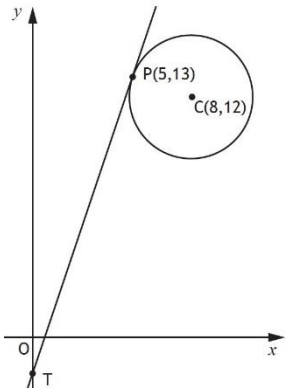
Circles

Go to the appropriate Past Paper for the answers

2019 Paper 2

15. A circle has centre C(8,12).

The point P(5,13) lies on the circle as shown.



(a) Find the equation of the tangent at P. **3**

The tangent from P meets the y-axis at the point T.

(b) (i) State the coordinates of T. **1**

(ii) Find the equation of the circle that passes through the points C, P and T. **3**

2019 Paper 1

16. The point P has coordinates $(4,k)$.

C is the centre of the circle with equation $(x-1)^2+(y+2)^2=25$.

(a) Show that the distance between the points P and C is given by $\sqrt{k^2+4k+13}$. **2**

(b) Hence, or otherwise, find the range of values of k such that P lies outside the circle. **4**

2019 Paper 1

3. Circle C_1 has equation $x^2+y^2-6x-2y-26=0$.

Circle C_2 has centre $(4,-2)$. **2**

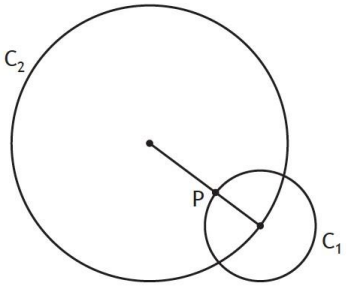
The radius of C_2 is equal to the radius of C_1.

Find the equation of circle C_2.

2018 Paper 2

12. Circle C_1 has equation $(x-13)^2+(y+4)^2=100$.

Circle C_2 has equation $x^2+y^2+14x-22y+c=0$.



(a) (i) Write down the coordinates of the centre of C_1. **1**

(ii) The centre of C_1 lies on the circumference of C_2. **1**

Show that $c=-455$.

The line joining the centres of the circles intersects C_1 at P.

(b) (i) Determine the ratio in which P divides the line joining the centres of the circles. **2**

(ii) Hence, or otherwise, determine the coordinates of P. **2**

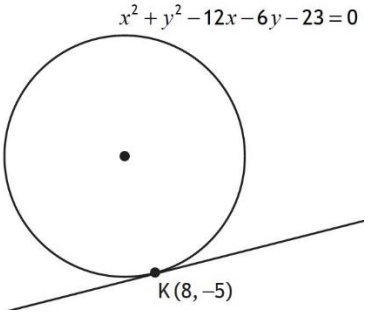
P is the centre of a third circle, C_3.

C_2 touches C_3 internally.

(c) Determine the equation of C_3. **1**

2018 Paper 1

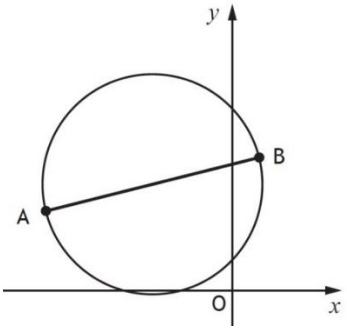
4. The point K (8, –5) lies on the circle with equation $x^2 + y^2 - 12x - 6y - 23 = 0$.
Find the equation of the tangent to the circle at K.



4

Specimen 5 Paper 1

2. A and B are the points (–7, 3) and (1, 5).
AB is a diameter of a circle.
Find the equation of this circle.



3

Specimen 5 Paper 1

7. Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact.

5

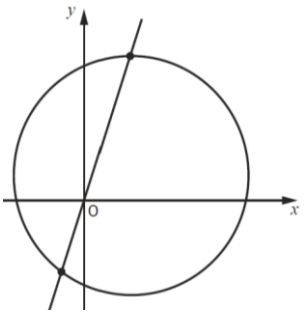
2017 Paper 1

2. The point P (–2, 1) lies on the circle $x^2 + y^2 - 8x - 6y - 15 = 0$.
Find the equation of the tangent to the circle at P.

4

2017 Paper 2

3. The line $y = 3x$ intersects the circle with equation $(x-2)^2 + (y-1)^2 = 25$.
Find the coordinates of the points of intersection.

