MATHEMATICS

Higher Grade Extended Unit Test - UNIT 1

Time allowed   -   50 minutes

Read Carefully

1.   Full credit will be given only where the solution contains appropriate working.
2.   **Calculators may be used.**
3.   Answers obtained by readings from scale drawings will not receive any credit.
4.   **This Unit Test contains questions graded at all levels.**

Section A

In this section the correct answer to each question is given by one of the alternatives A, B, C or D.
Indicate the correct answer by writing A, B, C or D opposite the number of the question.
Rough working may be done on the paper provided. 2 marks will be given for each correct answer.

1. If A is the point (-5, -2) and B is the point (-2, 4) then the gradient of AB is

    A $-\dfrac{7}{2}$

    B $\dfrac{1}{2}$

    C   0

    D   2

2. The derivative of $\dfrac{1}{2x^3}$ is

    A $\dfrac{1}{6x^2}$

    B $-\dfrac{3}{2x^4}$
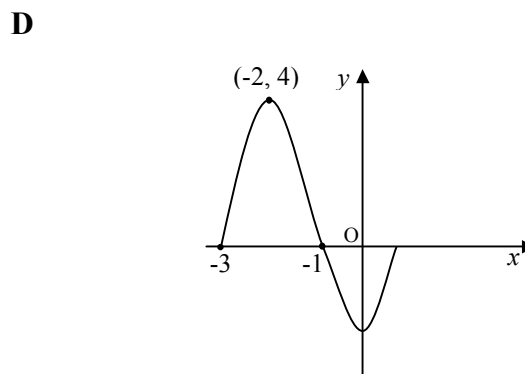
    C $-6x^2$

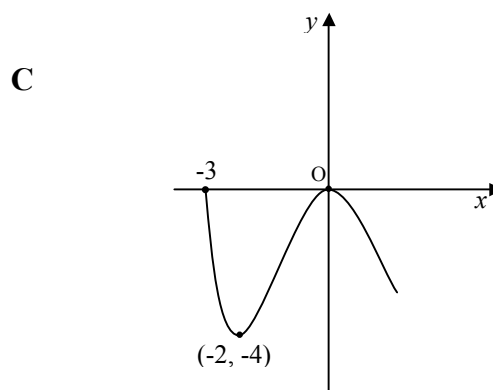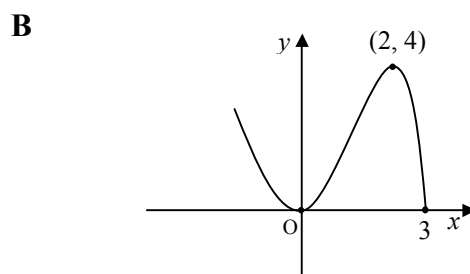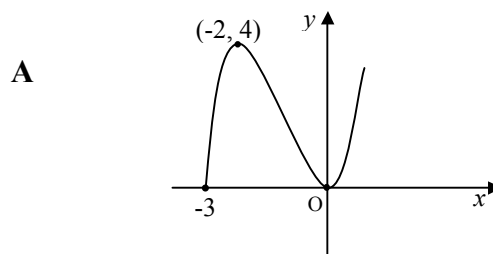   D $-\dfrac{3}{2x^2}$

3. The limit of the sequence defined by the recurrence relation $U_{n+1} = 0\cdot25U_n + 12$ is

    A   -16

    B   9·6

    C   16

    D   48

4. The rate of change of the function $f(x) = 3x^2$ when $x = 3$ is

    A   3

    B   18

    C   27

    D   54

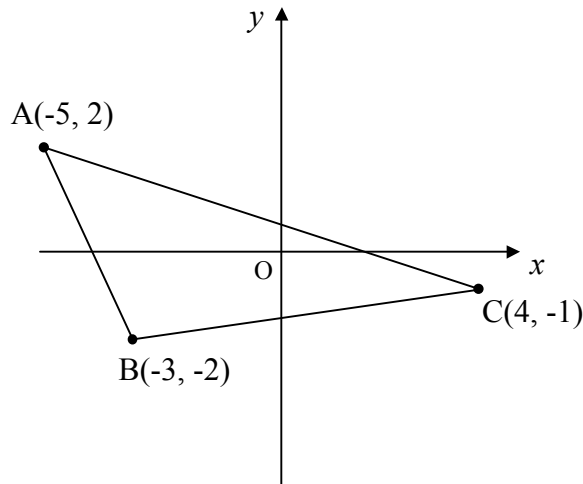**5.** Which graph is most likely to be that of the function $f(x) = x^2(x+3)$?

A

B

C

D

Section B
ALL QUESTIONS SHOULD BE ATTEMPTED
In this section credit will be given for all correct working.

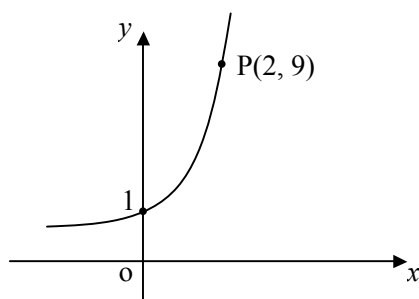**6.**   In the diagram A, B and C are the points (-5, 2), (-3, -2) and (4, -1) respectively.

    (a)   Find the equation of the line through C parallel to the line AB.   **3**

    (b)   Find the equation of the line perpendicular to BC which passes through the point A.   **3**

    (c)   Find the coordinates of T, the point of intersection of these two lines.   **4**

7.   Two functions are defined on suitable domains and are given as

$$f(x) = 3 - x \ \text{ and } \ g(x) = x - 3 .$$

    (a)   Find an expression, in its simplest form, for $g(f(x))$ **2**

    (b)   Show that $g(f(x)) - f(g(x)) = -6$ **2**

8. A recurrence relation is defined as $U_{n+1} = aU_n + b$, where a and b are constants.

    (a) Given that $U_2 = 14$, $U_3 = 9 \cdot 2$ and $U_4 = 5 \cdot 36$, find the values of the constants a and b.   **3**

    (b) Hence explain why this recurrence relation has a limit.   **1**

    (c) Establish the value of U_1.   **2**

9. Find the equation of the tangent to the curve $y = x^3 - 3x$ at the point where $x = 2$.   **5**

10. Express the function $f(x) = 3x^2 - 6x + 11$ in the form $p(x - q)^2 + r$.   **3**

11.   **(a)** The point $(125, k)$ lies on the graph of $y = \log_5 x$. Find the value of k.   **1**

    (b) The diagram show part of the graph of $y = a^x$. State the value of a.   **1**

12. The diagram shows an open top box with a square base of x cm and height h cm.

h cm

x cm

x cm
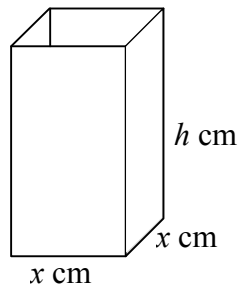
The box has to be made from $1350\,\text{cm}^2$ of card.

(a) Show that, in terms of x, the height, $h$cm, of the box can be expressed as

$$\frac{1350 - x^2}{4x}$$

2

(b) Show clearly that the volume of the box, in terms of x, can be expressed as:

$$V(x) = \tfrac{1}{4}x(1350 - x^2)$$

3

(c) Hence, or otherwise, find the value of x, so that the volume is a maximum, leaving your answer as a surd in its simplest form. **Justify your answer.**

5

END OF QUESTION PAPER