $\quad (0,0)$

$y = 3^4 - 4\times3^3 = -27$ $\quad \underline{(3,-27)}$

	$\rightarrow$	0	$\rightarrow$	3	$\rightarrow$
$\frac{dy}{dx}$	−	0	−	0	+
slope	\	—	\	—	/

Falling Point of inflection $\underline{(0,0)}$ $\qquad$ Min TP $\underline{(3,-27)}$

c) $y = x^3 - 3x + 2$

$\frac{dy}{dx} = 3x^2 - 3$

st pts occur when $\frac{dy}{dx} = 0$

$3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$3(x-1)(x+1) = 0$

$x = 1$ $\quad$ $x = -1$

$y = 1^3 - 3\times1 + 2 = 0$ $\quad \underline{(1,0)}$

$y = (-1)^3 - 3\times(-1) + 2 = 4$ $\quad \underline{(-1,4)}$

	$\rightarrow$	−1	$\rightarrow$	1	$\rightarrow$
$\frac{dy}{dx}$	+	0	−	0	+
slope	/	—	\	—	/

Max TP $\underline{(-1,4)}$ $\qquad$ Min TP $\underline{(1,0)}$

d) $y = 3x^4 + 16x^3$

$\frac{dy}{dx} = 12x^3 + 48x^2$

st pts occur when $\frac{dy}{dx} = 0$

$12x^3 + 48x^2 = 0$

$12x^2(x+4) = 0$

$x = 0$ $\quad$ $x = -4$

$y = 3\times0^4 + 16\times0^3 = 0$ $\quad \underline{(0,0)}$

$y = 3\times(-4)^4 + 16\times(-4)^3 = -256$ $\quad \underline{(-4,-256)}$

	$\rightarrow$	−4	$\rightarrow$	0	$\rightarrow$
$\frac{dy}{dx}$	−	0	+	0	+
slope	\	—	/	—	/

Min TP $\underline{(-4,-256)}$ $\qquad$ Rising P.O.I $\underline{(0,0)}$

11 a) $y = x^2 - 6x$

$\frac{dy}{dx} = 2x - 6$

st pts occur when $\frac{dy}{dx} = 0$

$2x - 6 = 0$

$2x = 6$

$x = 3$

$y = 3^2 - 6 \times 3 = -9$

$(3, -9)$

	$\rightarrow$	3	$\rightarrow$
$\frac{dy}{dx}$	$-$	0	$+$
slope	\	—	/

Min TP $(3, -9)$.

x int (Roots) at $y=0$ $\rightarrow$ $x^2 - 6x = 0$

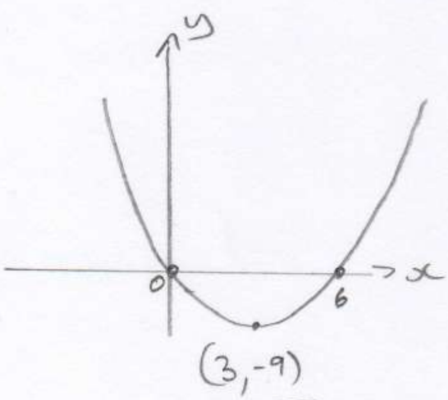
$x(x-6) = 0$

$x = 0 \quad x = 6$

$(0,0) \quad (6,0)$

y inter $(x=0)$ $\rightarrow$ $y = 0^2 - 6 \times 0 = 0$

$(0, 0)$



b) $y = x^3 - 3x^2$

$\frac{dy}{dx} = 3x^2 - 6x$

st pts occur when $\frac{dy}{dx} = 0$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$x = 0 \quad x = 2$

$y = 0^3 - 3 \times 0^2 = 0 \quad (0,0)$

$y = 2^3 - 3 \times 2^2 = -4 \quad (2,-4)$

	$\rightarrow$	0	$\rightarrow$	2	$\rightarrow$
$\frac{dy}{dx}$	$+$	0	$-$	0	$+$
Slope	/	—	\	—	/

