St Peter the Apostle High School
Maths Department

Higher Practice Questions

6. Differentiation

1 Find the derivative of the following:

(a) $f(x) = x^5$ **(b)** $y = x^9$ **(c)** $f(x) = x^{14}$

(d) $y = x^{-2}$ **(e)** $f(x) = x^{-8}$ **(f)** $y = x^{-20}$

2 Find the derivative of the following:

(a) $y = 2x^3$ **(b)** $y = 5x^4$ **(c)** $f(x) = \frac{1}{2}x^6$

(d) $f(x) = \frac{2}{3}x^9$ **(e)** $y = \frac{3}{x^3}$ **(f)** $y = \frac{25}{x^4}$

(g) $y = 9x^4$ **(h)** $f(x) = x^{\frac{2}{3}}$ **(i)** $f(x) = 10x^{\frac{2}{5}}$

(j) $f(x) = 4x^{\frac{3}{4}}$ **(k)** $y = \frac{1}{2x^4}$ **(l)** $y = \frac{3}{4x^5}$

3 Find the derivative of the following:

(a) $y = (x+1)(x+2)$ **(b)** $f(x) = (x+2)(x-3)$ **(c)** $y = (x+2)^2$

(d) $f(x) = (x-3)(x+4)$ **(e)** $y = (x-5)(2x-2)$ **(f)** $y = x(x-4)$

(g) $y = (2x-3)(x+4)$ **(h)** $f(x) = x^2(x-2)$ **(i)** $y = \frac{1}{x^2}(x^3+2x)$

(j) $f(x) = \frac{1}{x^2}(x^2+x)$ **(k)** $y = \left(\frac{1}{x}+1\right)^2$ **(l)** $y = \frac{1}{x^2}(x-5)^2$

4 Find the derivative of the following:

(a) $y = \dfrac{x^2+3x+5}{x}$ **(b)** $y = \dfrac{2x^3+x^2+x}{x}$ **(c)** $f(x) = \dfrac{x^4+x^3-6x}{x^2}$

(d) $y = \dfrac{3+x^2}{x^2}$ **(e)** $y = \dfrac{x+2}{\sqrt{x}}$ **(f)** $f(x) = \dfrac{3x^2+5x+1}{2x^2}$

5 For each of the following find $f'(x)$ and the rate of change of f at the given value of x:

(a) $f(x) = x^3 + 3x^2 + 5x, \quad x = 2$

(b) $f(x) = 3x^5 + 2x^4 - x, \quad x = 1$

(c) $f(x) = x^2 + 6x - 1, \quad x = -4$

(d) $f(x) = x^{\frac{2}{3}} + 4x^2, \quad x = 8$

(e) $f(x) = 3x^{\frac{1}{2}} - 2x^{-5}, \quad x = 1$

(f) $f(x) = 5x^{-2} - 3x^{\frac{1}{2}}, \quad x = 4$

(g) $f(x) = \frac{1}{2\sqrt[3]{x}} + x^2, \quad x = -1$

(h) $f(x) = 3x^7 - \frac{1}{5\sqrt[4]{x^3}}, \quad x = 1$

6 Find the equation of the tangent to the curve at the given point.

(a) $y = x^2$ at $(1, 4)$

(b) $y = x^3 + 3,$ at $(2, 11)$

(c) $y = x^2 - 6x + 5$ at $x = 2$

(d) $y = \sqrt{x} + 1$ at $x = 1$

(e) $y = x^2 + 2x - 3$ at $(1, 0)$

(f) $f(x) = 5x^2 - 6x^{\frac{1}{2}}$ at $x = 1$

7 Find the point on the curve $y = 5x^2 - 2x + 7$ at which the tangent has gradient 8. Hence find the equation of the tangent at this point.

8 A curve has equation $f(x) = 2x^2 + 8x - 3$

A tangent to this curve has a gradient of -4, find the equation of this tangent.

9 The gradient of the tangent to the curve $y = ax^2 + b$ is equal to 30 at the point (3, 1).

Find the values of a and b.

