

# Exponential & Logarithms

Go to the appropriate Past Paper for the answers

## 2019 Paper 1

14. (a) Evaluate  $\log_{10} 4 + 2\log_{10} 5$ .

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(b) Solve  $\log_2(7x-2) - \log_2 3 = 5$ ,  $x \geq 1$ .

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## 2019 Paper 2

9. Electricity on a spacecraft can be produced by a type of nuclear generator.  
The electrical power produced by this generator can be modelled by

$$P_t = 120e^{-0.0079t}$$

where  $P_t$  is the electrical power produced, in watts, after  $t$  years.

- (a) Determine the electrical power initially produced by the generator.  
(b) Calculate how long it takes for the electrical power produced by the generator to reduce by 15%.

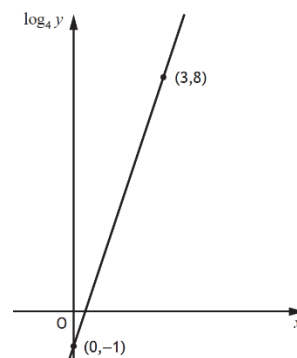
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## 2019 Paper 2

12. Two variables,  $x$  and  $y$ , are connected by the equation  $y = ab^x$ .  
The graph of  $\log_4 y$  against  $x$  is a straight line as shown.  
Find the values of  $a$  and  $b$ .

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## 2018 Paper 1

6. Find the value of  $\log_5 250 - \frac{1}{3}\log_5 8$ .

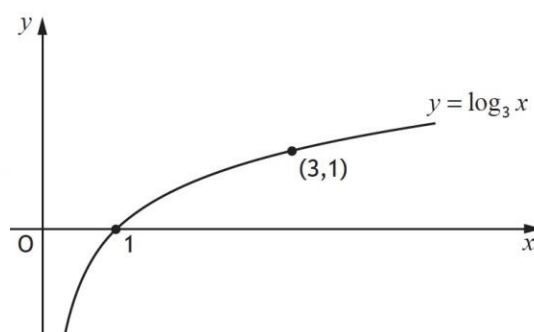
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## 2018 Paper 1

11. The diagram shows the curve with equation  $y = \log_3 x$ .  
(a) sketch the curve with equation  $y = 1 - \log_3 x$ .  
(b) Determine the exact value of the  $x$ -coordinate of the point of intersection of the two curves.

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## 2018 Paper 2

11. A supermarket has been investigating how long customers have to wait at the checkout. During any half hour period, the percentage,  $P\%$ , of customers who wait for less than  $t$  minutes, can be modelled by

$$P = 100(1 - e^{-kt}), \text{ where } k \text{ is a constant.}$$

- (a) If 50% of customers wait for less than 3 minutes, determine the value of  $k$ .  
(b) Calculate the percentage of customers who wait for 5 minutes or longer.

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## Specimen 5 Paper 1

10. (a) Evaluate  $\log_5 25$ .

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- (b) Hence solve  $\log_4 x + \log_4 (x - 6) = \log_5 25$ , where  $x > 6$ .

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## 2017 Paper 1

12. Given that  $\log_a 36 - \log_a 4 = \frac{1}{2}$ , find the value of  $a$ .

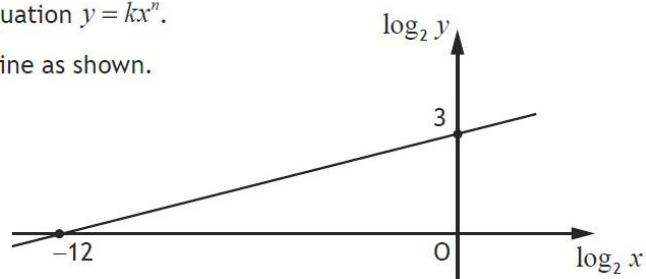
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## 2017 Paper 2

9. Two variables,  $x$  and  $y$ , are connected by the equation  $y = kx^n$ .

The graph of  $\log_2 y$  against  $\log_2 x$  is a straight line as shown.

Find the values of  $k$  and  $n$ .



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## 2016 Paper 1

14. (a) Evaluate  $\log_5 25$ .

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- (b) Hence solve  $\log_4 x + \log_4 (x - 6) = \log_5 25$ , where  $x > 6$ .

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## 2016 Paper 2

6. Scientists are studying the growth of a strain of bacteria. The number of bacteria present is given by the formula

$$B(t) = 200e^{0.107t},$$

where  $t$  represents the number of hours since the study began.

- (a) State the number of bacteria present at the start of the study.  
(b) Calculate the time taken for the number of bacteria to double.

