Polynomials

Go to the appropriate Past Paper for the answers

2019 Paper 2

10. (a) Show that $(x+3)$ is a factor of $3x^4 + 10x^3 + x^2 - 8x - 6$.

    2

    (b) Hence, or otherwise, factorise $3x^4 + 10x^3 + x^2 - 8x - 6$ fully.

    5

Specimen 5 Paper 1

8. For the polynomial, $x^3 - 4x^2 + ax + b$

    - $x - 1$ is a factor

    - -12 is the remainder when it is divided by $x - 2$

    (a) Determine the values of a and b.

    5

    (b) Hence solve $x^3 - 4x^2 + ax + b = 0$.

    3

Specimen 5 Paper 2

3. The diagram shows the curve with equation $y = f(x)$, where $f(x) = kx(x+a)(x+b)$.

   The curve passes through $(-1, 0)$, $(0, 0)$, $(1, 2)$ and $(2, 0)$.
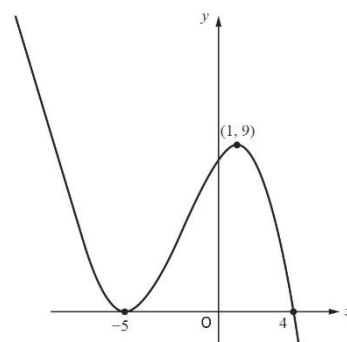
   Find the values of a, b and k.
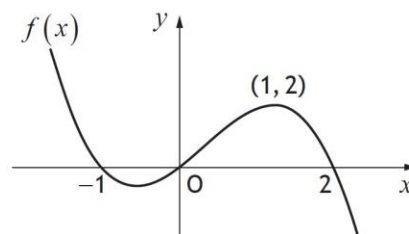
   3

2017 Paper 2

2. (a) Show that $(x-1)$ is a factor of $f(x) = 2x^3 - 5x^2 + x + 2$.

    2

    (b) Hence, or otherwise, solve $f(x) = 0$.

    3

2016 Paper 1

15. The diagram below shows the graph with equation $y = f(x)$, where $f(x) = k(x-a)(x-b)^2$.

    (a) Find the values of a, b and k.

    3

    (b) For the function $g(x) = f(x) - d$, where d is positive, determine the range of values of d for which $g(x)$ has exactly one real root.

    1

2016 Paper 2

3. (a) (i) Show that $(x+1)$ is a factor of $2x^3 - 9x^2 + 3x + 14$. **2**

(ii) Hence solve the equation $2x^3 - 9x^2 + 3x + 14 = 0$. **3**

New 2015 Paper 1

3. Show that $(x + 3)$ is a factor of $x^3 - 3x^2 - 10x + 24$ and hence factorise $x^3 - 3x^2 - 10x + 24$ fully. **3**

Specimen 4 Paper 1

2. Find the coordinates of the points of intersection of the curve $y = x^3 - 2x^2 + x + 4$ and the line $y = 4x + 4$. **5**

Specimen 4 Paper 1

7. (a) Show that $(x + 1)$ is a factor of $x^3 - 13x - 12$. **3**

(b) Factorise $x^3 - 13x - 12$ fully. **2**

Exemplar Paper 1

5. For the polynomial, $x^3 - 4x^2 + ax + b$

- $x - 1$ is a factor

- -12 is the remainder when it is divided by $x - 2$

(a) Determine the values of a and b. **4**

(b) Hence solve $x^3 - 4x^2 + ax + b = 0$. **4**

Exemplar Paper 1

2. The diagram shows the curve with equation $y = f(x)$, where $f(x) = kx(x + a)(x + b)$.

The curve passes through (-1,0), (0,0), (1,2) and (2,0).

Find the values of a, b and k. **3**
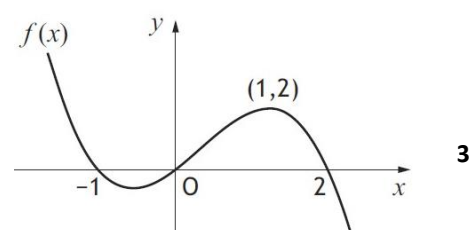
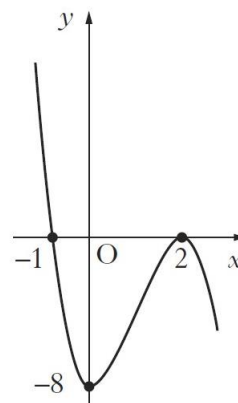## 2014 Paper 1

22. For the polynomial $6x^3 + 7x^2 + ax + b$,

- $x + 1$ is a factor
- 72 is the remainder when it is divided by $x - 2$.

(a) Determine the values of a and b. **4**

(b) Hence factorise the polynomial completely. **2**

2014 Paper 1

15. The diagram shows a cubic curve passing through $(-1, 0)$, $(2, 0)$ and $(0, -8)$.

    What is the equation of the curve?
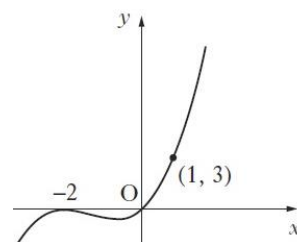
    2

2013 Paper 1

6. What is the remainder when $x^3 + 3x^2 - 5x - 6$ is divided by $(x - 2)$?

    2

2013 Paper 1

17. The diagram shows a curve with equation of the form $y = kx(x + a)^2$, which passes through the points $(-2, 0)$, $(0, 0)$ and $(1, 3)$.

