Wave Function

Go to the appropriate Past Paper for the answers

2019 Paper 2

6. (a) Express $2\cos x° - 3\sin x°$ in the form $k\cos(x+a)°$ where $k>0$ and $0 \le a < 360$. **4**

(b) Hence solve $2\cos x° - 3\sin x° = 3$ for $0 \le x < 360$. **3**

2018 Paper 2

8. (a) Express $2\cos x° - \sin x°$ in the form $k\cos(x-a)°$, $k>0$, $0<a<360$. **4**

(b) Hence, or otherwise, find

(i) the minimum value of $6\cos x° - 3\sin x°$ and **1**

(ii) the value of x for which it occurs where $0 \le x < 360$. **2**

2017 Paper 1

14. (a) Express $\sqrt{3}\sin x° - \cos x°$ in the form $k\sin(x-a)°$, where $k>0$ and $0<a<360$. **4**

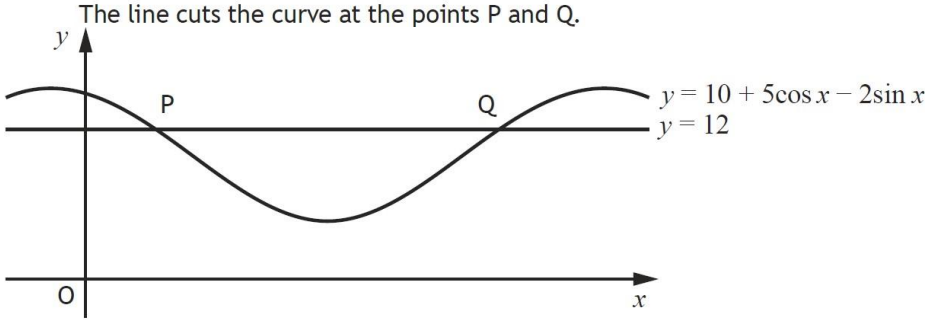
(b) Hence, or otherwise, sketch the graph with equation $y = \sqrt{3}\sin x° - \cos x°$, $0 \le x \le 360$. **3**

2016 Paper 2

8. (a) Express $5\cos x - 2\sin x$ in the form $k\cos(x+a)$, where $k>0$ and $0<a<2\pi$. **4**

(b) The diagram shows a sketch of part of the graph of $y = 10 + 5\cos x - 2\sin x$ and the line with equation $y = 12$.

The line cuts the curve at the points P and Q.



4

Find the x-coordinates of P and Q.

2015 Paper 2

9. The blades of a wind turbine are turning at a steady rate.

The height, h metres, of the tip of one of the blades above the ground at time, t seconds, is given by the formula

$$h = 36\sin(1{\cdot}5t) - 15\cos(1{\cdot}5t) + 65.$$

Express $36\sin(1{\cdot}5t) - 15\cos(1{\cdot}5t)$ in the form **8**

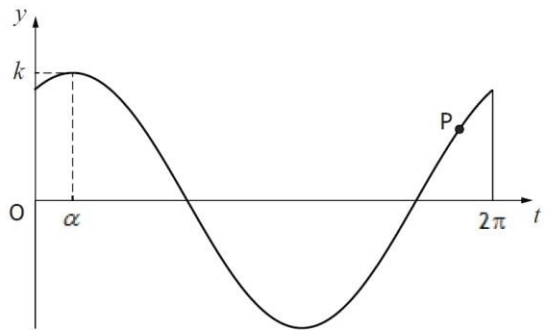
$k\sin(1{\cdot}5t - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$,

and hence find the **two** values of t for which the tip of this blade is at a height of 100 metres above the ground during the first turn.

Specimen 5 Paper 2

10. Two sound sources produce the waves $y = \sin t$ and $y = \sqrt{3}\cos t$.

An investigation into the addition of these two waves produces the graph shown, with equation $y = k\cos(t - \alpha)$ for $0 \le t \le 2\pi$.



(a) Calculate the values of k and α. **4**

The point P has a y-coordinate of 1·2.

(c) Hence calculate the value of the t-coordinate of point P. **4**

Exemplar Paper 1

9. The expression $\cos 4x - \sqrt{3}\sin 4x$ can be written in the form $k\cos(4x + a)$ where $k > 0$ and $0 \le a \le 2\pi$.

