Vectors

Go to the appropriate Past Paper for the answers

2019 Paper 1

5. (a) Show that the points A(1,5,−3), B(4,−1,0) and C(8,−9,4) are collinear. **3**

(b) State the ratio in which B divides AC. **1**

2019 Paper 1

9. Vectors **u** and **v** have components $\begin{pmatrix} p \\ -2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} 2p+16 \\ -3 \\ 6 \end{pmatrix}$, $p \in \mathbb{R}$.

(a) (i) Find an expression for **u.v**. **1**

(ii) Determine the values of p for which **u** and **v** are perpendicular. **3**

(b) Determine the value of p for which **u** and **v** are parallel. **2**
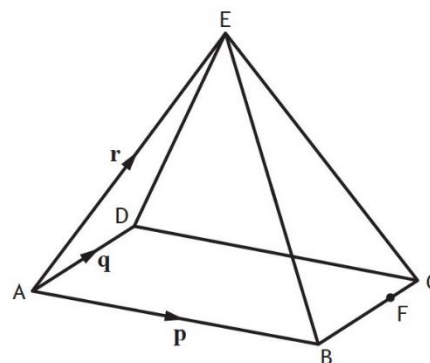
2019 Paper 2

3. E,ABCD is a rectangular based pyramid.
$\overrightarrow{AB} = \mathbf{p}$, $\overrightarrow{AD} = \mathbf{q}$ and $\overrightarrow{AE} = \mathbf{r}$.

(a) Express $\overrightarrow{BE}$ in terms of **p** and **r**. **1**

Point F divides BC in the ratio 3:1.

(b) Express vector $\overrightarrow{EF}$ in terms of **p**, **q** and **r**. **2**

2019 Paper 2

14. The vectors **u** and **v** are such that

- $|\mathbf{u}| = 4$
- $|\mathbf{v}| = 5$
- $\mathbf{u}.(\mathbf{u} + \mathbf{v}) = 21$

Determine the size of the angle between the vectors **u** and **v**. **4**

2018 Paper 1

5. A(−3,4,−7), B(5,t,5) and C(7,9,8) are collinear.

(a) State the ratio in which B divides AC. **1**

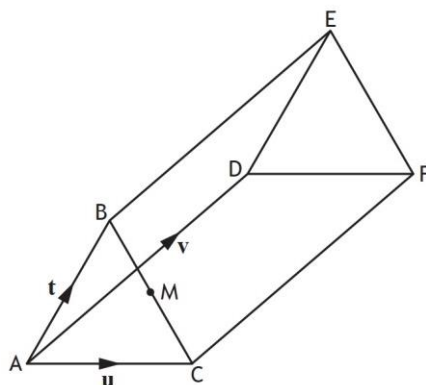(b) State the value of t. **1**

2018 Paper 1

9. The diagram shows a triangular prism ABC,DEF.

$\overrightarrow{AB} = \mathbf{t}$, $\overrightarrow{AC} = \mathbf{u}$ and $\overrightarrow{AD} = \mathbf{v}$.

(a) Express $\overrightarrow{BC}$ in terms of $\mathbf{u}$ and $\mathbf{t}$.

1

M is the midpoint of BC.

(b) Express $\overrightarrow{MD}$ in terms of $\mathbf{t}$, $\mathbf{u}$ and $\mathbf{v}$.

2

2018 Paper 1

12. Vectors $\mathbf{a}$ and $\mathbf{b}$ are such that $\mathbf{a} = 4\mathbf{i} - 2\mathbf{j} + 2\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + \mathbf{j} + p\mathbf{k}$.

(a) Express $2\mathbf{a} + \mathbf{b}$ in component form.

1

(b) Hence find the values of p for which $|2\mathbf{a} + \mathbf{b}| = 7$.

1

2018 Paper 2

2. Vectors $\mathbf{u}$ and $\mathbf{v}$ are defined by $\mathbf{u} = \begin{pmatrix} -1 \\ 4 \\ -3 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -7 \\ 8 \\ 5 \end{pmatrix}$.

