

Integration

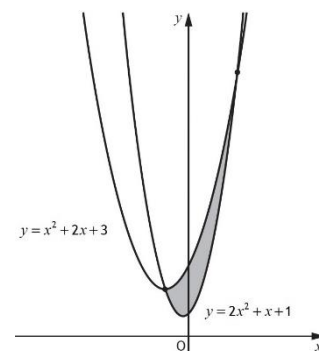
Go to the appropriate Past Paper for the answers

2019 Paper 1

8. The graphs of $y = x^2 + 2x + 3$ and $y = 2x^2 + x + 1$ are shown below.

The graphs intersect at the points where $x = -1$ and $x = 2$.

- (a) Express the shaded area, enclosed between the curves, as an integral.
(b) Evaluate the shaded area.



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2019 Paper 2

2. Find $\int (6\sqrt{x} - 4x^{-3} + 5) dx$.

4

2019 Paper 2

13. For a function, f , defined on the set of real numbers, \mathbb{R} , it is known that

- the rate of change of f with respect to x is given by $3x^2 - 16x + 11$
- the graph with equation $y = f(x)$ crosses the x -axis at $(7, 0)$.

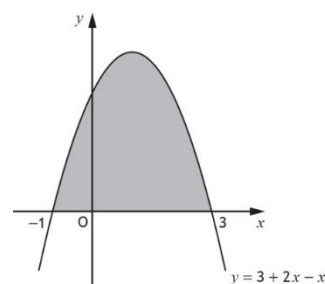
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Express $f(x)$ in terms of x .

2018 Paper 2

1. The diagram shows the curve with equation $y = 3 + 2x - x^2$.

Calculate the shaded area.



4

Specimen 5 Paper 1

4. Evaluate $\int_1^2 \frac{1}{6}x^{-2} dx$.

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Specimen 5 Paper 2

2. Find $\int \frac{4x^3 + 1}{x^2} dx, x \neq 0$.

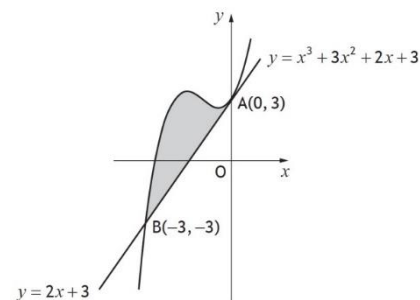
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Specimen 5 Paper 2

5. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at $A(0, 3)$, as shown.

The line meets the curve again at $B(-3, -3)$.

Find the area enclosed by the line and the curve.

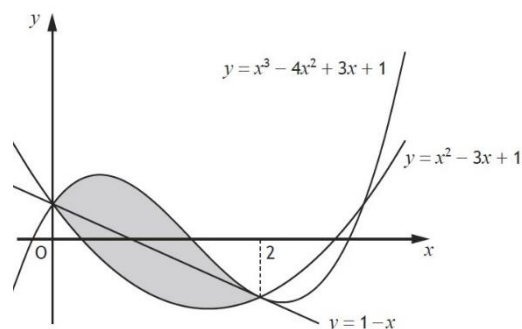


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2017 Paper 1

10. Two curves with equations $y = x^3 - 4x^2 + 3x + 1$ and $y = x^2 - 3x + 1$ intersect as shown in the diagram.

- (a) Calculate the shaded area.



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The line passing through the points of intersection of the curves has equation $y = 1 - x$.

- (b) Determine the fraction of the shaded area which lies below the line $y = 1 - x$.

4

2017 Paper 1

15. A quadratic function, f , is defined on \mathbb{R} , the set of real numbers.

Diagram 1 shows part of the graph with equation $y = f(x)$.

The turning point is $(2, 3)$.

Diagram 2 shows part of the graph with equation $y = h(x)$.

The turning point is $(7, 6)$.

- (a) Given that $h(x) = f(x+a) + b$.
Write down the values of a and b .

- (b) It is known that $\int_1^3 f(x) dx = 4$.
Determine the value of $\int_6^8 h(x) dx$.

- (c) Given $f'(1) = 6$, state the value of $h'(8)$.