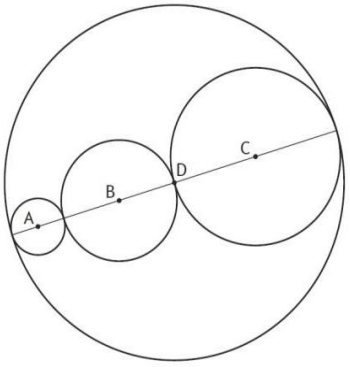


5

2017 Paper 2

10. (a) Show that the points A(–7, –2), B(2, 1) and C(17, 6) are collinear. **3**

Three circles with centres A, B and C are drawn inside a circle with centre D as shown.



The circles with centres A, B and C have radii r_A, r_B and r_C respectively.

- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

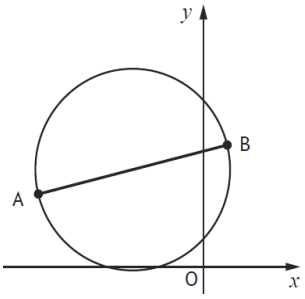
(b) Determine the equation of the circle with centre D. **4**

2016 Paper 1

4. A and B are the points $(-7, 3)$ and $(1, 5)$.

AB is a diameter of a circle.

Find the equation of this circle. **3**



2016 Paper 1

8. Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact. **5**

2016 Paper 2

4. Circles C_1 and C_2 have equations $(x+5)^2 + (y-6)^2 = 9$ and $x^2 + y^2 - 6x - 16 = 0$ respectively.

(a) Write down the centres and radii of C_1 and C_2. **4**

(b) Show that C_1 and C_2 do not intersect. **3**

New 2015 Paper 1

14. The circle with equation $x^2 + y^2 - 12x - 10y + k = 0$ meets the coordinate axes at exactly three points.

What is the value of k? **2**

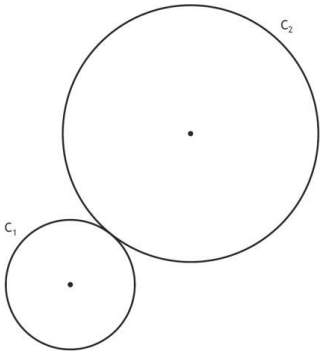
New 2015 Paper 1

5. Circle C_1 has equation $x^2 + y^2 + 6x + 10y + 9 = 0$.

The centre of circle C_2 is $(9, 11)$.

Circles C_1 and C_2 touch externally.

(a) Determine the radius of C_2. **4**



A third circle, C_3, is drawn such that:

- both C_1 and C_2 touch C_3 internally
- the centres of C_1, C_2 and C_3 are collinear.

(b) Determine the equation of C_3. **4**

New 2015 Paper 2

11. T$(-2, -5)$ lies on the circumference of the circle with equation

$$(x + 8)^2 + (y + 2)^2 = 45.$$

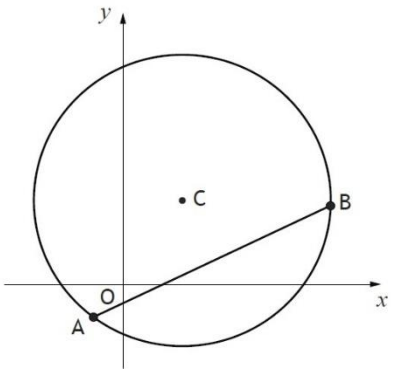
(a) Find the equation of the tangent to the circle passing through T. **4**

(b) This tangent is also a tangent to a parabola with equation $y = -2x^2 + px + 1 - p$, where $p > 3$.

Determine the value of p. **6**

Specimen 4 Paper 2

5.



Points A$(-1, -1)$ and B$(7, 3)$ lie on the circumference of a circle with centre C, as shown in the diagram.

(a) Find the equation of the perpendicular bisector of AB. **4**

CB is parallel to the x-axis.

(b) Find the equation of the circle, passing through A and B, with centre C. **4**

Exemplar Paper 2

2. (a) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x-y+5=0$ intersecting the circle $x^2+y^2-6x-2y-30=0$ at the points P and Q.

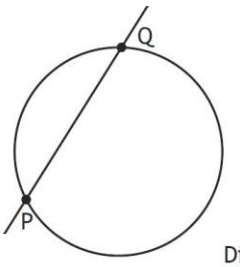


Diagram 1

Find the coordinates of P and Q. **6**

(b) Diagram 2 shows the circle from (a) and a second congruent circle, which also passes through P and Q.

