Mathematics

Higher Mini-Prelim 4

Assessing Unit 3 + revision from Units 1 & 2

Time allowed   -   1 hour 10 minutes

Read carefully

1. **Calculators may be used in this paper.**

2. Full credit will be given only where the solution contains appropriate working.

3. Answers obtained from readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae:
$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$
$$\sin 2A = 2 \sin A \cos A$$
$$\cos 2A = \cos^2 A - \sin^2 A$$
$$= 2 \cos^2 A - 1$$
$$= 1 - 2 \sin^2 A$$

Scalar Product: $\quad a \cdot b = |a||b| \cos\theta$, where θ is the angle between a and b.

or

$$a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3 \text{ where } a = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \text{ and } b = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\, dx$
$\sin ax$	$-\dfrac{1}{a} \cos ax + C$
$\cos ax$	$\dfrac{1}{a} \sin ax + C$

SECTION A

*In this section the correct answer to each question is given by one of the alternatives **A**, **B**, **C** or **D**.*
*Indicate the correct answer by writing **A**, **B**, **C** or **D** opposite the number of the question on your answer paper.*
Rough working may be done on the paper provided. 2 marks will be given for each correct answer.

1. A vector is defined as

$$\mathbf{v} = \begin{pmatrix} 2 \\ 0 \\ \sqrt{5} \end{pmatrix}.$$

The magnitude of this vector is

A   3
B $2 + \sqrt{5}$
C   9
D   unknown

2. $\displaystyle\int_{0}^{\frac{\pi}{2}} \cos 2x \; dx$ is equal to

A $\frac{1}{2}$
B -1
C 0
D 2

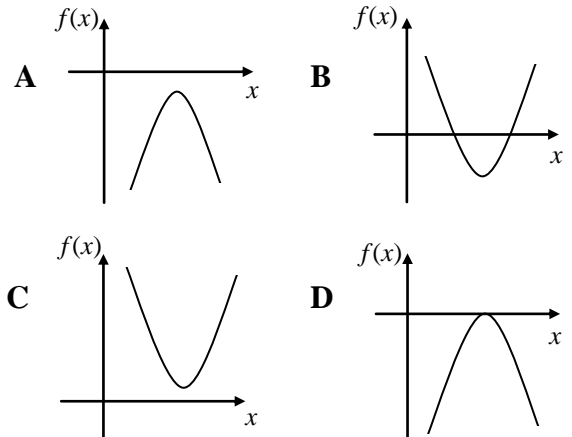**3.** The maximum value of $2\cos x - 3\sin x$ is

A 5
B -1
C $\sqrt{5}$
D $\sqrt{13}$

4. The exact value of $\log_9 27$ is

A $\frac{1}{3}$
B $\frac{2}{3}$
C $\frac{3}{2}$
D 3

5. A quadratic function f, where $f(x) = ax^2 + bx + c$, is such that $a < 0$ and $b^2 - 4ac < 0$.

Which of the following could be a possible sketch of the graph of this function?

6. What value of x makes the vectors

$\begin{pmatrix} -2 \\ 4 \\ 10 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 6 \\ x \end{pmatrix}$ perpendicular to each other?

A 15
B -3
C 9
D   no possible value

7. If $f(x) = \sin^3 x$ then $f'(x)$ equals

A $3\sin^2 x$
B $3\cos^2 x$
C $3\sin^2 x \cos x$
D $-3\sin^2 x \cos x$
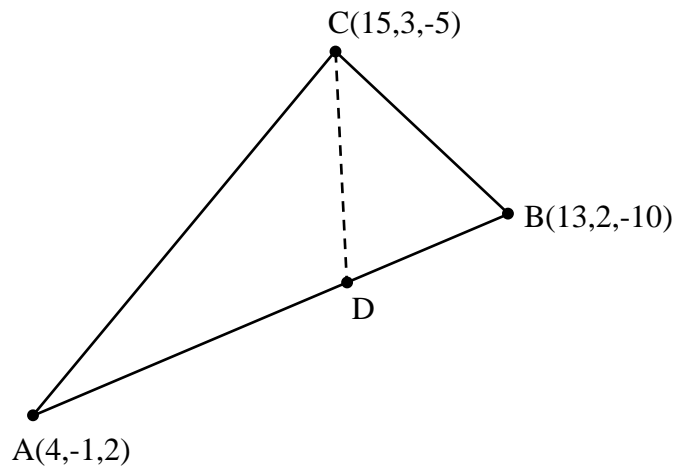
8. The graph of $y = \log_2 x$ cuts the x-axis at

A $(0,0)$
B $(1,0)$
C $(2,0)$
D $(0,1)$

[END OF SECTION A]

9. Triangle ABC has vertices A(4,-1,2), B(13,2,-10) and C(15,3,-5) as shown.

Point D lies on side AB.

C(15,3,-5)

B(13,2,-10)

D

A(4,-1,2)

(a) Given that D divides the line AB in the ratio 2:1, find the coordinates of D. **3**

(b) Hence calculate the size of angle CDA. **5**

10. Given $f(x) = \dfrac{9}{1 - 4x}$ where $x \neq \dfrac{1}{4}$, find the value of $f'(1)$. **4**

11. The noise level, N decibels, emitting from a siren as it slowly gains and loses volume is given by the formula

$$N = 2\cos t° + 4\sqrt{2}\sin t° + 30,$$

where t is the time elapsed, in seconds, from switch on.

(a) Express N in the form $N = k\sin(t+\alpha)° + 30$, where $k > 0$ and $0 \leq \alpha \leq 90$. **4**

(b) Hence calculate how many seconds the siren takes to first reach a noise level of 34 decibels.
Give your answer correct to 3 significant figures. **3**

12. (a) Given that $3\log_x y = \log_x y^2 + 2$, find a relationship connecting x and y. **4**

(b) Hence find the two values of y when $x = y - 2$. **3**

13. (a) A **linear** function, f, is such that $f(-1) = -3$ and $f(4) = 7$.

Find a formula for this function in terms of x. **3**

(b) Given that a second function, g, has as its formula $g(x) = x^3$, evaluate

$$\int_1^2 g(f(x)) \, dx$$

5

[END OF SECTION B]

[END OF QUESTION PAPER]