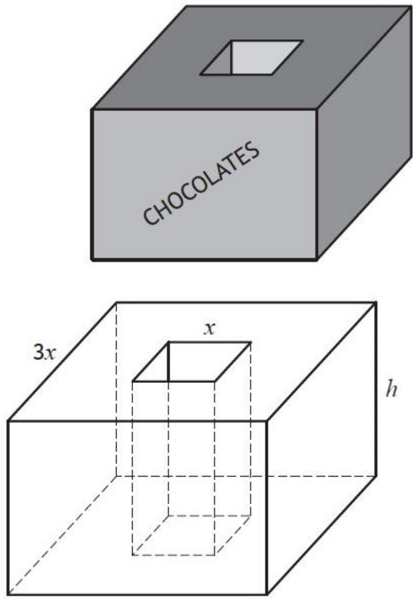
Differentiation

Go to the appropriate Past Paper for the answers

2019 Paper 1



The box is a cuboid with a cuboid shaped tunnel through it.

- The height of the box is h centimetres
- The top of the box is a square of side $3x$ centimetres
- The end of the tunnel is a square of side x centimetres
- The volume of the box is $2000\ \text{cm}^3$

(a) Show that the total surface area, $A\ \text{cm}^2$, of the box is given by 3

$$A = 16x^2 + \frac{4000}{x}.$$

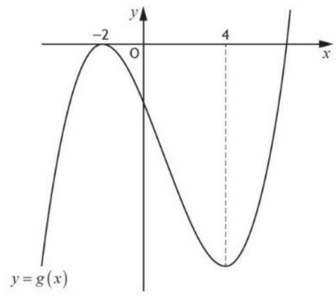
(b) To minimise the cost of production, the surface area, A, of the box should be as small as possible. 6

Find the minimum value of A.

2019 Paper 2

5. The diagram below shows the graph of a cubic function $y = g(x)$, with stationary points at $x = -2$ and $x = 4$.

sketch the graph of $y = g'(x)$. 2



2019 Paper 1

1. Find the x-coordinates of the stationary points on the curve with equation $y = \frac{1}{2}x^4 - 2x^3 + 6$. 4

2018 Paper 2

9. A sector with a particular fixed area has radius x cm.

The perimeter, P cm, of the sector is given by 6

$$P = 2x + \frac{128}{x}.$$

Find the minimum value of P.

2018 Paper 1

15. A cubic function, f, is defined on the set of real numbers.

- $(x+4)$ is a factor of $f(x)$
- $x=2$ is a repeated root of $f(x)$
- $f'(-2)=0$
- $f'(x)>0$ where the graph with equation $y=f(x)$ crosses the y-axis

Sketch a possible graph of $y=f(x)$ **4**

2018 Paper 1

10. Given that

- $\frac{dy}{dx}=6x^2-3x+4$, and
- $y=14$ when $x=2$,

express y in terms of x. **4**

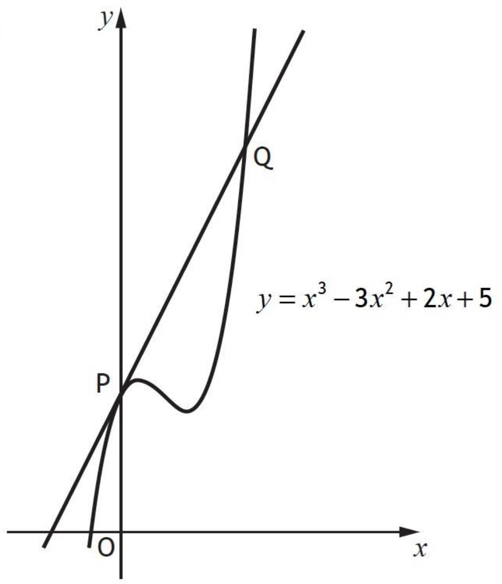
2018 Paper 1

7. The curve with equation $y=x^3-3x^2+2x+5$ is shown

(a) Write down the coordinates of P, the point where the curve crosses the y-axis . **1**