(a) Find $\mathbf{u.v}$.

1

(b) Calculate the acute angle between $\mathbf{u}$ and $\mathbf{v}$.

4

Specimen 5 Paper 1

5. The points $A(0, 9, 7)$, $B(5, -1, 2)$, $C(4, 1, 3)$ and $D(x, -2, 2)$ are such that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CD}$.

4

Determine the value of x.

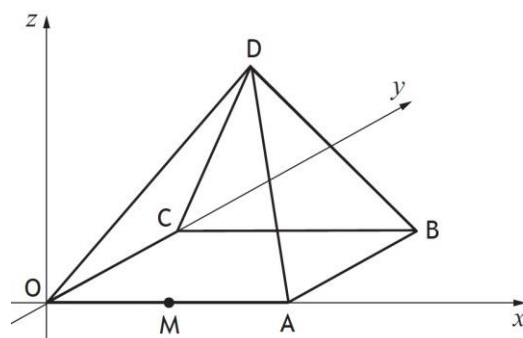## Specimen Paper 2

4. D,OABC is a square-based pyramid as shown.

- O is the origin and OA = 4 units.

- M is the mid-point of OA.

- $\overrightarrow{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

(a) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form.

3

(b) Find the size of angle BDM.

5

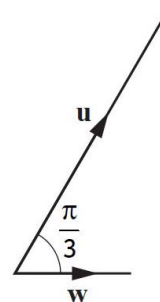## 2017 Paper 1

5. Vectors **u** and **v** are $\begin{pmatrix} 5 \\ 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -8 \\ 6 \end{pmatrix}$ respectively.

   (a) Evaluate **u**.**v**.

   1

   (b) Vector **w** makes an angle of $\frac{\pi}{3}$ with **u** and $|\mathbf{w}| = \sqrt{3}$.
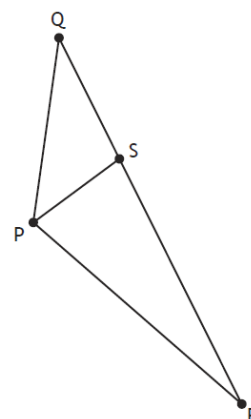   Calculate **u**.**w**.

   3

2017 Paper 2

5. In the diagram, $\overrightarrow{PR} = 9\mathbf{i} + 5\mathbf{j} + 2\mathbf{k}$ and $\overrightarrow{RQ} = -12\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$.

   (a) Express $\overrightarrow{PQ}$ in terms of **i**, **j** and **k**.

   2

   The point S divides QR in the ratio 1:2.

   (b) Show that $\overrightarrow{PS} = \mathbf{i} - \mathbf{j} + 4\mathbf{k}$.

   2

   (c) Hence, find the size of angle QPS.

   5

2016 Paper 1

7. Three vectors can be expressed as follows: $\overrightarrow{FG} = -2\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}$

   (a) Find $\overrightarrow{FH}$.

   $\overrightarrow{GH} = 3\mathbf{i} + 9\mathbf{j} - 7\mathbf{k}$

   2

   (b) Hence, or otherwise, find $\overrightarrow{FE}$.

   $\overrightarrow{EH} = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$

   2

2016 Paper 1

11. (a) A and C are the points $(1, 3, -2)$ and $(4, -3, 4)$ respectively.
    Point B divides AC in the ratio $1 : 2$.
    Find the coordinates of B.

    2

    (b) $k\overrightarrow{AC}$ is a vector of magnitude 1, where $k > 0$.
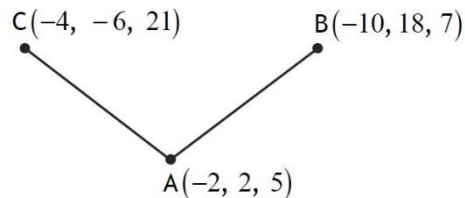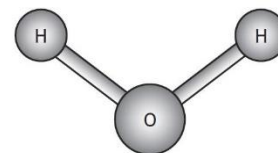   Determine the value of k.