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## New 2015 Paper 1

6. Evaluate  $\log_6 12 + \frac{1}{3} \log_6 27$ .

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## Specimen 4 Paper 2

7. Given that  $P(t) = 30e^{t-2}$  decide whether each of the statements below is true or false. Justify your answers.

Statement A  $P(0) = 30$ .

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Statement B When  $P(t) = 15$ , the only possible value of  $t$  is 1.3 to one decimal place.

## Specimen 4 Paper 2

4. (a) Express  $y = \log_4 2x$  in the form  $y = \log_4 x + k$ , clearly stating the value of  $k$ .

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- (b) Hence, or otherwise, describe the relationship between the graphs of  $y = \log_4 2x$  and  $y = \log_4 x$ .

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- (c) Determine the coordinates of the point where the graph of  $y = \log_4 2x$  intersects the  $x$ -axis.

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- (d) Sketch and annotate the graph of  $y = f^{-1}(x)$ , where  $f(x) = \log_4 2x$ .

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## Exemplar Paper 2

7. The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation.

where:  $P_t = P_0 e^{-kt}$

- $P_0$  is the initial concentration;
- $P_t$  is the concentration at time  $t$ ;
- $t$  is the time, in days, after the application of the pesticide.

Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

- (a) If the half-life of *Xpesto* is 25 days, find the value of  $k$  to 2 significant figures.

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On all *Xpesto* packaging, the manufacturer states that 80 days after application the concentration of *Xpesto* in the soil will have decreased by over 90%.

- (b) Is this statement correct? Justify your answer.

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## 2014 Paper 1

3. If  $\log_4 12 - \log_4 x = \log_4 6$ , what is the value of  $x$ ?

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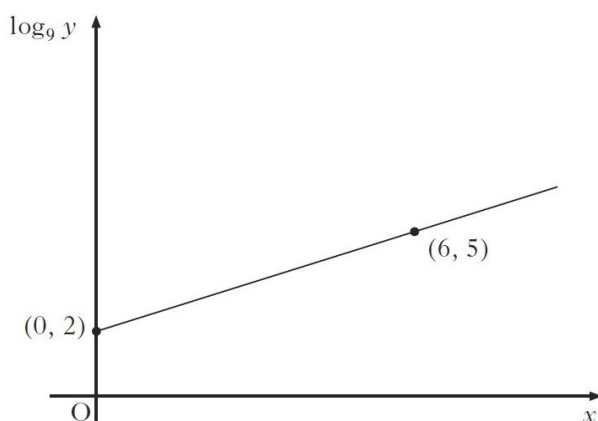
## 2014 Paper 1

20. Evaluate  $2 - \log_5 \frac{1}{25}$ .

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## 2014 Paper 1

24. Two variables,  $x$  and  $y$ , are related by the equation  $y = ka^x$ .



When  $\log_9 y$  is plotted against  $x$ , a straight line passing through the points  $(0, 2)$  and  $(6, 5)$  is obtained, as shown in the diagram.

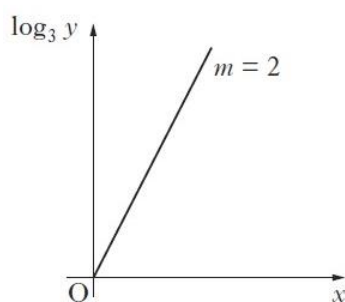
Find the values of  $k$  and  $a$ .

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## 2013 Paper 1

20. The graph of  $\log_3 y$  plotted against  $x$  is a line through the origin with gradient 2, as shown.

Express  $y$  in terms of  $x$ .



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## 2013 Paper 2

5. Solve the equation

$$\log_5(3 - 2x) + \log_5(2 + x) = 1, \text{ where } x \text{ is a real number.}$$

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## 2012 Paper 1

20. Simplify  $\frac{\log_b 9a^2}{\log_b 3a}$ , where  $a > 0$  and  $b > 0$ .

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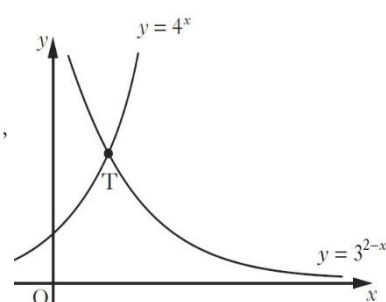
## 2012 Paper 2

7. The diagram shows the curves with equations  $y = 4^x$  and  $y = 3^{2-x}$ .

The graphs intersect at the point T.

- (a) Show that the  $x$ -coordinate of T can be written in the form  $\frac{\log_a p}{\log_a q}$ , for all  $a > 1$ .

- (b) Calculate the  $y$ -coordinate of T.