(b) Determine the equation of the tangent to the curve at P. **3**

(c) Find the coordinates of Q, the point where this tangent meets the curve again. **4**



2018 Paper 2

6. (a) Express $3x^2+24x+50$ in the form $a(x+b)^2+c$. **3**

(b) Given that $f(x)=x^3+12x^2+50x-11$, find $f'(x)$. **2**

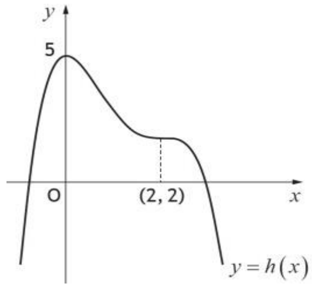
(c) Hence, or otherwise, explain why the curve with equation $y=f(x)$ is strictly increasing for all values of x. **2**

Specimen 5 Paper 2

7. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $(0, 5)$ and $(2, 2)$.

sketch the graphs of: $y = h'(x)$. 3

When checking the answers look at part (b)



Specimen Paper 1

13. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x. 4

2017 Paper 2

4. (a) Express $3x^2 + 24x + 50$ in the form $a(x+b)^2 + c$. 3

(b) Given that $f(x) = x^3 + 12x^2 + 50x - 11$, find $f'(x)$. 2

(c) Hence, or otherwise, explain why the curve with equation $y = f(x)$ is strictly increasing for all values of x. 2

2017 Paper 2

7. (a) Find the x-coordinate of the stationary point on the curve with equation $y = 6x - 2\sqrt{x^3}$. 4

(b) Hence, determine the greatest and least values of y in the interval $1 \leq x \leq 9$. 3

2017 Paper 1

3. Given $y = (4x-1)^{12}$, find $\frac{dy}{dx}$. 2

2017 Paper 1

8. Calculate the rate of change of $d(t) = \frac{1}{2t}$, $t \neq 0$, when $t = 5$. 3

2016 Paper 1

2. Given that $y = 12x^3 + 8\sqrt{x}$, where $x > 0$, find $\frac{dy}{dx}$. 3

2016 Paper 1

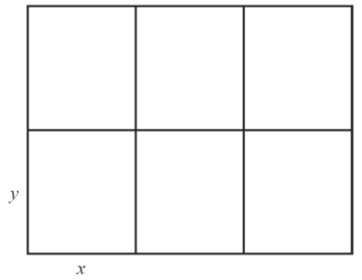
9. (a) Find the x-coordinates of the stationary points on the graph with equation $y = f(x)$, where $f(x) = x^3 + 3x^2 - 24x$. **4**

(b) Hence determine the range of values of x for which the function f is strictly increasing. **2**

2016 Paper 2

7. A council is setting aside an area of land to create six fenced plots where local residents can grow their own food.

Each plot will be a rectangle measuring x metres by y metres as shown in the diagram.



(a) The area of land being set aside is 108 m^2. **3**

Show that the total length of fencing, L metres, is given by

$$L(x) = 9x + \frac{144}{x}.$$

(b) Find the value of x that minimises the length of fencing required. **6**

New 2015 Paper 1

2. Find the equation of the tangent to the curve $y = 2x^3 + 3$ at the point where $x = -2$. **4**

New 2015 Paper 1

7. A function f is defined on a suitable domain by $f(x) = \sqrt{x}\left(3x - \frac{2}{x\sqrt{x}}\right)$. **4**

Find $f'(4)$.

New 2015 Paper 2

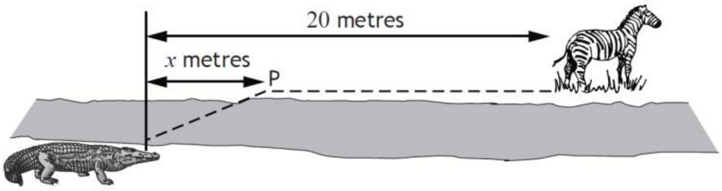
8. A crocodile is stalking prey located 20 metres further upstream on the opposite bank of a river.

Crocodiles travel at different speeds on land and in water.