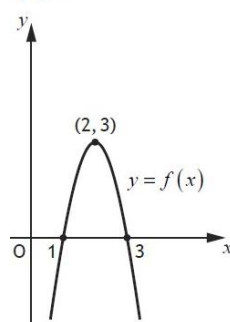


Diagram 1

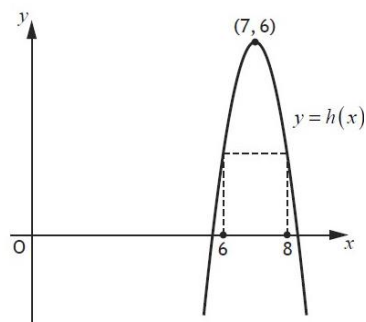


Diagram 2

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2016 Paper 2

9. For a function f , defined on a suitable domain, it is known that:

$$f'(x) = \frac{2x+1}{\sqrt{x}}$$

$$f(9) = 40$$

4

Express $f(x)$ in terms of x .

2016 Paper 2

3. (a) (i) Show that $(x+1)$ is a factor of $2x^3 - 9x^2 + 3x + 14$.

2

(ii) Hence solve the equation $2x^3 - 9x^2 + 3x + 14 = 0$.

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(b) The diagram below shows the graph with equation $y = 2x^3 - 9x^2 + 3x + 14$.

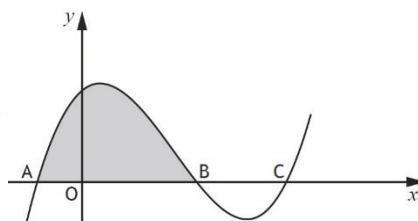
The curve cuts the x -axis at A, B and C.

(i) Write down the coordinates of the points A and B.

1

(ii) Hence calculate the shaded area in the diagram.

4



New 2015 Paper 2

4. A wall plaque is to be made to commemorate the 150th anniversary of the publication of "*Alice's Adventures in Wonderland*".

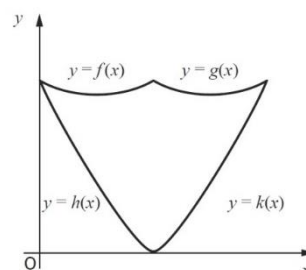
The edges of the wall plaque can be modelled by parts of the graphs of four quadratic functions as shown in the sketch.

- $f(x) = \frac{1}{4}x^2 - \frac{1}{2}x + 3$

- $g(x) = \frac{1}{4}x^2 - \frac{3}{2}x + 5$

- $h(x) = \frac{3}{8}x^2 - \frac{9}{4}x + 3$

- $k(x) = \frac{3}{8}x^2 - \frac{3}{4}x$



(a) Find the x -coordinate of the point of intersection of the graphs with equations $y = f(x)$ and $y = g(x)$.

2

The graphs of the functions $f(x)$ and $h(x)$ intersect on the y -axis.

The plaque has a vertical line of symmetry.

(b) Calculate the area of the wall plaque.

7

2015 Paper 1

15. The rate of change of the temperature, T °C of a mug of coffee is given by

$$\frac{dT}{dt} = \frac{1}{25}t - k, \quad 0 \leq t \leq 50$$

- t is the elapsed time, in minutes, after the coffee is poured into the mug
- k is a constant
- initially, the temperature of the coffee is 100 °C
- 10 minutes later the temperature has fallen to 82 °C.

6

Express T in terms of t .

Specimen 4 Paper 2

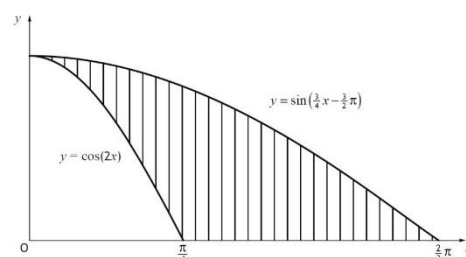
9. A sea-life visitor attraction has a new logo in the shape of a shark fin.

The outline of the logo can be represented by parts of

- the x axis
- the curve with equation $y = \cos(2x)$
- the curve with equation $y = \sin\left(\frac{3}{4}x - \frac{3}{2}\pi\right)$

as shown in the diagram.

Calculate the shaded area.



6

2014 Paper 2

7. Land enclosed between a path and a railway line is being developed for housing.

This land is represented by the shaded area shown in Diagram 1.

- The path is represented by a parabola with equation $y = 6x - x^2$.
- The railway is represented by a line with equation $y = 2x$.
- One square unit in the diagram represents 300 m^2 of land.

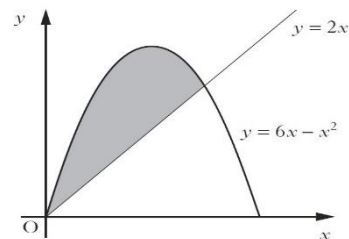


Diagram 1

- (a) Calculate the area of land being developed.

- (b) A road is built parallel to the railway line and is a tangent to the path as shown in Diagram 2.