Max TP $(0,0)$ Min TP $(2,-4)$

x int $(y=0)$ $\rightarrow$ $x^3 - 3x^2 = 0$

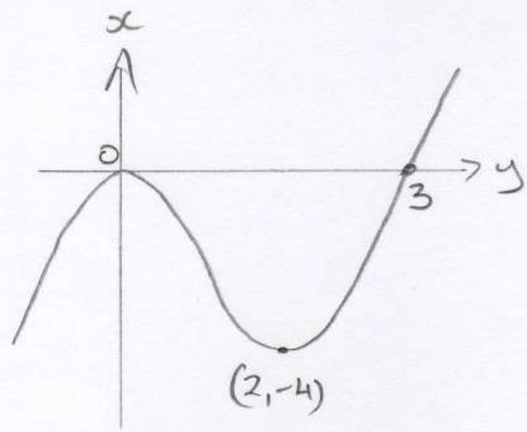
$x^2(x-3) = 0$

$x = 0 \quad x = 3$

$(0,0) \quad (3,0)$

y int $(x=0)$ $\rightarrow$ $y = 0^3 - 3 \times 0^2 = 0$

$(0,0)$



c) $y = (x-1)^2(x+2)$

$y = (x^2-2x+1)(x+2)$

$y = x^3 + 2x^2 - 2x^2 - 4x + x + 2$

$y = x^3 - 3x + 2$

$\frac{dy}{dx} = 3x^2 - 3$

st pts occur when $\frac{dy}{dx} = 0$

$3x^2 - 3 = 0$

$3(x^2-1) = 0$

$3(x-1)(x+1) = 0$

$\underline{\underline{x = 1}}$ $\underline{\underline{x = -1}}$

$y = 1^3 - 3\times1 + 2 = 0$ $\underline{\underline{(1,0)}}$

$y = (-1)^3 - 3\times(-1) + 2 = 4$ $\underline{\underline{(-1,4)}}$

[* NATURE TABLE BELOW *]

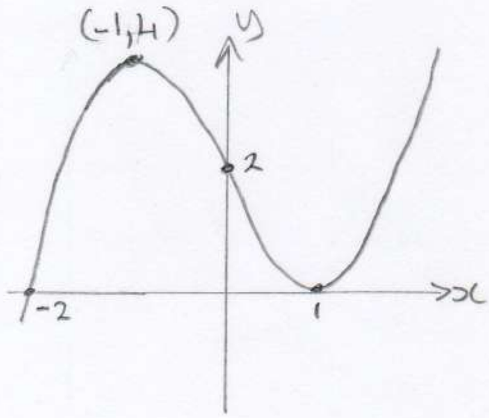
x int (y=0) $\Rightarrow$ $(x-1)^2(x+2) = 0$

$x = 1$ $x = -2$

$\underline{\underline{(1,0)}}$ $\underline{\underline{(-2,0)}}$

y-int (x=0) $y = 0^3 - 3\times0 + 2 = 2$

$\underline{\underline{(0,2)}}$



*

	→	−1	→	1	→
$\frac{dy}{dx}$	+	0	−	0	+
slope	/	—	\	_	/

Max TP $\underline{\underline{(-1,4)}}$ Min TP $\underline{\underline{(1,0)}}$

d) $y = x^2(3-x)$

$y = 3x^2 - x^3$

$\frac{dy}{dx} = 6x - 3x^2$

st pts occur when $\frac{dy}{dx} = 0$

$6x - 3x^2 = 0$

$3x(2-x) = 0$

$\underline{\underline{x = 0}}$ $\underline{\underline{x = 2}}$

$y = 3\times0^2 - 0^3 = 0$ $\underline{\underline{(0,0)}}$

$y = 3\times(2)^2 - (2)^3 = 4$ $\underline{\underline{(2,4)}}$

	→	0	→	2	→
$\frac{dy}{dx}$	−	0	+	0	−
slope	\	_	/	—	\

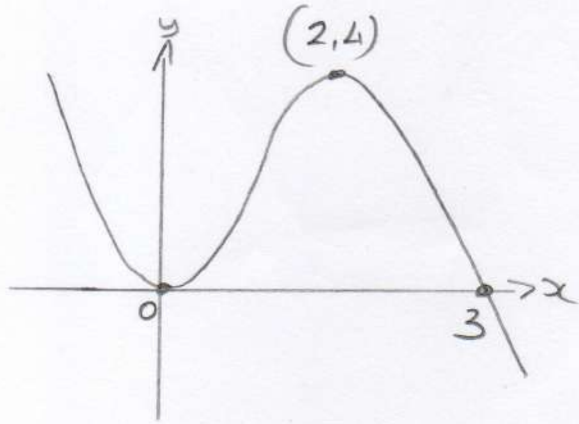
Min TP $\underline{\underline{(0,0)}}$ Max TP $\underline{\underline{(2,4)}}$