The time taken for the crocodile to reach its prey can be minimised if it swims to a particular point, P, x metres upstream on the other side of the river as shown in the diagram.

The time taken, T, measured in tenths of a second, is given by

$$T(x) = 5\sqrt{36 + x^2} + 4(20 - x)$$



(a) (i) Calculate the time taken if the crocodile does not travel on land. **1**

(ii) Calculate the time taken if the crocodile swims the shortest distance possible. **1**

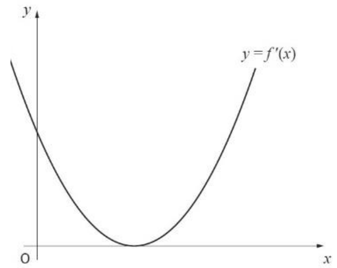
(b) Between these two extremes there is one value of x which minimises the time taken. Find this value of x and hence calculate the minimum possible time. **8**

Specimen 4 Paper 1

11. The diagram shows the graph of $y = f'(x)$. The x-axis is a tangent to this graph.

(a) Explain why the function $f(x)$ is never decreasing. 1

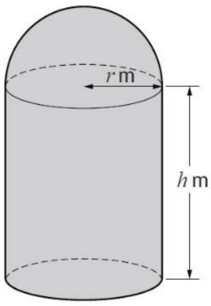
(b) On a graph of $y = f(x)$, the y-coordinate of the stationary point is negative.
Sketch a possible graph for $y = f(x)$. 2



Specimen 4 Paper 2

8. A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres.

The volume of the **cylindrical** part of the container needs to be 100 cubic metres.



(a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by: 3

$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$

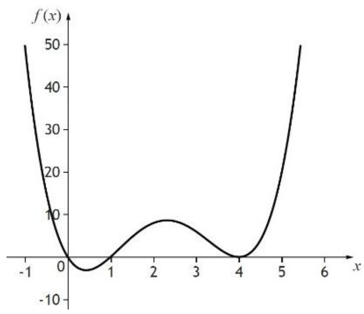
(b) Determine the value of r which minimises the amount of metal needed to build the container. 6

Specimen 4 Paper 2

3. The diagram shows the graph of $f(x) = x(x-p)(x-q)^2$.

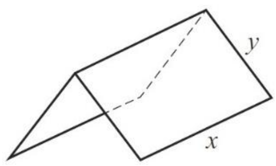
(a) Determine the values of p and q. 1

(b) Find the equation of the tangent to the curve when $x = 1$. 4



Exemplar Paper 2

9. A manufacturer is asked to design an open-ended shelter, as shown:



The frame of the shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges; • y metres for each sloping edge.

The total length, L metres, of the rods used in a shelter is given b $L = 3x + \frac{48}{x}$

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(a) Find the value of x for which L is a minimum. 5

The rods used for the frame cost £8·25 per metre.

The manufacturer claims that the minimum cost of a frame is less than £195.

(b) Is this claim correct? Justify your answer. 2

2014 Paper 2

2. A curve has equation $y = x^4 - 2x^3 + 5$.

Find the equation of the tangent to this curve at the point where $x = 2$. **4**

2014 Paper 1

21. A curve has equation $y = 3x^2 - x^3$.

(*a*) Find the coordinates of the stationary points on this curve and determine their nature. **6**

(*b*) State the coordinates of the points where the curve meets the coordinate axes and sketch the curve. **2**

2013 Paper 2

3. (*a*) Given that $(x - 1)$ is a factor of $x^3 + 3x^2 + x - 5$, factorise this cubic fully. **4**

(*b*) Show that the curve with equation

$$y = x^4 + 4x^3 + 2x^2 - 20x + 3$$

has only one stationary point.