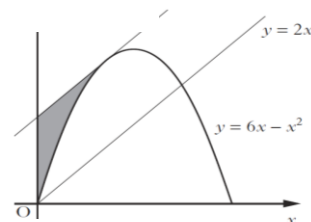


Diagram 2

It is decided that the land, represented by the shaded area in Diagram 2, will become a car park.

Calculate the area of the car park.

2013 Paper 1

7. Find $\int x(3x + 2) dx$.

2012 Paper 1

2012 Paper 1

21. (a) (i) Show that $(x - 4)$ is a factor of $x^3 - 5x^2 + 2x + 8$.

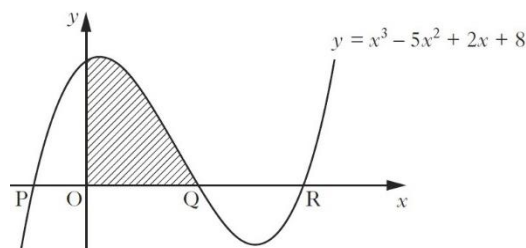
- (ii) Factorise $x^3 - 5x^2 + 2x + 8$ fully.

- (iii) Solve $x^3 - 5x^2 + 2x + 8 = 0$.

- (b) The diagram shows the curve with equation $y = x^3 - 5x^2 + 2x + 8$.

The curve crosses the x -axis at P, Q and R.

Determine the shaded area.



2011 Paper 1

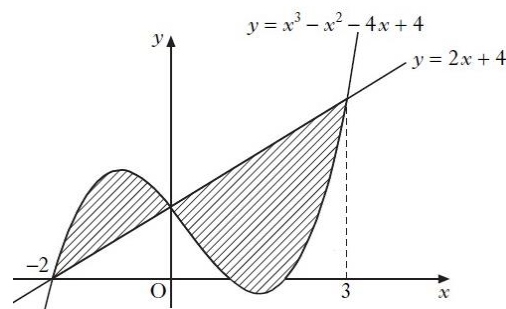
11. Find $\int \left(\frac{1}{6x^2} \right) dx, x \neq 0$.

2011 Paper 2

4. The diagram shows the curve with equation $y = x^3 - x^2 - 4x + 4$ and the line with equation $y = 2x + 4$.

The curve and the line intersect at the points $(-2, 0)$, $(0, 4)$ and $(3, 10)$.

Calculate the total shaded area.

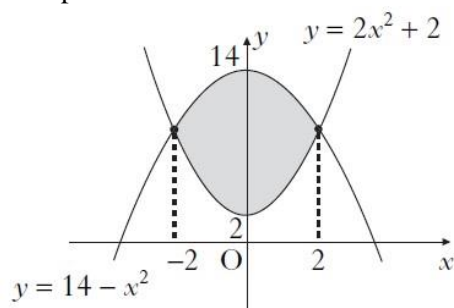


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2010 Paper 1

14. The diagram shows graphs with equations $y = 14 - x^2$ and $y = 2x^2 + 2$.

Write a definite integral to represent the shaded area.

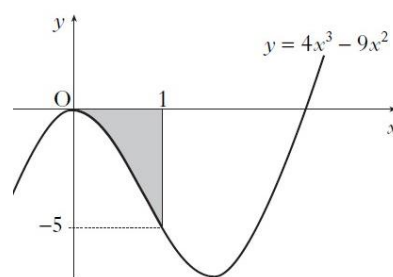


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2009 Paper 1

16. The graph of $y = 4x^3 - 9x^2$ is shown in the diagram.

Write a definite integral to represent the shaded area.



2

2008 Paper 1

14. Find $\int 4 \sin(2x + 3) dx$.

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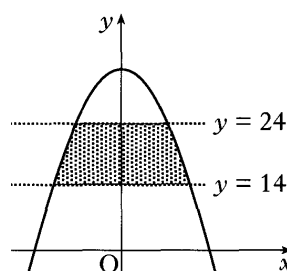
2008 Paper 2

7. The parabola shown in the diagram has equation

$$y = 32 - 2x^2.$$

The shaded area lies between the lines $y = 14$ and $y = 24$.

Calculate the shaded area.

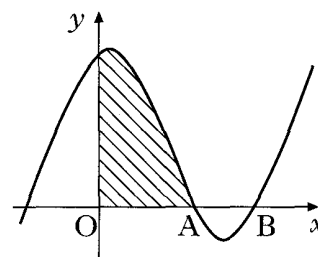


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2007 Paper 1

8. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.