(a) Calculate the values of k and a. **4**

(b) Find the points of intersection of the graph of $y = \cos 4x - \sqrt{3}\sin 4x$ with the x axis, in the interval $0 \le x \le \frac{\pi}{2}$. **3**

Exemplar Paper 1

11. Functions f and g are defined on suitable domains by $f(x) = x^3 - 1$ and $g(x) = 3x + 1$.

(a) Find an expression for $k(x)$, where $k(x) = g(f(x))$. **2**

(b) If $h(k(x)) = x$, find an expression for $h(x)$. **3**

2014 Paper1

4. If $3\sin x - 4\cos x$ is written in the form $k\cos(x - a)$, what are the values of $k\cos a$ and $k\sin a$? **2**

2013 Paper 1

23. (*a*) The expression $\sqrt{3}\sin x° - \cos x°$ can be written in the form $k\sin(x - a)°$, where $k > 0$ and $0 \leq a < 360$. **4**

Calculate the values of k and a.

(*b*) Determine the maximum value of $4 + 5\cos x° - 5\sqrt{3}\sin x°$, where $0 \leq x < 360$. **2**

2012 Paper 1

22. (*a*) The expression $\cos x - \sqrt{3}\sin x$ can be written in the form $k\cos(x + a)$ where $k > 0$ and $0 \leq a < 2\pi$.

Calculate the values of k and a. **4**

(*b*) Find the points of intersection of the graph of $y = \cos x - \sqrt{3}\sin x$ with the x and y axes, in the interval $0 \leq x \leq 2\pi$. **3**

2011 Paper 2

6. (*a*) The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x + a)$ where $R > 0$ and $0 \leq a < 2\pi$.

Calculate the values of R and a. **4**

(*b*) Hence find the value of t, where $0 \leq t \leq 2$, for which

$$\int_0^t (3\cos x + 5\sin x)\, dx = 3.$$ **7**

2010 Paper 2

2. (*a*) $12\cos x° - 5\sin x°$ can be expressed in the form $k\cos(x + a)°$, where $k > 0$ and $0 \leq a < 360$.

Calculate the values of k and a. **4**

(*b*) (i) Hence state the maximum and minimum values of $12\cos x° - 5\sin x°$.

(ii) Determine the values of x, in the interval $0 \leq x < 360$, at which these maximum and minimum values occur. **3**

2009 Paper 1

13. k and a are given by

$$k \sin a^\circ = 1$$
$$k \cos a^\circ = \sqrt{3}$$

where $k > 0$ and $0 \leq a < 90$.

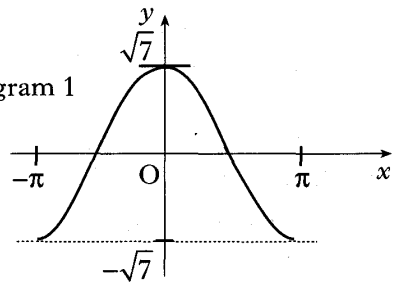
What are the values of k and a? **2**

2008 Paper 2

3. (*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of p.

Diagram 1

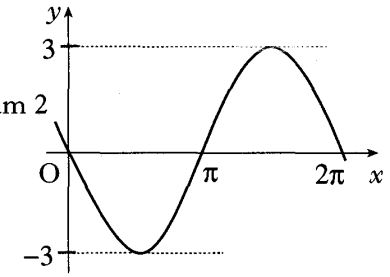


2

(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

Write down the value of q.

Diagram 2



(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. **2**

2007 Paper 2

11. (*a*) Express $f(x) = \sqrt{3}\cos x + \sin x$ in the form $k\cos(x - a)$, where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*b*) Hence or otherwise sketch the graph of $y = f(x)$ in the interval $0 \leq x \leq 2\pi$. **4**

2006 Paper 2

10. A curve has equation $y = 7\sin x - 24\cos x$.

(*a*) Express $7\sin x - 24\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq \frac{\pi}{2}$. **4**

(*b*) Hence find, in the interval $0 \leq x \leq \pi$, the x-coordinate of the point on the curve where the gradient is 1. **3**

2005 Paper 1

10. (*a*) Express $\sin x - \sqrt{3}\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq 2\pi$. **4**

(*b*) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin x - \sqrt{3}\cos x$ in the interval $0 \leq x \leq 2\pi$. **5**

2014 Paper 2

6. (*a*) Express $3\cos(x°) + 5\sin(x°)$ in the form $k\cos(x° - a°)$ where $k > 0$ and $0 \leq a \leq 90$. 4

(*b*) Hence solve the equation $3\cos(x°) + 5\sin(x°) = 4$ for $0 \leq x \leq 90$. 3