Find the x-coordinate and determine the nature of this point. **5**

2013 Paper 1

2. The point P (5, 12) lies on the curve with equation $y = x^2 - 4x + 7$.

What is the gradient of the tangent to this curve at P? **2**

2012 Paper 1

8. The volume of a sphere is given by the formula $V = \frac{4}{3}\pi r^3$.

What is the rate of change of V with respect to r, at $r = 2$? **2**

2012 Paper 1

2. What is the gradient of the tangent to the curve with equation $y = x^3 - 6x + 1$ at the point where $x = -2$? **2**

2012 Paper 1

6. If $y = 3x^{-2} + 2x^{\frac{3}{2}}$, $x > 0$, determine $\dfrac{dy}{dx}$. **2**

2012 Paper 1

18. The graph of $y = f(x)$ shown has stationary points at $(0, p)$ and (q, r).

Here are two statements about $f(x)$:

(1) $f(x) < 0$ for $s < x < t$;

(2) $f'(x) < 0$ for $x < q$.

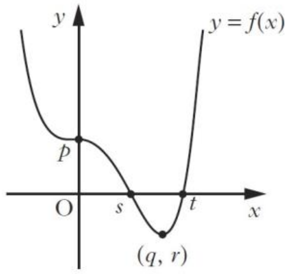
Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.



2

2012 Paper 2

3. A function f is defined on the domain $0 \leq x \leq 3$ by $f(x) = x^3 - 2x^2 - 4x + 6$.

Determine the maximum and minimum values of f. 7

2011 Paper 1

4. A tangent to the curve with equation $y = x^3 - 2x$ is drawn at the point $(2, 4)$.

What is the gradient of this tangent? 2

2011 Paper 1

22. A function f is defined on the set of real numbers by $f(x) = (x - 2)(x^2 + 1)$.

(*a*) Find where the graph of $y = f(x)$ cuts:

(i) the x-axis;

(ii) the y-axis. 2

(*b*) Find the coordinates of the stationary points on the curve with equation $y = f(x)$ and determine their nature. 8

(*c*) On separate diagrams sketch the graphs of:

(i) $y = f(x)$;

(ii) $y = -f(x)$. 3

2010 Paper 1

15. The derivative of a function f is given by $f'(x) = x^2 - 9$.

Here are two statements about f:

(1) f is increasing at $x = 1$;

(2) f is stationary at $x = -3$.

Which of the following is true? 2

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

2010 Paper 1

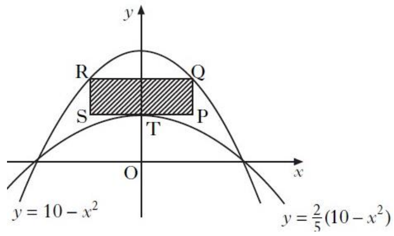
17. If $s(t) = t^2 - 5t + 8$, what is the rate of change of s with respect to t when $t = 3$? **2**

2010 Paper 2

5. The parabolas with equations $y = 10 - x^2$ and $y = \frac{2}{5}(10 - x^2)$ are shown in the diagram below.

A rectangle PQRS is placed between the two parabolas as shown, so that:

- Q and R lie on the upper parabola;
- RQ and SP are parallel to the x-axis;
- T, the turning point of the lower parabola, lies on SP.



(*a*) (i) If TP = x units, find an expression for the length of PQ.

(ii) Hence show that the area, A, of rectangle PQRS is given by **3**

$$A(x) = 12x - 2x^3.$$

(*b*) Find the maximum area of this rectangle. **6**

2009 Paper 1

4. A curve has equation $y = 5x^3 - 12x$.

What is the gradient of the tangent at the point $(1, -7)$? **2**

2009 Paper 1

8. What is the derivative of $\frac{1}{4x^3}$, $x \neq 0$? **2**

2009 Paper 1

20. $A = 2\pi r^2 + 6\pi r$.

What is the rate of change of A with respect to r when $r = 2$? **2**

2009 Paper 2

1. Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature. **8**

2009 Paper 2

2. Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

(*a*) (i) Find $p(x)$ where $p(x) = f(g(x))$.