    3

2016 Paper 2

5. The picture shows a model of a water molecule.

Relative to suitable coordinate axes, the oxygen atom is positioned at point $A(-2, 2, 5)$.

The two hydrogen atoms are positioned at points $B(-10, 18, 7)$ and $C(-4, -6, 21)$ as shown in the diagram below.

$C(-4, -6, 21)$ $B(-10, 18, 7)$

(a) Express $\overrightarrow{AB}$ and $\overrightarrow{AC}$ in component form.   **2**

(b) Hence, or otherwise, find the size of angle BAC. $A(-2, 2, 5)$ **4**

New 2015 Paper 1

1. Vectors $\mathbf{u} = 8\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{v} = -3\mathbf{i} + t\mathbf{j} - 6\mathbf{k}$ are perpendicular.   **2**

Determine the value of t.
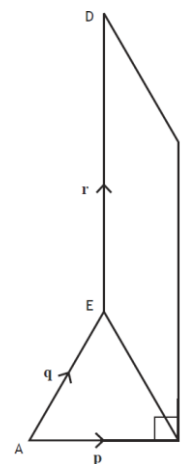
New 2015 Paper 2

6. Vectors $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$ are represented on the diagram as shown.

- BCDE is a parallelogram
- ABE is an equilateral triangle
- $|\mathbf{p}| = 3$
- Angle ABC $= 90°$

(a) Evaluate $\mathbf{p}.(\mathbf{q}+\mathbf{r})$.   **3**

(b) Express $\overrightarrow{EC}$ in terms of $\mathbf{p}$, $\mathbf{q}$ and $\mathbf{r}$.   **1**

(c) Given that $\overrightarrow{AE}.\overrightarrow{EC} = 9\sqrt{3} - \dfrac{9}{2}$, find $|\mathbf{r}|$.   **3**
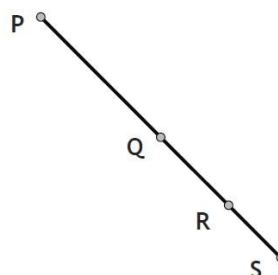
Specimen 4 Paper 1

3. In the diagram, P has coordinates $(-6, 3, 9)$,

$\overrightarrow{PQ} = 6\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}$ and $\overrightarrow{PQ} = 2\overrightarrow{QR} = 3\overrightarrow{RS}$.   **5**

Find the coordinates of S.

Specimen 4 Paper 1

4. Given that $2x^2 + px + p + 6 = 0$ has no real roots, find the range of values for p, where $p \in \mathbb{R}$.   **4**

Specimen 4 Paper 2

6. The points A(0, 9, 7), B(5, −1, 2), C(4, 1, 3) and D(x, −2, 2) are such that AB is perpendicular to CD.

Determine the value of x.
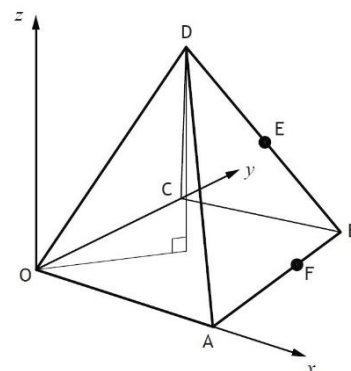
5

Specimen 4 Paper 2

1. A square based right pyramid is shown in the diagram.

Square OABC has a side length of 60 units with edges OA and OC lying on the x-axis and y-axis respectively.

The coordinates of D are (30, 30, 80).

E is the midpoint of BD and F divides AB in the ratio 2:1.

(a) Find the coordinates of E and F.

2

(b) Calculate $\overrightarrow{ED}.\overrightarrow{EF}$.

2

(c) Hence, or otherwise, calculate the size of angle DEF.