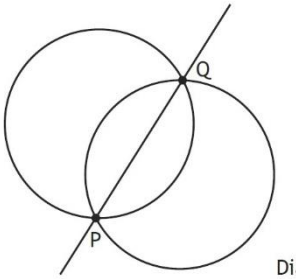


Diagram 2

Determine the equation of this second circle. **6**

2014 Paper 2

8. Given that the equation

$$x^2 + y^2 - 2px - 4py + 3p + 2 = 0$$ **5**

represents a circle, determine the range of values of p.

2014 Paper 1

23. (*a*) Find P and Q, the points of intersection of the line $y = 3x - 5$ and the circle C_1 with equation $x^2 + y^2 + 2x - 4y - 15 = 0$. **4**

(*b*) T is the centre of C_1.

Show that PT and QT are perpendicular. **3**

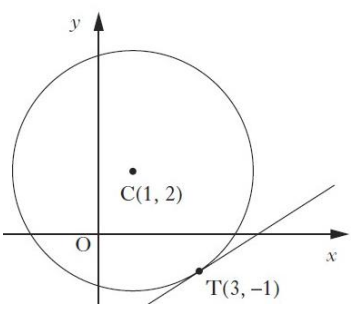
(*c*) A second circle C_2 passes through P, Q and T.

Find the equation of C_2. **3**

2014 Paper 1

2. The diagram shows a circle with centre C(1, 2) and the tangent at T(3, –1).

What is the gradient of this tangent? **2**



2013 Paper 1

22. A circle C_1 has equation $x^2 + y^2 + 2x + 4y - 27 = 0$.

(a) Write down the centre and calculate the radius of C_1. **2**

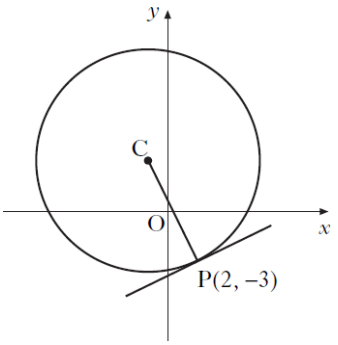
(b) The point P(3, 2) lies on the circle C_1.
Find the equation of the tangent at P. **3**

(c) A second circle C_2 has centre (10, –1). The radius of C_2 is half of the radius of C_1.
Show that the equation of C_2 is $x^2 + y^2 - 20x + 2y + 93 = 0$. **3**

(d) Show that the tangent found in part (b) is also a tangent to circle C_2. **4**

2011 Paper 1

6. The point P(2, –3) lies on the circle with centre C as shown.
The gradient of CP is –2.
What is the equation of the tangent at P? **2**

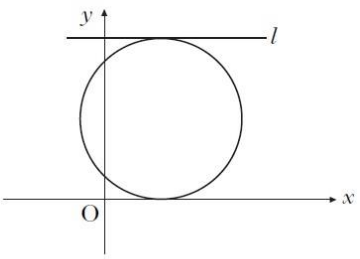


2011 Paper 2

7. Circle C_1 has equation $(x + 1)^2 + (y - 1)^2 = 121$.
A circle C_2 with equation $x^2 + y^2 - 4x + 6y + p = 0$ is drawn inside C_1.
The circles have no points of contact.
What is the range of values of p? **9**

2010 Paper 1

8. The equation of the circle shown in the diagram is $x^2 + y^2 - 6x - 10y + 9 = 0$.
The x-axis and the line l are parallel tangents to the circle.

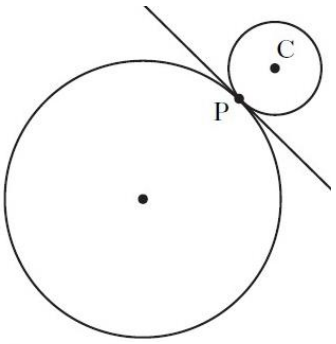


What is the equation of line l? **2**

2010 Paper 2

3. (*a*) (i) Show that the line with equation $y = 3 - x$ is a tangent to the circle with equation $x^2 + y^2 + 14x + 4y - 19 = 0$.

(ii) Find the coordinates of the point of contact, P. **5**



(*b*) Relative to a suitable set of coordinate axes, the diagram below shows the circle from (*a*) and a second smaller circle with centre C.

The line $y = 3 - x$ is a common tangent at the point P.

The radius of the larger circle is three times the radius of the smaller circle.