(ii) Find $q(x)$ where $q(x) = g(f(x))$. **3**

(*b*) Solve $p'(x) = q'(x)$. **3**

2008 Paper 1

21. A function f is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

(*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. **6**

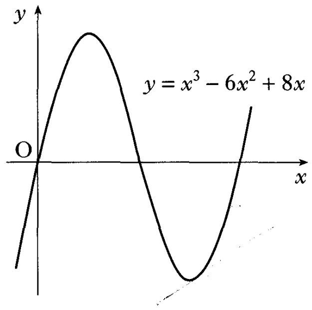
(*b*) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. **5**

(*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. **4**

2008 Paper 1

22. The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.



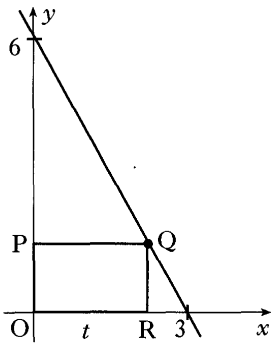
(*a*) Find the coordinates of the points on the curve where the gradient of the tangent is -1. **5**

(*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. **2**

2008 Paper 2

6. In the diagram, Q lies on the line joining (0, 6) and (3, 0).

OPQR is a rectangle, where P and R lie on the axes and OR = t.

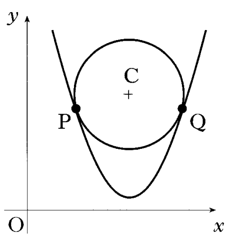


(*a*) Show that QR = $6 - 2t$. **3**

(*b*) Find the coordinates of Q for which the rectangle has a maximum area. **6**

2007 Paper 2

5. A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q.



(*a*) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q. **5**

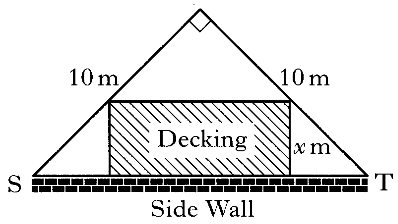
(*b*) Find the coordinates of P. **2**

(*c*) Find the coordinates of C, the centre of the circle. **2**

2007 Paper 2

6. A householder has a garden in the shape of a right-angled isosceles triangle.

It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



(*a*) (i) Find the exact value of ST. 3

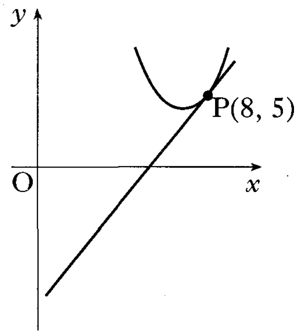
(ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

$$A = \left(10\sqrt{2}\right)x - 2x^2.$$

(*b*) Find the dimensions of the decking which maximises its area. 5

2006 Paper 2

3. The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8, 5).

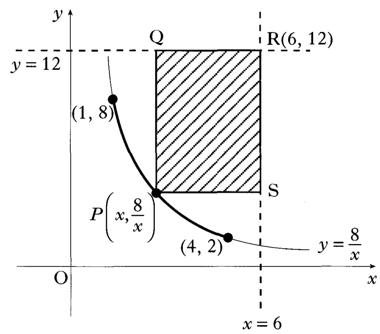


(*a*) Find the equation of this tangent. 4

2006 Paper 2

12. PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

(ii) Hence show that the area, A square units, of PQRS is given by 3

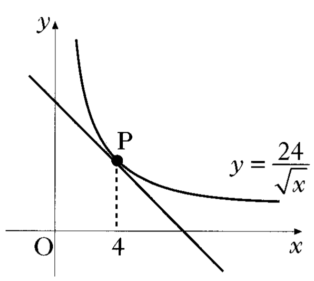
$$A = 80 - 12x - \frac{48}{x}.$$

(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. 8

2005 Paper 2

6. The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, $x > 0$.

Find the equation of the tangent at P, where $x = 4$. 6



2004 Paper 2

5. The point P(x, y) lies on the curve with equation $y = 6x^2 - x^3$.

   (a) Find the value of x for which the gradient of the tangent at P is 12. **5**

   (b) Hence find the equation of the tangent at P. **2**

2004 Paper 2

7. The graph of the cubic function $y = f(x)$ is shown in the diagram. There are turning points at (1, 1) and (3, 5).

   Sketch the graph of $y = f'(x)$. **3**