4

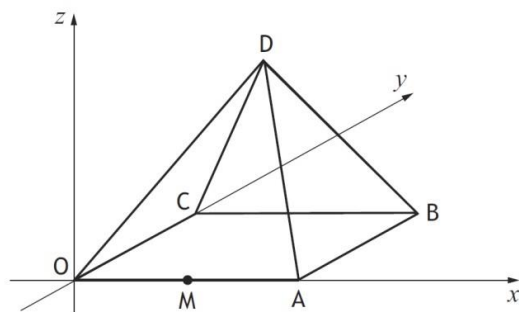## Exemplar Paper 2

5. D,OABC is a square-based pyramid as shown.

O is the origin and OA = 4 units.

M is the mid-point of OA.

$$\overrightarrow{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$$

(a) Express $\overrightarrow{OB}$ in terms of $\mathbf{i}$ and $\mathbf{j}$ and $\mathbf{k}$.

1

(b) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form.

3

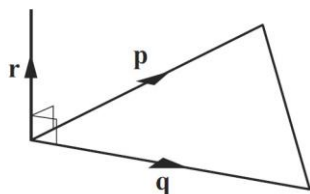(c) Find the size of angle BDM.

5

Exemplar Paper 2

6. An equilateral triangle with sides of length 3 units is shown.

4

Vector $\mathbf{r}$ is 2 units long and is perpendicular to both vectors $\mathbf{p}$ and $\mathbf{q}$.

Calculate the value of the scalar product $\mathbf{p}.(\mathbf{p}+\mathbf{q}+\mathbf{r})$.

2014 Paper 1

6. Given that $u = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, find $2u - 3v$ in component form.

2

2014 Paper 1

16. The unit vectors a and b are such that $a.b = \frac{2}{3}$. Determine the value of $a.(a + 2b)$.

2

2014 Paper 1

14. The vectors $u = \begin{pmatrix} 1 \\ k \\ k \end{pmatrix}$ and $v = \begin{pmatrix} -6 \\ 2 \\ 5 \end{pmatrix}$ are perpendicular.
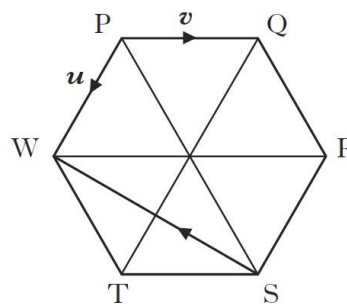
2

What is the value of k?

2014 Paper 1

19. The diagram shows a regular hexagon PQRSTW.
    $\overrightarrow{PW}$ and $\overrightarrow{PQ}$ represent vectors u and v respectively.

    What is $\overrightarrow{SW}$ in terms of u and v?

2

2013 Paper 1

12. If $\mathbf{f} = 3\mathbf{i} + 2\mathbf{k}$ and $\mathbf{g} = 2\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, find $|\mathbf{f} + \mathbf{g}|$.

2

2013 Paper 1

14. Given that $|\mathbf{a}| = 3$, $|\mathbf{b}| = 2$ and $\mathbf{a}.\mathbf{b} = 5$, what is the value of $\mathbf{a}.(\mathbf{a} + \mathbf{b})$?

2

2013 Paper 1

24. (a) (i) Show that the points A(–7, –8, 1), T(3, 2, 5) and B(18, 17, 11) are collinear.

4

    (ii) Find the ratio in which T divides AB.

    (b) The point C lies on the x-axis.
        If TB and TC are perpendicular, find the coordinates of C.
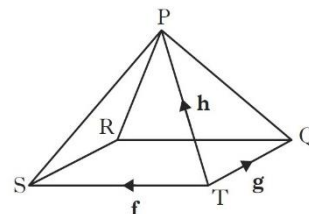
5

2012 Paper 1

7. If $\mathbf{u} = \begin{pmatrix} -3 \\ 1 \\ 2t \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} 1 \\ t \\ -1 \end{pmatrix}$ are perpendicular, what is the value of t?

2

2012 Paper 1

10. The diagram shows a square-based pyramid P,QRST.

$\overrightarrow{TS}$, $\overrightarrow{TQ}$ and $\overrightarrow{TP}$ represent $\mathbf{f}$, $\mathbf{g}$ and $\mathbf{h}$ respectively.