10 Find the coordinates of the stationary points and determine their nature.

(a) $y = x^3 - 6x^2 + 9x$

(b) $y = x^4 - 4x^3$

(c) $y = x^3 - 3x + 2$

(d) $y = 3x^4 + 16x^3$

11 Sketch the graph of the following functions, showing clearly the intersections with the axes and any stationary points.

(a) $y = x^2 - 6x$

(b) $y = x^3 - 3x^2$

(c) $y = (x-1)^2(x+2)$

(d) $y = x^2(3-x)$

12 Show that the function

(a) $y = x^3 - x^2 + x$ is never decreasing

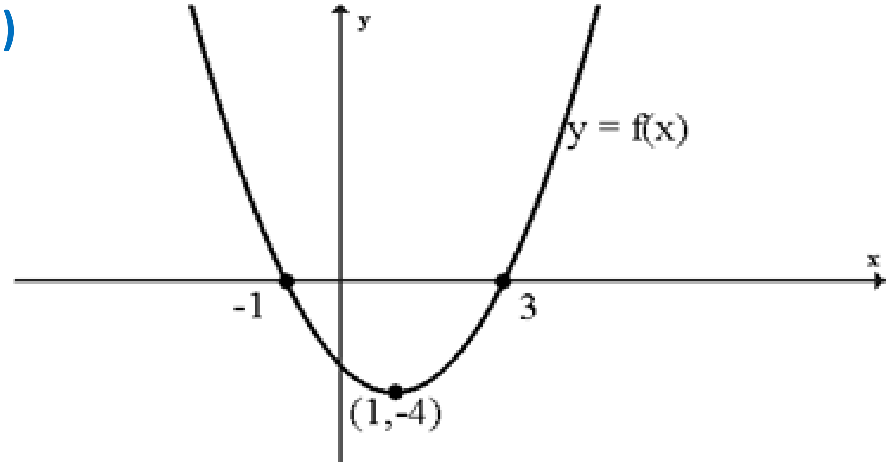
(b) $y = 2x^5 + 5$ is never decreasing

(c) $y = -x^3 - 3x^2 - 3x$ is never increasing

13

For each graph of $f(x)$, sketch its derivative, $f'(x)$.
Mark all the significant points clearly.

(a)



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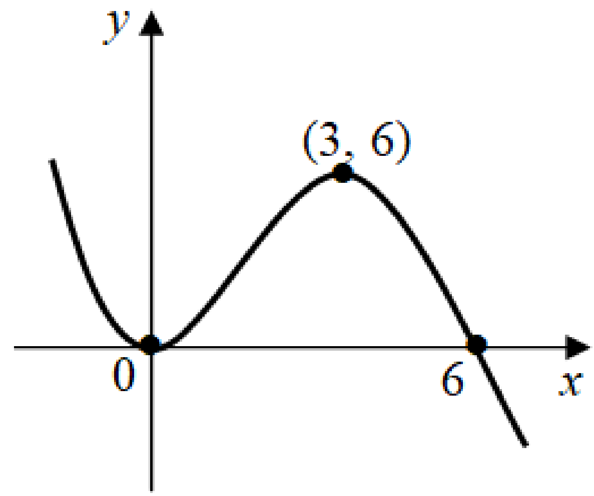

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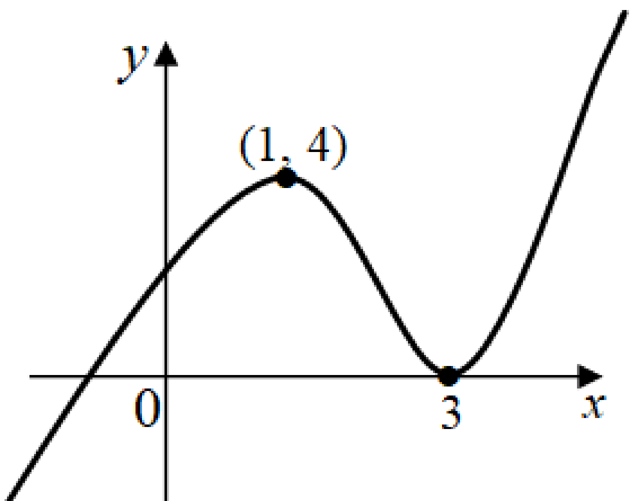