Find the equation of the smaller circle. **6**

2009 Paper 1

2. A circle has equation $x^2 + y^2 + 8x + 6y - 75 = 0$.

What is the radius of this circle? **2**

2009 Paper 1

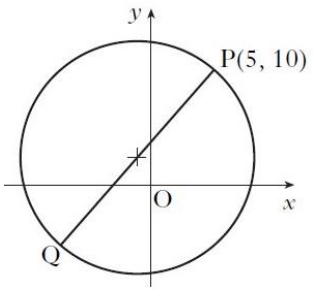
9. The line with equation $y = 2x$ intersects the circle with equation $x^2 + y^2 = 5$ at the points J and K.

What are the x-coordinates of J and K? **2**

2009 Paper 2

4. (*a*) Show that the point P(5, 10) lies on circle C_1 with equation $(x + 1)^2 + (y - 2)^2 = 100$. **1**

(*b*) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. **5**



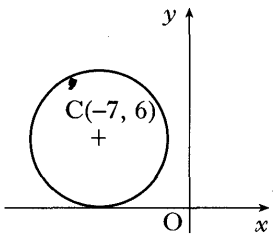
(*c*) Two circles, C_2 and C_3, touch circle C_1 at Q.

The radius of each of these circles is twice the radius of circle C_1.

Find the equations of circles C_2 and C_3. **4**

2008 Paper 1

2. The x-axis is a tangent to a circle with centre (–7, 6) as shown in the diagram.



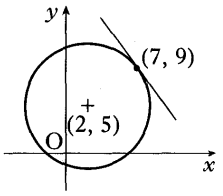
2

What is the equation of the circle?

2008 Paper 1

5. The diagram shows a circle, centre (2, 5) and a tangent drawn at the point (7, 9).

What is the equation of this tangent?



2

2008 Paper 2

4. (*a*) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. 2

(*b*) A second circle has equation $(x - 4)^2 + (y - 6)^2 = 26$.

Find the distance between the centres of these two circles and hence show that the circles intersect. 4

(*c*) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles. 5

Find the coordinates of the points of intersection of the two circles.

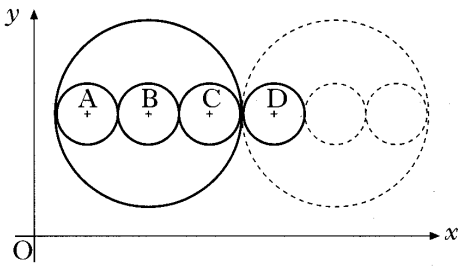
2007 Paper 1

5. The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x-axis.

This pattern is continued, as shown in the diagram.

Find the equation of the circle with centre D.



5

2007 Paper 2

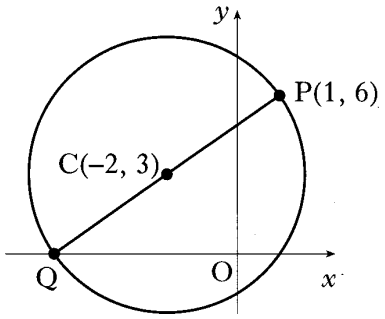
3. Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle. 6

2006 Paper 1

2. A circle has centre C(–2, 3) and passes through P(1, 6).

(*a*) Find the equation of the circle. 2

(*b*) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q. 4



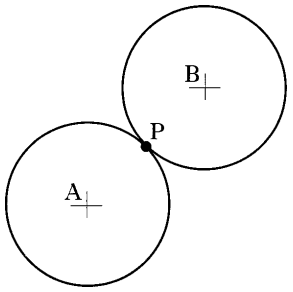
2006 Paper 2

4. The circles with equations $(x-3)^2+(y-4)^2=25$ and $x^2+y^2-kx-8y-2k=0$ have the same centre. **5**

Determine the radius of the larger circle.

2005 Paper 1

2. Two congruent circles, with centres A and B, touch at P.



Relative to suitable axes, their equations are

$x^2+y^2+6x+4y-12=0$ and
$x^2+y^2-6x-12y+20=0$.

(*a*) Find the coordinates of P. **3**

(*b*) Find the length of AB. **2**

2005 Paper 1

11. (*a*) A circle has centre $(t, 0)$, $t > 0$, and radius 2 units.

Write down the equation of the circle. **1**

(*b*) Find the exact value of t such that the line $y = 2x$ is a tangent to the circle. **5**