Express $\overrightarrow{RP}$ in terms of $\mathbf{f}$, $\mathbf{g}$ and $\mathbf{h}$.

2

2012 Paper 1

15. If $\mathbf{u} = k \begin{pmatrix} 3 \\ -1 \\ 0 \end{pmatrix}$, where $k>0$ and $\mathbf{u}$ is a unit vector, determine the value of k.

2

2012 Paper 1

17. Given that $\mathbf{a} = \begin{pmatrix} 3 \\ 4 \\ 0 \end{pmatrix}$ and $\mathbf{a} \cdot (\mathbf{a} + \mathbf{b}) = 7$, what is the value of $\mathbf{a} \cdot \mathbf{b}$?

2

2012 Paper 2

5. A is the point (3, −3, 0), B is (2, −3, 1) and C is (4, k, 0).

   (a) (i) Express $\overrightarrow{BA}$ and $\overrightarrow{BC}$ in component form.

7

       (ii) Show that $\cos \hat{ABC} = \dfrac{3}{\sqrt{2(k^2 + 6k + 14)}}$.

   (b) If angle ABC = 30°, find the possible values of k.

5

2011 Paper 1

1. Given that $\mathbf{p} = \begin{pmatrix} 2 \\ 5 \\ -7 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -4 \\ 2 \\ 0 \end{pmatrix}$, express $2\mathbf{p} - \mathbf{q} - \frac{1}{2}\mathbf{r}$ in component form.

2

2011 Paper 1

15. Given that the points S(−4, 5, 1), T(−16, −4, 16) and U(−24, −10, 26) are colline: calculate the ratio in which T divides SU.

2

2011 Paper 1

14. An equilateral triangle of side 3 units is shown.

    The vectors **p** and **q** are as represented in the diagram.
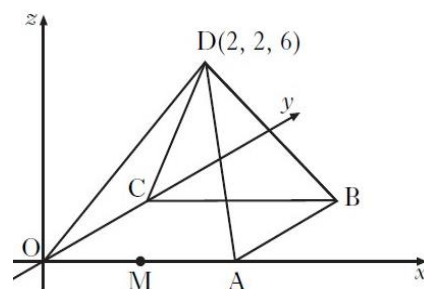
    What is the value of **p.q**?
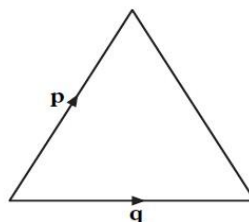
    2

2011 Paper 2

1. D,OABC is a square based pyramid as shown in the diagram below.

   O is the origin, D is the point (2, 2, 6) and OA = 4 units.

   M is the mid-point of OA.

   (*a*) State the coordinates of B.

   1

   (*b*) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form.

   3

   (*c*) Find the size of angle BDM.

   5

2010 Paper 1

3. Given that $\mathbf{u} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{v} = \begin{pmatrix} -1 \\ 2 \\ 4 \end{pmatrix}$, find $3\mathbf{u} - 2\mathbf{v}$ in component form.

   2

2010 Paper 1

10. The vectors $x\mathbf{i} + 5\mathbf{j} + 7\mathbf{k}$ and $-3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ are perpendicular.

    What is the value of x?

    2
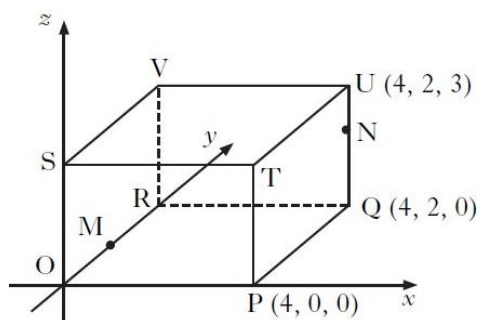
2010 Paper 2

1. The diagram shows a cuboid OPQR,STUV relative to the coordinate axes.

   P is the point (4, 0, 0),
   Q is (4, 2, 0) and U is (4, 2, 3).

