Further Calculus

Go to the appropriate Past Paper for the answers

2019 Paper 1

17. (a) Express $(\sin x - \cos x)^2$ in the form $p + q \sin rx$ where p, q and r are integers.

3

(b) Hence, find $\int (\sin x - \cos x)^2 \, dx$.

2

2019 Paper 1

11. Evaluate $\displaystyle\int_0^{\frac{\pi}{9}} \cos\left(3x - \frac{\pi}{6}\right) dx$.

4

2019 Paper 1

6. Given that $y = \dfrac{1}{(1-3x)^5}$, $x \neq \dfrac{1}{3}$, find $\dfrac{dy}{dx}$.

3

2018 Paper 1

14. Evaluate $\displaystyle\int_{-4}^{9} \frac{1}{\sqrt[3]{(2x+9)^2}} \, dx$.

5

2018 Paper 1

3. Given $h(x) = 3\cos 2x$, find the value of $h'\left(\dfrac{\pi}{6}\right)$.

3

Specimen 5 Paper 2

10. Given that

calculate the value of a. $\displaystyle\int_{\frac{\pi}{8}}^{a} \sin\left(4x - \frac{\pi}{2}\right) dx = \frac{1}{2}, \quad 0 \leq a < \frac{\pi}{2},$

6

Specimen 5 Paper 1

11. Find the rate of change of the function $f(x) = 4\sin^3 x$ when $x = \dfrac{5\pi}{6}$.

3

2017 Paper 1

13. Find $\displaystyle\int \frac{1}{(5-4x)^{\frac{1}{2}}}\,dx, \quad x < \frac{5}{4}.$ **4**

2017 Paper 2

11. (a) Show that $\displaystyle\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x = \sin^3 x$, where $0 < x < \frac{\pi}{2}$.   **3**

     (b) Hence, differentiate $\displaystyle\frac{\sin 2x}{2\cos x} - \sin x \cos^2 x$, where $0 < x < \frac{\pi}{2}$.   **3**

2016 Paper 1

5. Find $\displaystyle\int 8\cos(4x+1)\,dx.$ **2**

2016 Paper 2

10. (a) Given that $y = (x^2 + 7)^{\frac{1}{2}}$, find $\displaystyle\frac{dy}{dx}$.   **2**

     (b) Hence find $\displaystyle\int \frac{4x}{\sqrt{x^2+7}}\,dx.$ **1**

2016 Paper 2

11. (a) Show that $\sin 2x \tan x = 1 - \cos 2x$, where $\frac{\pi}{2} < x < \frac{3\pi}{2}$.   **4**

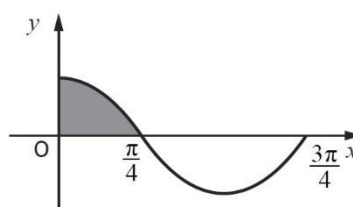   (b) Given that $f(x) = \sin 2x \tan x$, find $f'(x)$.   **2**

New 2015 Paper 1

12. The diagram shows part of the graph of $y = a\cos bx$.

     The shaded area is $\frac{1}{2}$ unit^2.

     What is the value of $\displaystyle\int_0^{\frac{3\pi}{4}} (a\cos bx)\,dx$?   **2**

New 2015 Paper 2

7. (a) Find $\displaystyle\int (3\cos 2x + 1)\,dx.$ **2**

     (b) Show that $3\cos 2x + 1 = 4\cos^2 x - 2\sin^2 x$.   **2**

     (c) Hence, or otherwise, find $\displaystyle\int \left(\sin^2 x - 2\cos^2 x\right)\,dx.$ **2**

Specimen 4 Paper 1

1. Find $\int \frac{3x^3+1}{2x^2}\,dx,\ x \neq 0$. **4**

Specimen 4 Paper 1

10. Find the rate of change of the function $f(x) = 4\sin^3 x$ when $x = \frac{5\pi}{6}$. **3**

Exemplar Paper 2

10. Acceleration is defined as the rate of change of velocity.

    An object is travelling in a straight line. The velocity, v m/s, of this object,

    t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$.

    (a) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. **3**

    (b) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. **2**

    (c) Velocity is defined as the rate of change of displacement. **3**

    Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$.

Exemplar Paper 2

8. Given that $\int_{\frac{\pi}{8}}^{a} 5\sin(4x - \frac{\pi}{2})dx = \frac{10}{4}$, $0 \leq a < \frac{\pi}{2}$, calculate the value of a. **6**

Exemplar Paper 1

10. The gradient of a tangent to a curve is given by $\frac{dy}{dx} = 3\cos 2x$.

    The curve passes through the point $\left(\frac{7\pi}{6},\ \sqrt{3}\right)$.

    Find y in terms of x. **4**

2014 Paper 2

9. Acceleration is defined as the rate of change of velocity.

    An object is travelling in a straight line. The velocity, v m/s, of this object,

    t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$.

    (a) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. **3**

    (b) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. **2**

    (c) Velocity is defined as the rate of change of displacement. **3**

    Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$.