x int (y=0) $\Rightarrow$ $x^2(3-x) = 0$

$x = 0$ $x = 3$

$\underline{\underline{(0,0)}}$ $\underline{\underline{(3,0)}}$

y int (x=0) $\Rightarrow$ $y = 0^2(3-0) = 0$ $\underline{\underline{(0,0)}}$



12. a) $y = x^3 - x^2 + x$

$\frac{dy}{dx} = 3x^2 - 2x + 1$

As $3x^2 > 2x$

function is never decreasing (Also if $x = 0$ func is increasing)

b) $y = 2x^5 + 5$

$\frac{dy}{dx} = 10x^4$

As x^4 is always positive (even power) it is never decreasing (Also if $x = 0$ $\frac{dy}{dx} = 0$ so not decreasing)

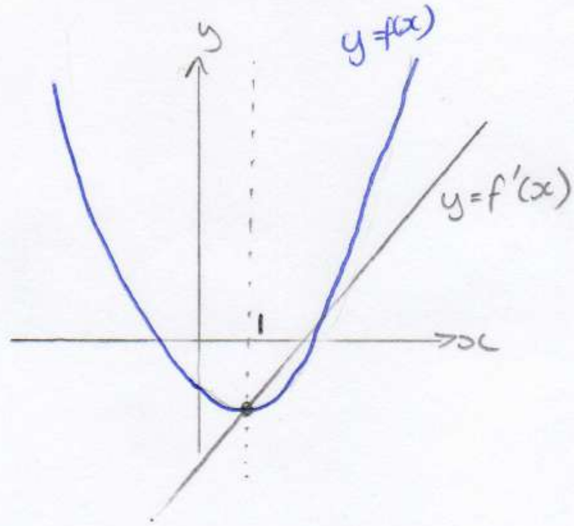
c) $y = -x^3 - 3x^2 - 3x$

$\frac{dy}{dx} = -3x^2 - 6x - 3$

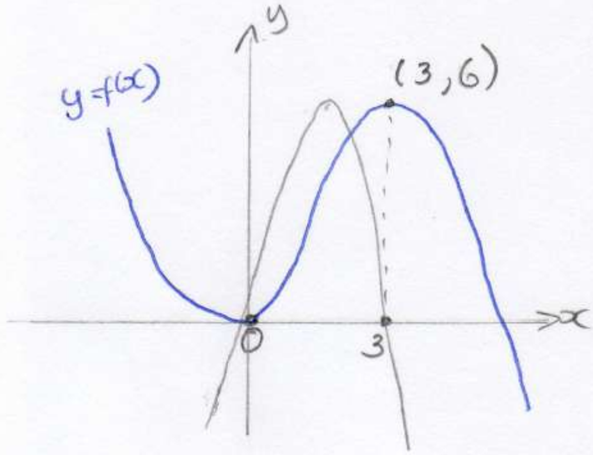
$= -3(x^2 + 2x + 1)$

As $-3 \times (\quad)$ then function can never be increasing.

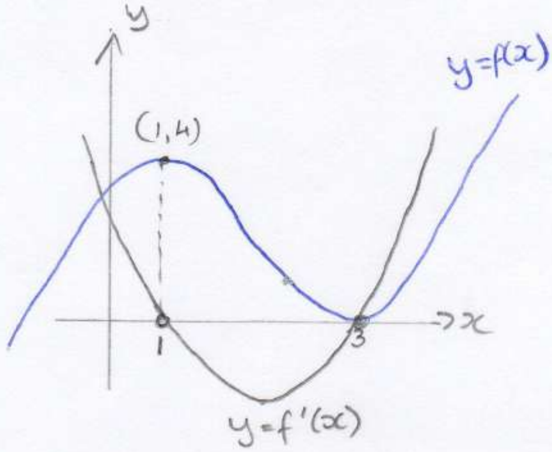
13 a)


y
y = f(x)
y = f'(x)
1
x

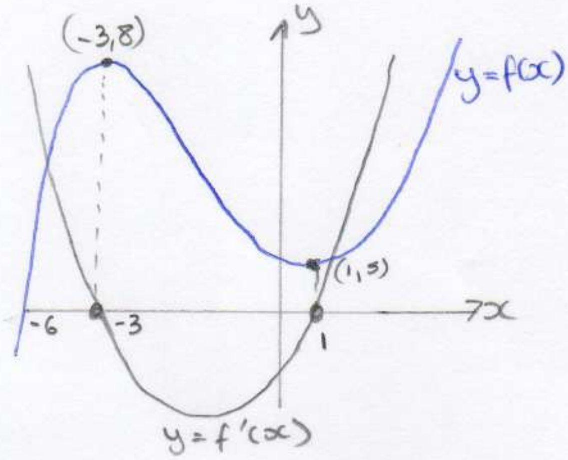
b)