   M is the midpoint of OR.

   N is the point on UQ such that
   $UN = \frac{1}{3}UQ$.

   (*a*) State the coordinates of M and N.

   (*b*) Express $\overrightarrow{VM}$ and $\overrightarrow{VN}$ in component form.

   1

   (*c*) Calculate the size of angle MVN.

   2

   5

2009 Paper 1

17. The vector u has components $\begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$. Find a unit vector which is parallel to u.

2

2009 Paper 1

22. D, E and F have coordinates (10, −8, −15), (1, −2, −3) and (−2, 0, 1) respectively.

   (a) (i) Show that D, E and F are collinear.

      (ii) Find the ratio in which E divides DF.

4
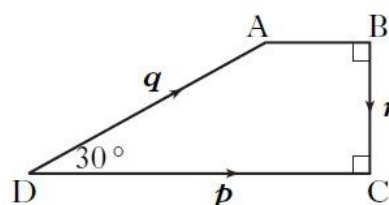
   (b) G has coordinates (k, 1, 0).

      Given that DE is perpendicular to GE, find the value of k.

4

2009 Paper 2

7. Vectors p, q and r are represented on the diagram shown where angle ADC = 30°.

   It is also given that $|p| = 4$ and $|q| = 3$.

   (a) Evaluate $p.(q + r)$ and $r.(p − q)$.

   (b) Find $|q + r|$ and $|p − q|$.

6

4

2008 Paper 1

3. The vectors $u = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix}$ and $v = \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix}$ are perpendicular.

2

   What is the value of k?

2008 Paper 1

11. E(−2, −1, 4), P(1, 5, 7) and F(7, 17, 13) are three collinear points.
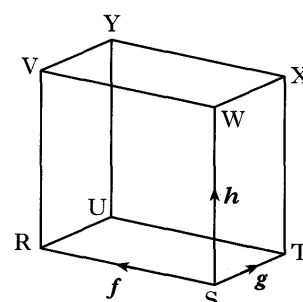
   P lies between E and F.

2

   What is the ratio in which P divides EF?

2008 Paper 1

12. In the diagram RSTU, VWXY represents a cuboid.

   $\overrightarrow{SR}$ represents vector f, $\overrightarrow{ST}$ represents vector g and $\overrightarrow{SW}$ represents vector h.

   Express $\overrightarrow{VT}$ in terms of f, g and h.

2

2008 Paper 1

18. Vectors p and q are such that $|p| = 3$, $|q| = 4$ and $p.q = 10$.

Find the value of $q.(p + q)$.

2

2008 Paper 2

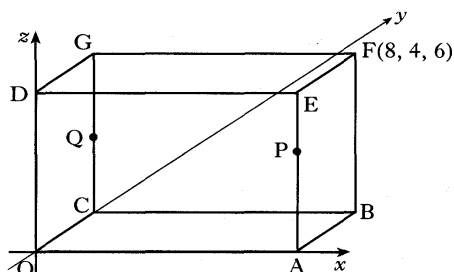**2.** The diagram shows a cuboid OABC, DEFG.

F is the point (8, 4, 6).

P divides AE in the ratio 2:1.

Q is the midpoint of CG.

(*a*) State the coordinates of P and Q.

2

(*b*) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$.
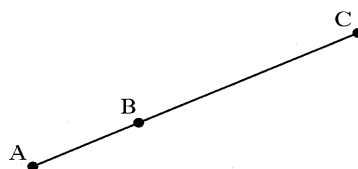
2

(*c*) Find the size of angle QPA.

5

2007 Paper 1

2. Relative to a suitable coordinate system A and B are the points (−2, 1, −1) and (1, 3, 2) respectively.

A, B and C are collinear points and C is positioned such that BC = 2AB.

4

Find the coordinates of C.