2014 Paper 2

5. Given that $\int_{4}^{t} (3x+4)^{-\frac{1}{2}} \, dx = 2$, find the value of t.

5

2014 Paper 2

8. What is the derivative of $\left(4 - 9x^4\right)^{\frac{1}{2}}$?

2

2014 Paper 1

5. Find $\int (2x+9)^5 \, dx$.

2

2013 Paper 2

6. Given that $\int_{0}^{a} 5\sin 3x \, dx = \dfrac{10}{3}$, $0 \leq a < \pi$, calculate the value of a.

2

2013 Paper 1

18. Given that $y = \sin(x^2 - 3)$, find $\dfrac{dy}{dx}$.

2

2013 Paper 1

16. If $y = 3\cos^4 x$, find $\dfrac{dy}{dx}$.

2

2012 Paper 1

16. If $y = 3\cos^4 x$, find $\dfrac{dy}{dx}$.

2

2012 Paper 1

14. Find $\int (2x-1)^{\frac{1}{2}} \, dx$ where $x > \dfrac{1}{2}$.

2

2011 Paper 2

6. (a) The expression $3\sin x - 5\cos x$ can be written in the form $R\sin(x+a)$ where $R > 0$ and $0 \leq a < 2\pi$.

   Calculate the values of R and a.

4

   (b) Hence find the value of t, where $0 \leq t \leq 2$, for which

$$\int_{0}^{t} (3\cos x + 5\sin x) \, dx = 3.$$

7

2011 Paper 1

20. On a suitable domain, D, a function g is defined by $g(x) = \sin^2 \sqrt{x-2}$.

    Which of the following gives the real values of x in D and the corresponding values of $g(x)$?

2

2011 Paper 1

13. Given that $f(x) = 4\sin 3x$, find $f'(0)$.

2

2010 Paper 2

6. (a) A curve has equation $y = (2x-9)^{\frac{1}{2}}$.

5

   Show that the equation of the tangent to this curve at the point where $x = 9$ is $y = \frac{1}{3}x$.

   (b) Diagram 1 shows part of the curve and the tangent.

1

   The curve cuts the x-axis at the point A.
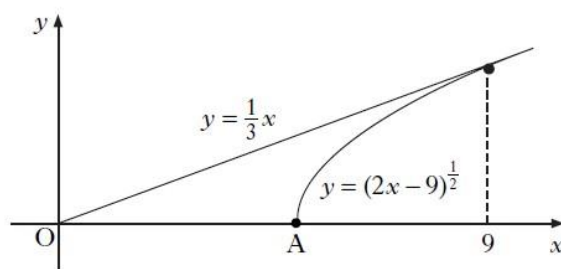
Diagram 1

   Find the coordinates of point A.
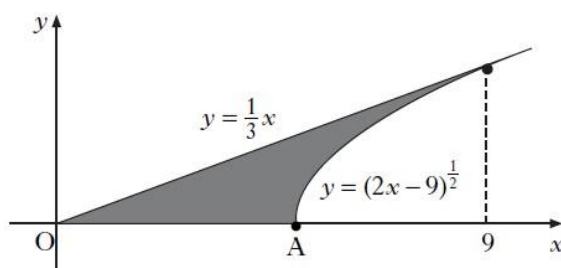
   (c) Calculate the shaded area shown in diagram 2.

7

Diagram 2

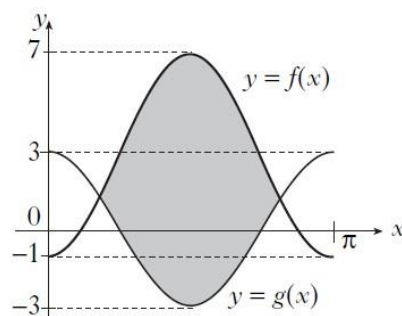## 2011 Paper 2

9. Find $\int (2x^{-4} + \cos 5x)\, dx$.

2

2009 Paper 2

5. The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.

   $f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$.

   (a) Write down the values of m and n. **1**

   (b) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \le x \le \pi$. **5**

   (c) Calculate the shaded area. **6**

2009 Paper 1

18. Given that $f(x) = (4 - 3x^2)^{-\frac{1}{2}}$ on a suitable domain, find $f'(x)$. **2**

2008 Paper 1

15. What is the derivative of $(x^3 + 4)^2$? **2**
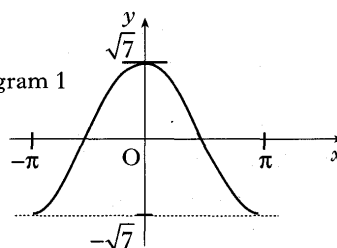
2008 Paper 2

3. (a) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

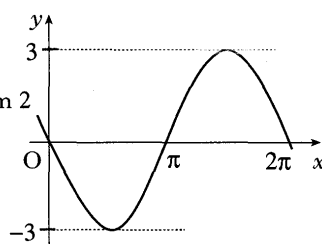   Write down the value of p. Diagram 1 **1**

   (ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

      Write down the value of q. Diagram 2 **1**

   (b) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \dfrac{\pi}{2}$. **4**

   (c) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. **2**

2007 Paper 1

10. Given that $y = \sqrt{3x^2 + 2}$, find $\dfrac{dy}{dx}$. **3**

2007 Paper 2

7. Find the value of $\int_0^2 \sin(4x+1)\,dx.$

4

2006 Paper 1

5. A function f is defined by $f(x) = (2x-1)^5.$

   Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature.

7

2006 Paper 2

9. If $y = \dfrac{1}{x^3} - \cos 2x,\ x \neq 0,$ find $\dfrac{dy}{dx}.$

4

2005 Paper 1

5. Differentiate $(1 + 2\sin x)^4$ with respect to $x.$

2

2004 Paper 1

6. Given that $y = 3\sin(x) + \cos(2x),$ find $\dfrac{dy}{dx}.$

3

2004 Paper 1

7. Find $\int_0^2 \sqrt{4x+1}\ dx.$

5