y
y = f(x)
(3, 6)
0
3
x

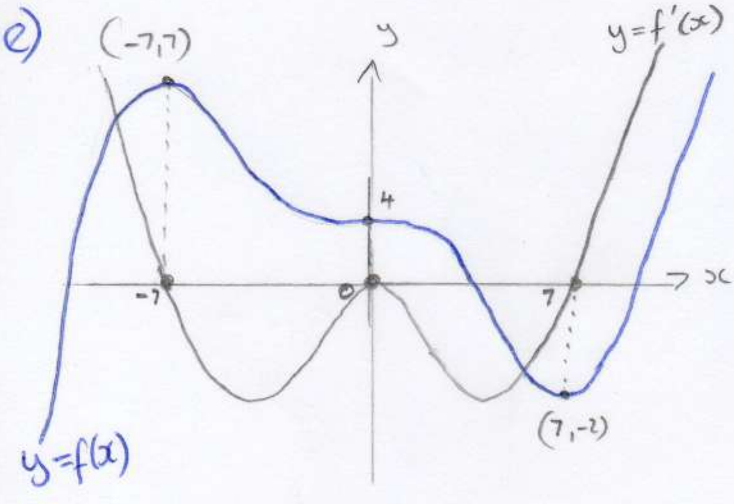
c)


y
y = f(x)
(1, 4)
1
3
x
y = f'(x)

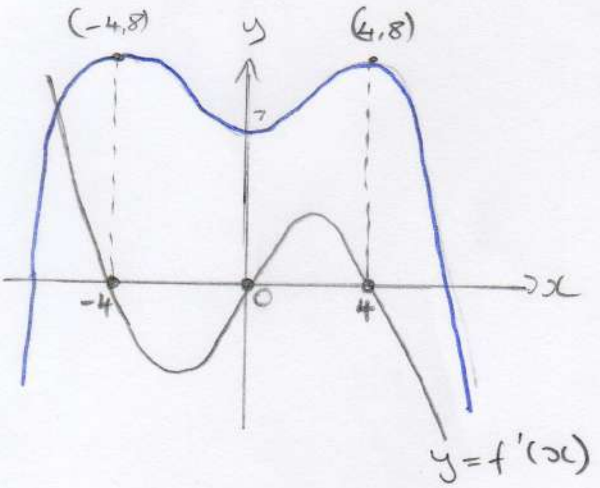
d)


(-3, 8)
y
y = f(x)
(1, 5)
-6
-3
1
x
y = f'(x)

e)


(-7, 7)
y
y = f'(x)
4
-7
0
7
x
(7, -2)
y = f(x)

f)


(-4, 8)
y
(4, 8)
7
-4
0
4
x
y = f'(x)

14 a) $f(x) = x^3 - 3x^2 - 9x + 27 \qquad -4 \le x \le 3$

$f'(x) = 3x^2 - 6x - 9$

$3(x^2 - 2x - 3)$

$3(x+1)(x-3) = 0$

st points at $x = -1 \quad x = 3$.

st pts

$f(-1) = (-1)^3 - 3(-1)^2 - 9\times(-1) + 27 = 32 \longrightarrow \underline{\text{Max}}$

$f(3) = (3)^3 - 3(3)^2 - 9\times(3) + 27 = -0\ 9$

extremities

$f(-4) = (-4)^3 - 3(-4)^2 - 9(-4) + 27 = -49 \longrightarrow \underline{\text{Min}}$

$f(3) = 0\ 9$

b) $f(x) = x^3 - 3x \qquad -2 \le x \le 3$

$f'(x) = 3x^2 - 3$

$= 3(x^2 - 1)$

$= 3(x-1)(x+1)$

ST points at $x = 1$ and $x = -1$

ST PTS

$f(1) = (1)^3 - 3(1) = -2$

$f(-1) = (-1)^3 - 3(-1) = 2$

extremities

$f(-2) = (-2)^3 - 3(-2) = -2$

$\underline{\text{Min}}$

$f(3) = (3)^3 - 3(3) = 18 \longrightarrow \underline{\text{Max}}$

c) $f(x) = 5x^3 - 3x^5 + 3 \qquad 0 \le x \le 5$

$f'(x) = 15x^2 - 15x^4$

$= 15x^2(1 - x^2)$

$= 15x^2(1-x)(1+x)$

St pts at $x = 0$, $x = -1$ and $x = 1$.