- (a) Show that the graph cuts the x -axis at $(3, 0)$.
 (b) Hence or otherwise find the coordinates of A.
 (c) Find the shaded area.



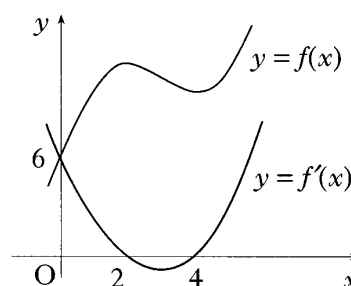
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2007 Paper 2

10. The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.

Both graphs pass through the point $(0, 6)$.

The graph of $y = f'(x)$ also passes through the points $(2, 0)$ and $(4, 0)$.



- (a) Given that $f'(x)$ is of the form $k(x - a)(x - b)$:
 (i) write down the values of a and b ;
 (ii) find the value of k .
 (b) Find the equation of the graph of the cubic function $y = f(x)$.

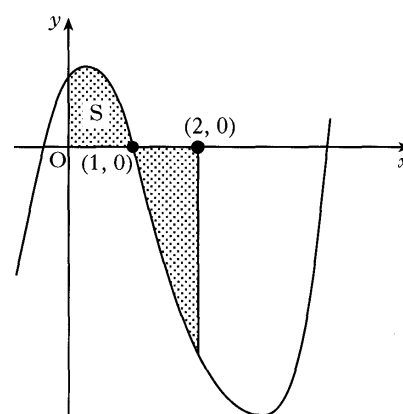
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2006 Paper 1

6. The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.

The total shaded area is bounded by the curve, the x -axis, the y -axis and the line $x = 2$.

- (a) Calculate the shaded area labelled S.
 (b) Hence find the total shaded area.



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2006 Paper 2

5. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x .

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2005 Paper 2

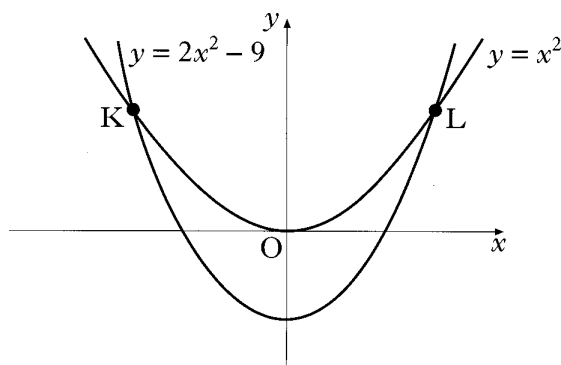
1. Find $\int \frac{4x^3 - 1}{x^2} dx$, $x \neq 0$.

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2005 Paper 2

5. The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves.



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2004 Paper 1

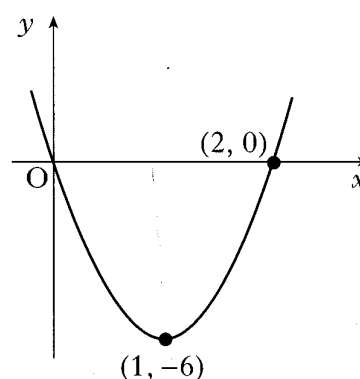
11. The diagram shows a parabola passing through the points $(0, 0)$, $(1, -6)$ and $(2, 0)$.

(a) The equation of the parabola is of the form $y = ax(x - b)$.

Find the values of a and b .

(b) This parabola is the graph of $y = f'(x)$.

Given that $f(1) = 4$, find the formula for $f(x)$.



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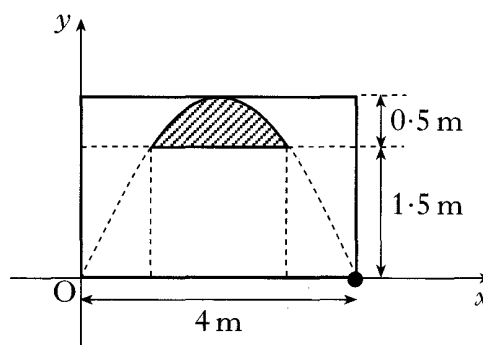
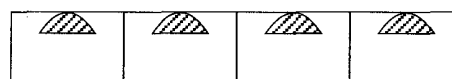
2004 Paper 2

11. An architectural feature of a building is a wall with arched windows. The curved edge of each window is parabolic.

The second diagram shows one such window. The shaded part represents the glass.

The top edge of the window is part of the parabola with equation $y = 2x - \frac{1}{2}x^2$.

Find the area in square metres of the glass in one window.



8