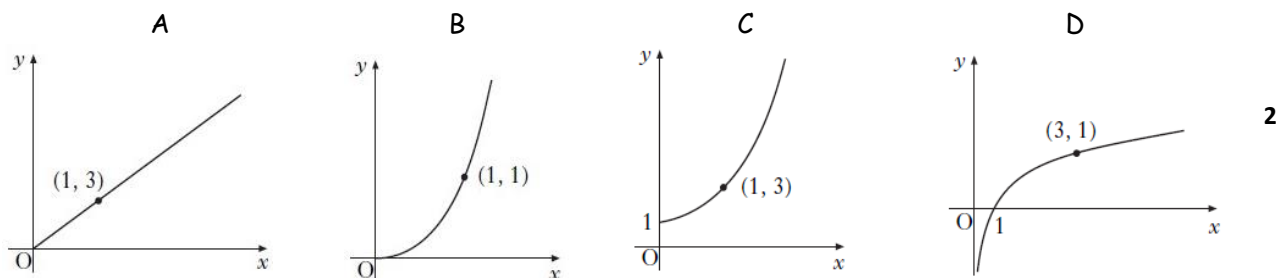


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## 2011 Paper 1

19. Which of the following diagrams represents the graph with equation  $\log_3 y = x$ ?

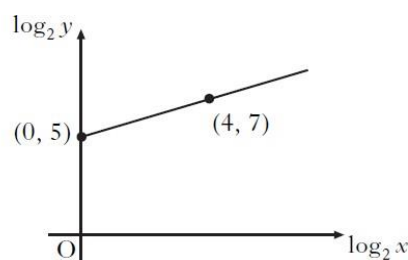


## 2011 Paper 2

5. Variables  $x$  and  $y$  are related by the equation  $y = kx^n$ .

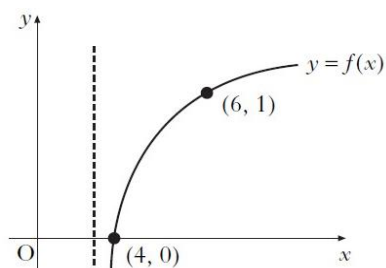
The graph of  $\log_2 y$  against  $\log_2 x$  is a straight line through the points  $(0, 5)$  and  $(4, 7)$ , as shown in the diagram.

Find the values of  $k$  and  $n$ .



## 2010 Paper 1

19. The diagram shows the graph of  $y = f(x)$  where  $f$  is a logarithmic function. What is  $f(x)$ ?



## 2010 Paper 2

7. (a) Given that  $\log_4 x = P$ , show that  $\log_{16} x = \frac{1}{2}P$ .

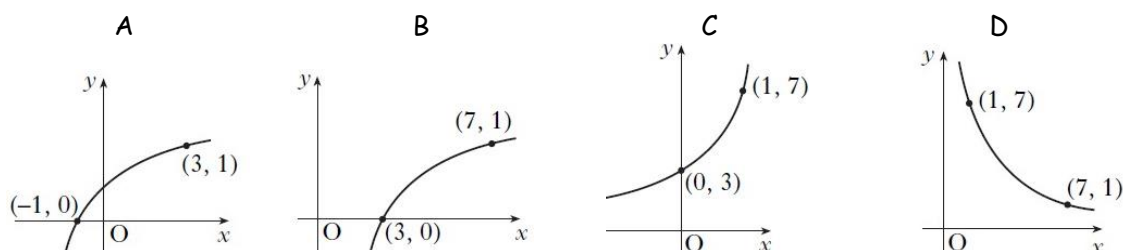
- (b) Solve  $\log_3 x + \log_9 x = 12$ .

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## 2009 Paper 1

10. Which of the following graphs has equation  $y = \log_5(x - 2)$ ?



## 2009 Paper 2

3. (a) (i) Show that  $x = 1$  is a root of  $x^3 + 8x^2 + 11x - 20 = 0$ .  
 (ii) Hence factorise  $x^3 + 8x^2 + 11x - 20$  fully.  
 (b) Solve  $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$ .

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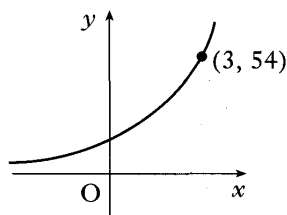
## 2009 Paper 2

6. The size of the human population,  $N$ , can be modelled using the equation  $N = N_0 e^{rt}$  where  $N_0$  is the population in 2006,  $t$  is the time in years since 2006, and  $r$  is the annual rate of increase in the population.
- (a) In 2006 the population of the United Kingdom was approximately 61 million, with an annual rate of increase of 1.6%. Assuming this growth rate remains constant, what would be the population in 2020?
- (b) In 2006 the population of Scotland was approximately 5.1 million, with an annual rate of increase of 0.43%.  
 Assuming this growth rate remains constant, how long would it take for Scotland's population to double in size?

