# Recurrence Relations

## Go to the appropriate Past Paper for the answers

### 2019 Paper 1

**4.** A sequence is generated by the recurrence relation

$$u_{n+1} = mu_n + c,$$

where the first three terms of the sequence are 6, 9 and 11.

(a) Find the values of $m$ and $c$. **3**

(b) Hence, calculate the fourth term of the sequence. **1**

### 2019 Paper 2

**4.** In a forest, the population of a species of mouse is falling by 2·7% each year.

To increase the population scientists plan to release 30 mice into the forest at the end of March each year.

(a) $u_n$ is the estimated population of mice at the start of April, $n$ years after the population was first estimated.

It is known that $u_n$ and $u_{n+1}$ satisfy the recurrence relation $u_{n+1} = au_n + b$. **1**

State the values of $a$ and $b$.

The scientists continue to release this species of mouse each year.

(b) (i) Explain why the estimated population of mice will stabilise in the long term. **1**

(ii) Calculate the long term population to the nearest hundred. **2**

### 2018 Paper 2

**7.** (a) (i) Show that $(x-2)$ is a factor of $2x^3 - 3x^2 - 3x + 2$. **2**

(ii) Hence, factorise $2x^3 - 3x^2 - 3x + 2$ fully. **2**

The fifth term, $u_5$, of a sequence is $u_5 = 2a - 3$.

The terms of the sequence satisfy the recurrence relation $u_{n+1} = au_n - 1$.

(b) Show that $u_7 = 2a^3 - 3a^2 - a - 1$. **1**

For this sequence, it is known that

- $u_7 = u_5$
- a limit exists.

(c) (i) Determine the value of $a$. **3**

(ii) Calculate the limit. **1**

## Specimen 5 Paper 1

9. A sequence is generated by the recurrence relation $u_{n+1} = mu_n + 6$ where $m$ is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of $m$. **2**

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$. **1**

(ii) Calculate this limit. **2**

## 2017 Paper 2

8. Sequences may be generated by recurrence relations of the form $u_{n+1} = ku_n - 20$, $u_0 = 5$ where $k \in \mathbb{R}$.

(a) Show that $u_2 = 5k^2 - 20k - 20$. **2**

(b) Determine the range of values of $k$ for which $u_2 < u_0$. **4**

## 2016 Paper 1

3. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 10$ with $u_3 = 6$.

(a) Find the value of $u_4$. **1**

(b) Explain why this sequence approaches a limit as $n \to \infty$. **1**

(c) Calculate this limit. **2**

## New 2015 Paper 2

3. A version of the following problem first appeared in print in the 16th Century.

A frog and a toad fall to the bottom of a well that is 50 feet deep.
Each day, the frog climbs 32 feet and then rests overnight. During the night, it slides down $\frac{2}{3}$ of its height above the floor of the well.
The toad climbs 13 feet each day before resting.
Overnight, it slides down $\frac{1}{4}$ of its height above the floor of the well.

Their progress can be modelled by the recurrence relations:

- $f_{n+1} = \frac{1}{3}f_n + 32, \qquad f_1 = 32$
- $t_{n+1} = \frac{3}{4}t_n + 13, \qquad t_1 = 13$

where $f_n$ and $t_n$ are the heights reached by the frog and the toad at the end of the $n$th day after falling in.

(a) Calculate $t_2$, the height of the toad at the end of the second day. **1**

(b) Determine whether or not either of them will eventually escape from the well. **5**

## Specimen 4 Paper 2

2. A wildlife reserve has introduced conservation measures to build up the population of an endangered mammal. Initially the reserve population of the mammal was 2000. By the end of the first year there were 2500 and by the end of the second year there were 2980.

It is believed that the population can be modelled by the recurrence relation:

$$u_{n+1} = au_n + b,$$

where $a$ and $b$ are constants and $n$ is the number of years since the reserve was set up.

(a) Use the information above to find the values of $a$ and $b$. **4**

(b) Conservation measures will end if the population stabilises at over 13 000. Will this happen? Justify your answer. **3**

## Exemplar Paper 2

1. A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.

(a) Determine the values of $u_1$ and $u_2$. **1**

(b) A second sequence is given by 4, 5, 7, 11, . . . .

It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.