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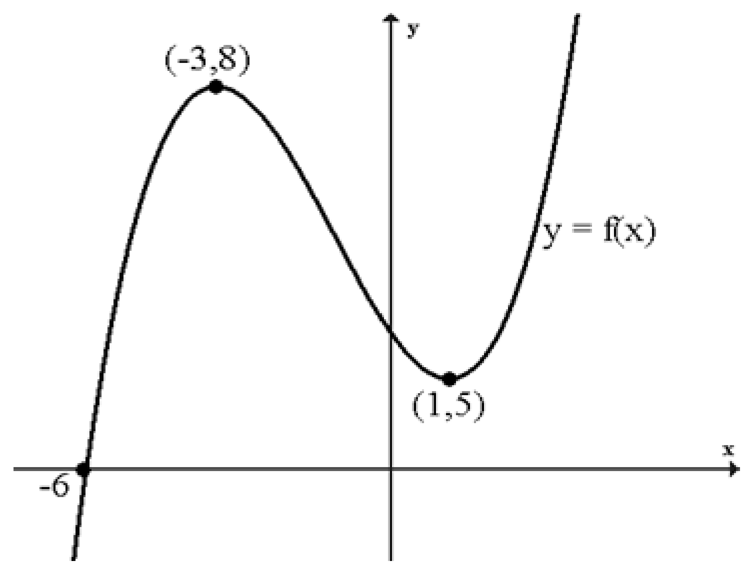

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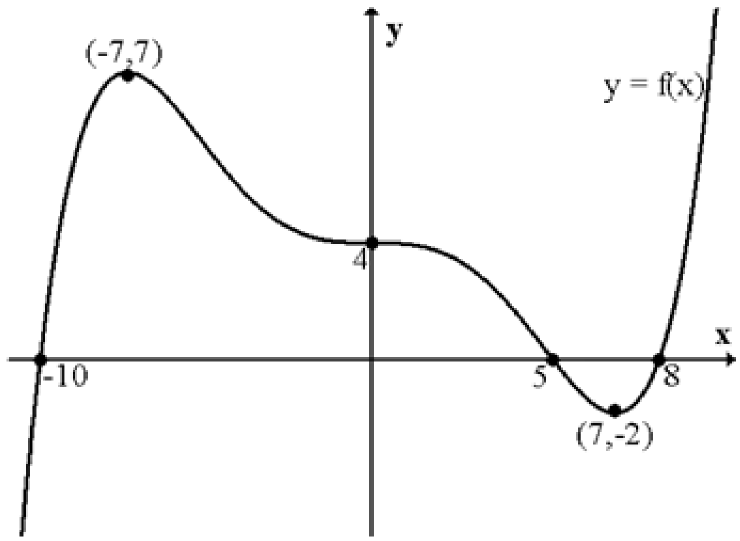
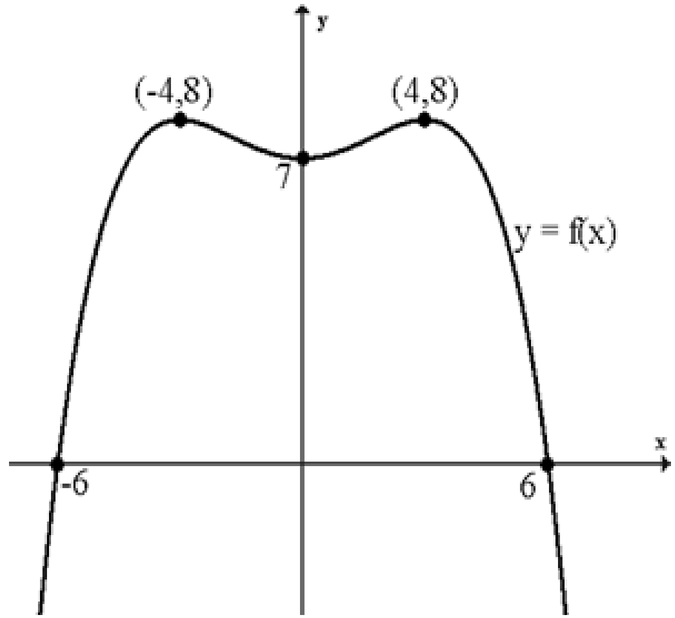