St pts

$f(0) = 5\times(0)^3 - 3\times(0)^5 + 3 = 3$

$f(-1) =$ This is outside range so ignore

$f(1) = 5\times(1)^3 - 3\times(1)^5 + 3 = 5 \longrightarrow \underline{\text{max}}$

extremities

$f(0) = 3$

$f(5) = 5\times(5)^3 - 3\times(5)^5 + 3 = -8747 \longrightarrow \underline{\text{min}}$

15. a) $V = l \times b \times h$

$125 = x \times x \times h$

$125 = x^2 h$

$h = \dfrac{125}{x^2}$

$SA = 2(\underset{\substack{\text{Base}\\\text{Top}}}{x \times x}) + 2(\underset{\substack{\text{Left}\\\text{Right}}}{x \times h}) + 2(\underset{\substack{\text{Front}\\\text{Back}}}{x \times h})$

$= 2x^2 + 4(\cancel{x} \times \dfrac{125}{x^{\cancel{2}}})$

$= 2x^2 + \dfrac{500}{x}$

as required

b) $A(x) = 2x^2 + \dfrac{500}{x}$

$A(x) = 2x^2 + 500x^{-1}$

$A'(x) = 4x - 500x^{-2}$

$= 4x - \dfrac{500}{x^2}$

St pts occur when $A'(x) = 0$

$4x - \dfrac{500}{x^2} = 0$

$4x = \dfrac{500}{x^2}$

$4x^3 = 500$

$x^3 = 125$

$x = \sqrt[5]{125}$

$x = 5$

	$\rightarrow$	5	$\rightarrow$
$A'(x)$	$-$	0	$+$
slope	\	—	/

Min TP at $x = 5$

16. a) $V = Ah$

$V = \frac{1}{2}bh \times h$

$256{,}000 = \frac{1}{2} \times x \times x \times h$

$256{,}000 = \frac{1}{2}x^2 h$

$h = \dfrac{256000}{\frac{1}{2}x^2}$

$h = \dfrac{512000}{x^2}$

$SA = 2\left(\frac{1}{2} \times x \times x\right) + 2(x \times h)$

$= x^2 + 2x \times \dfrac{512000}{x^2}$

$= x^2 + \dfrac{1024000}{x}$

as required

b) $A(x) = x^2 + \dfrac{1024000}{x}$

$A(x) = x^2 + 1024000x^{-1}$

$A'(x) = 2x - 1024000x^{-2}$

$= 2x - \dfrac{1024000}{x^2}$

st pts occur when $A'(x) = 0$

$2x - \dfrac{1024000}{x^2} = 0$

$2x = \dfrac{1024000}{x^2}$

$2x^3 = 1024000$

$x^3 = 512000$

$x = \sqrt{512000}$

$x = 80$

	$\rightarrow$	80	$\rightarrow$
$A(x)$	$-$	0	$+$
slope	\	—	/

min TP at $x = 80$

17. a) $SA = 2(x \times x) + 2(x \times l)$

$96 = 2x^2 + 2xl$

$96 - 2x^2 = 2xl$

$l = \dfrac{96 - 2x^2}{2x}$

$l = \dfrac{48 - x^2}{x}$

$V = l \times b \times h$

$= \dfrac{48 - x^2}{\cancel{x}} \times \cancel{x} \times x$

$= x(48 - x^2$

as required

b) $V(x) = x(48 - x^2)$

$= 48x - x^3$

$V'(x) = 48 - 3x^2$

st pt occur when $V'(x) = 0$

$48 - 3x^2 = 0$

$3x^2 = 48$

$x^2 = 16$

$x = \pm 4$ (discard -4 as you can't have a -ve length)

$x = 4$

	$\rightarrow$	4	$\rightarrow$
$V'(x)$	$+$	0	$-$
slope	/	—	\

max TP at $x = 4$

Dimensions

8m × 4m × 4m

$l = \dfrac{48 - x^2}{x}$

$= \dfrac{48 - 4^2}{4}$

$= 8$