2007 Paper 2

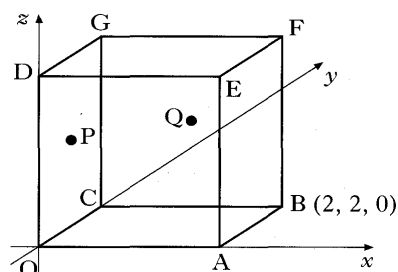**1.** OABCDEFG is a cube with side 2 units, as shown in the diagram.

B has coordinates (2, 2, 0).

P is the centre of face OCGD and Q is the centre of face CBFG.

(*a*) Write down the coordinates of G.

1

(*b*) Find p and q, the position vectors of points P and Q.
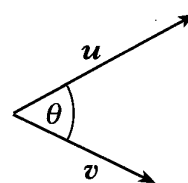
2

(*c*) Find the size of angle POQ.

5

2006 Paper 1

9. u and v are vectors given by $u = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $v = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.

(*a*) If $u.v = 1$, show that $k^3 + 3k^2 - k - 3 = 0$.

2

(*b*) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully.

5

(*c*) Deduce the only possible value of k.

(*d*) The angle between u and v is θ. Find the exact value of $\cos \theta$.

1

3

2006 Paper 2

6. P is the point (–1, 2, –1) and Q is (3, 2, –4).

   (*a*) Write down $\overrightarrow{PQ}$ in component form. **1**

   (*b*) Calculate the length of $\overrightarrow{PQ}$. **1**

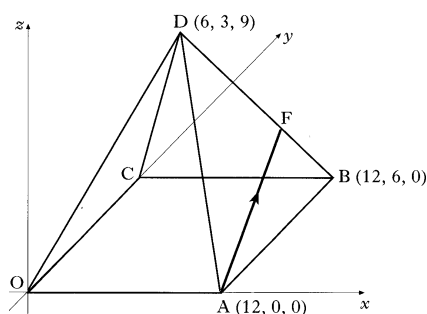   (*c*) Find the components of a unit vector which is parallel to $\overrightarrow{PQ}$. **1**

2005 Paper 1

3. D,OABC is a pyramid. A is the point (12, 0, 0), B is (12, 6, 0) and D is (6, 3, 9).

   F divides DB in the ratio 2:1.

   (*a*) Find the coordinates of the point F. **4**

   (*b*) Express $\overrightarrow{AF}$ in component form. **1**
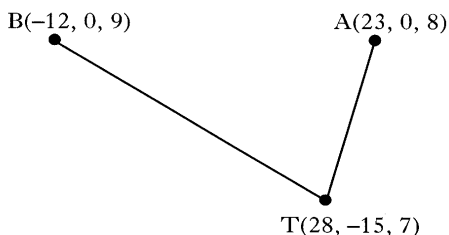

2005 Paper 2

4. The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops.

   Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(–12, 0, 9) and T(28, –15, 7).

   In the dark, Andrew and Bob locate Tracy using heat-seeking beams.

   (*a*) Express the vectors $\overrightarrow{TA}$ and $\overrightarrow{TB}$ in component form. **2**
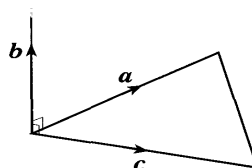
   (*b*) Calculate the angle between these two beams. **5**

2005 Paper 2

10. Vectors ***a*** and ***c*** are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram.

    Vector ***b*** is 2 units long and ***b*** is perpendicular to both ***a*** and ***c***.

    Evaluate the scalar product ***a***.(***a*** + ***b*** + ***c***). **4**

2004 Paper 1

5. A, B and C have coordinates (–3, 4, 7), (–1, 8, 3) and (0, 10, 1) respectively.

   (*a*) Show that A, B and C are collinear. **3**

   (*b*) Find the coordinates of D such that $\overrightarrow{AD} = 4\overrightarrow{AB}$. **2**

2004 Paper 2

2. P, Q and R have coordinates (1, 3, –1), (2, 0, 1) and (–3, 1, 2) respectively.

   (*a*) Express the vectors $\overrightarrow{QP}$ and $\overrightarrow{QR}$ in component form. **2**

   (*b*) Hence or otherwise find the size of angle PQR. **5**