(f)



14 For each function determine the maximum and minimum values within the given limits.

(a) $f(x) = x^3 - 3x^2 - 9x + 27 \quad \text{for} \ -4 \leq x \leq 3$

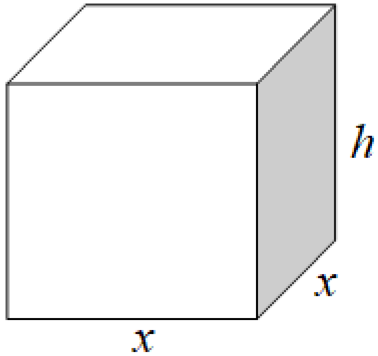
(b) $f(x) = x^3 - 3x \quad \text{for} \ -2 \leq x \leq 3$

(c) $f(x) = 5x^3 - 3x^5 + 3 \quad \text{for} \ \ 0 \leq x \leq 5$

15

A cuboid measures x by x by h units.

Its volume is 125 units^2.



(a) Show that the surface area of this cuboid is given by

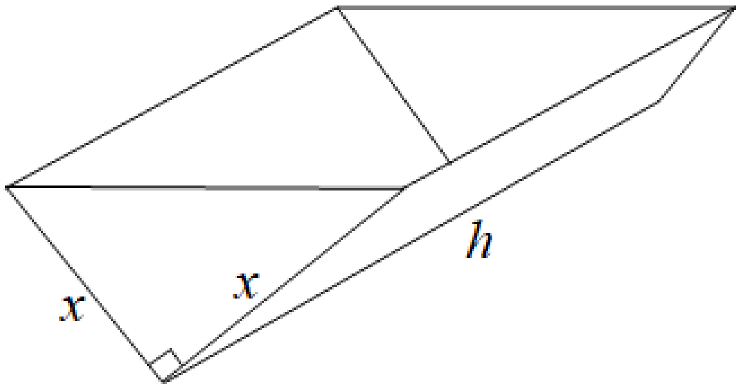
$$A(x) = 2x^2 + \frac{500}{x}$$

(b) Find the value of x such that the surface area is minimised

16

An open trough is in the shape of a triangular prism.

The trough has a capacity of 256 000 cm^3.



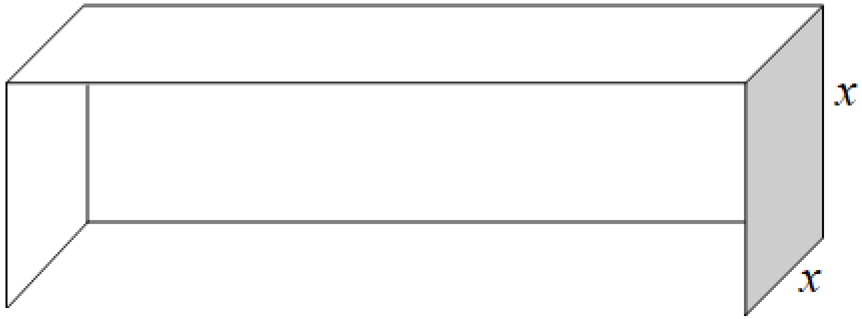
(a) Show that the surface area of this trough is given by

$$A(x) = x^2 + \frac{1\,024\,000}{x}$$

(b) Find the value of x such that the surface area is minimised

17

A shelter consists of two square sides (x metres long), a rectangular top and back. The total amount of material used to make the shelter is 96 m^2.



(a) Show that the volume of the shelter is given by

$$V(x) = x(48 - x^2)$$

(b) Find the dimensions of the shelter with the maximum volume.