    What are the values of a and k?

    2

2013 Paper 2

3. (a) Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully.

    4

2012 Paper 1

13. A parabola intersects the axes at $x = -2$, $x = -1$ and $y = 6$, as shown in the diagram.

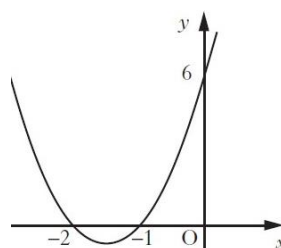   What is the equation of the parabola?

    A $y = 6(x - 1)(x - 2)$

    B $y = 6(x + 1)(x + 2)$

    C $y = 3(x - 1)(x - 2)$

    D $y = 3(x + 1)(x + 2)$

    2

2011 Paper 1

7. A function f is defined on the set of real numbers by $f(x) = x^3 - x^2 + x + 3$.
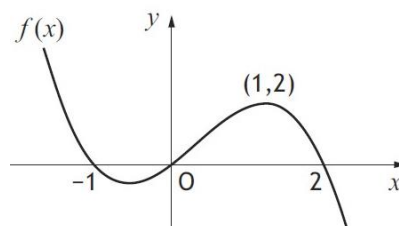
    What is the remainder when $f(x)$ is divided by $(x - 1)$?

    2

2011 Paper 1

17. The diagram shows the graph of a cubic.

    What is the equation of this cubic?

    2

2011 Paper 2

2. Functions f, g and h are defined on the set of real numbers by

    - $f(x) = x^3 - 1$
    - $g(x) = 3x + 1$
    - $h(x) = 4x - 5$.

    (a) Find $g(f(x))$.

    2

    (b) Show that $g(f(x)) + xh(x) = 3x^3 + 4x^2 - 5x - 2$.

    1

    (c) (i) Show that $(x - 1)$ is a factor of $3x^3 + 4x^2 - 5x - 2$.

    (ii) Factorise $3x^3 + 4x^2 - 5x - 2$ fully.
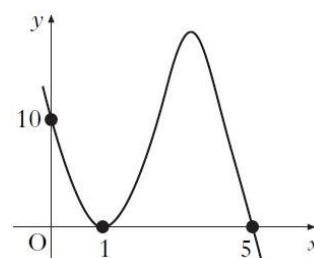
    5

    (d) Hence solve $g(f(x)) + xh(x) = 0$.

    1

2010 Paper 1

16. The diagram shows the graph with equation $y = k(x - 1)^2(x + t)$.

    What are the values of k and t?

    2

2010 Paper 1

22. (a) (i) Show that $(x - 1)$ is a factor of $f(x) = 2x^3 + x^2 - 8x + 5$.

    (ii) Hence factorise $f(x)$ fully.

    5

    (b) Solve $2x^3 + x^2 - 8x + 5 = 0$.

    1

    (c) The line with equation $y = 2x - 3$ is a tangent to the curve with equation $y = 2x^3 + x^2 - 6x + 2$ at the point G.

    Find the coordinates of G.

    5

    (d) This tangent meets the curve again at the point H.

    Write down the coordinates of H.

    1

2008 Paper 1

21. A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

    (*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature.   **6**

    (*b*)   (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

        (ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully.   **5**

    (*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve.   **4**

2007 Paper 2

10. The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.

Both graphs pass through the point (0, 6).

The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).
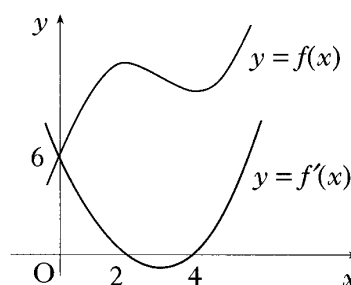
    (*a*) Given that $f'(x)$ is of the form $k(x - a)(x - b)$:

        (i) write down the values of a and b;

        (ii) find the value of k.   **3**

2005 Paper 1

8. A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

    (*a*) Show that $(x - 3)$ is a factor of $f(x)$, and hence factorise $f(x)$ fully.   **5**

    (*b*) Find the coordinates of the points where the curve with equation $y = f(x)$ crosses the x- and y-axes.   **2**

    (*c*) Find the greatest and least values of f in the interval $-2 \le x \le 2$.   **5**

2005 Paper 2

11. (*a*) Show that $x = -1$ is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$.   **1**

    (*b*) Hence find the range of values of p for which all the roots of the cubic equation are real.   **7**

2004 Paper 1

2. $f(x) = x^3 - x^2 - 5x - 3$.

    (*a*)   (i) Show that $(x + 1)$ is a factor of $f(x)$.   **5**

        (ii) Hence or otherwise factorise $f(x)$ fully.

    (*b*) One of the turning points of the graph of $y = f(x)$ lies on the x-axis. Write down the coordinates of this turning point.   **1**