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## 2008 Paper 1

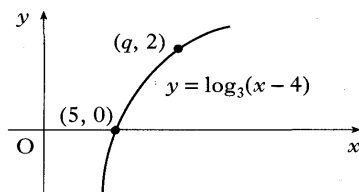
19. The diagram shows part of the graph whose equation is of the form  $y = 2m^x$ .  
 What is the value of  $m$ ?



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## 2008 Paper 1

20. The diagram shows part of the graph of  $y = \log_3(x - 4)$ .  
 The point  $(q, 2)$  lies on the graph.



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What is the value of  $q$ ?

## 2008 Paper 1

23. Functions  $f$ ,  $g$  and  $h$  are defined on suitable domains by

$$f(x) = x^2 - x + 10, \quad g(x) = 5 - x \quad \text{and} \quad h(x) = \log_2 x.$$

- (a) Find expressions for  $h(f(x))$  and  $h(g(x))$ .  
 (b) Hence solve  $h(f(x)) - h(g(x)) = 3$ .

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## 2007 Paper 2

8. The curve with equation  $y = \log_3(x - 1) - 2.2$ , where  $x > 1$ , cuts the  $x$ -axis at the point  $(a, 0)$ .

Find the value of  $a$ .

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## 2007 Paper 2

11. Two variables  $x$  and  $y$  satisfy the equation  $y = 3 \times 4^x$ .

(a) Find the value of  $a$  if  $(a, 6)$  lies on the graph with equation  $y = 3 \times 4^x$ .

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(b) If  $(-\frac{1}{2}, b)$  also lies on the graph, find  $b$ .

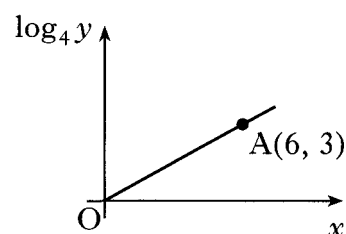
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(c) A graph is drawn of  $\log_{10}y$  against  $x$ . Show that its equation will be of the form  $\log_{10}y = Px + Q$  and state the gradient of this line.

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## 2006 Paper 1

10. Two variables,  $x$  and  $y$ , are connected by the law  $y = a^x$ . The graph of  $\log_4 y$  against  $x$  is a straight line passing through the origin and the point A(6, 3). Find the value of  $a$ .



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## 2006 Paper 2

11. It is claimed that a wheel is made from wood which is over 1000 years old.

To test this claim, carbon dating is used.

The formula  $A(t) = A_0 e^{-0.000124t}$  is used to determine the age of the wood, where  $A_0$  is the amount of carbon in any living tree,  $A(t)$  is the amount of carbon in the wood being dated and  $t$  is the age of the wood in years.

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For the wheel it was found that  $A(t)$  was 88% of the amount of carbon in a living tree.

Is the claim true?

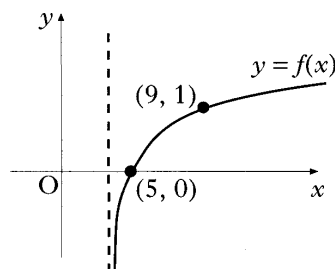
## 2005 Paper 1

7. The function  $f$  is of the form  $f(x) = \log_b(x - a)$ .

The graph of  $y = f(x)$  is shown in the diagram.

(a) Write down the values of  $a$  and  $b$ .

(b) State the domain of  $f$ .



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## 2005 Paper 2

7. Solve the equation  $\log_4(5 - x) - \log_4(3 - x) = 2$ ,  $x < 3$ .

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## 2005 Paper 2

9. The value  $V$  (in £ million) of a cruise ship  $t$  years after launch is given by the formula  $V = 252e^{-0.06335t}$ .

(a) What was its value when launched?

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(b) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold?

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## 2004 Paper 1

9. Solve the equation  $\log_2(x + 1) - 2\log_2(3) = 3$ .

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## 2004 Paper 2

10. The amount  $A_t$  micrograms of a certain radioactive substance remaining after  $t$  years decreases according to the formula  $A_t = A_0e^{-0.002t}$ , where  $A_0$  is the amount present initially.

(a) If 600 micrograms are left after 1000 years, how many micrograms were present initially?

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(b) The half-life of a substance is the time taken for the amount to decrease to half of its initial amount. What is the half-life of this substance?

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