Find the values of $p$ and $q$. **3**

(c) Either the sequence in (a) or the sequence in (b) has a limit.

(i) Calculate this limit.

(ii) Why does this other sequence not have a limit? **5**

## 2014 Paper 1

1. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 1$, with $u_2 = 15$. **2**

What is the value of $u_4$?

## 2014 Paper 1

10. A sequence is defined by the recurrence relation

$u_{n+1} = (k-2)u_n + 5$ with $u_0 = 3$. **2**

For what values of $k$ does this sequence have a limit as $n \rightarrow \infty$?

## 2013 Paper 2

1. The first three terms of a sequence are 4, 7 and 16.

The sequence is generated by the recurrence relation **4**

$$u_{n+1} = mu_n + c, \text{ with } u_1 = 4.$$

Find the values of $m$ and $c$.

## 2012 Paper 1

1. A sequence is defined by the recurrence relation $u_{n+1} = 3u_n + 4$, with $u_0 = 1$.
   Find the value of $u_2$. **2**

## 2011 Paper 2

3. (*a*) A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.
   Write down the values of $u_1$ and $u_2$. **1**

   (*b*) A second sequence is given by 4, 5, 7, 11, . . . .
   It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.
   Find the values of $p$ and $q$. **3**

   (*c*) Either the sequence in (*a*) or the sequence in (*b*) has a limit.

   (i) Calculate this limit. **3**

   (ii) Why does the other sequence not have a limit?

## 2010 Paper 1

2. A sequence is defined by the recurrence relation $u_{n+1} = 2u_n + 3$ and $u_0 = 1$.
   What is the value of $u_2$? **2**

## 2010 Paper 1

7. A sequence is generated by the recurrence relation $u_{n+1} = \frac{1}{4}u_n + 7$, with $u_0 = -2$.
   What is the limit of this sequence as $n \to \infty$? **2**

## 2009 Paper 1

1. A sequence is defined by $u_{n+1} = 3u_n + 4$ with $u_1 = 2$.
   What is the value of $u_3$? **2**

## 2009 Paper 1

6. A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}7u_n + 10$.
   What is the limit of this sequence as $n \to \infty$? **2**

## 2008 Paper 1

1. A sequence is defined by the recurrence relation

$$u_{n+1} = 0{\cdot}3u_n + 6 \text{ with } u_{10} = 10.$$ **2**

What is the value of $u_{12}$?

## 2008 Paper 1

4. A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}4u_n - 240$.

   What is the limit of this sequence as $n \to \infty$? **2**

## 2007 Paper 1

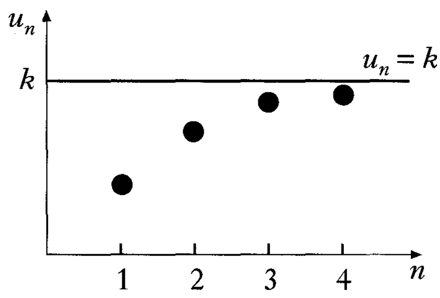
7. A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{4}u_n + 16, \; u_0 = 0.$$

   (*a*) Calculate the values of $u_1$, $u_2$ and $u_3$. **3**

   Four terms of this sequence, $u_1$, $u_2$, $u_3$ and $u_4$ are plotted as shown in the graph.

   As $n \to \infty$, the points on the graph approach the line $u_n = k$, where $k$ is the limit of this sequence.




   (*b*) (i) Give a reason why this sequence has a limit.

   (ii) Find the exact value of $k$. **3**

## 2006 Paper 1

4. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}8u_n + 12$, $u_0 = 4$.

   (*a*) State why this sequence has a limit. **1**

   (*b*) Find this limit. **2**

## 2005 Paper 1

6. (*a*) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of $k$ which produces a sequence with a limit of 4. **2**

   (*b*) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$.

   (i) Express $u_1$ and $u_2$ in terms of $m$.

   (ii) Given that $u_2 = 7$, find the value of $m$ which produces a sequence with no limit. **5**

## 2004 Paper 2

4. A sequence is defined by the recurrence relation $u_{n+1} = ku_n + 3$.

   (*a*) Write down the condition on $k$ for this sequence to have a limit. **1**

   (*b*) The sequence tends to a limit of 5 as $n \to \infty$. Determine the value of $k$. **3**