Higher Prelim Revision 7 Paper 1 ANSWERS - Section A

1	A
2	C
3	B
4	C
5	A
6	A
7	D
8	A
9	B
10	B
11	D
12	C
13	C
14	A
15	B
16	D
17	B
18	D
19	A
20	B

	A	B	C	D
1	■	□	□	□
2	□	□	■	□
3	□	■	□	□
4	□	□	■	□
5	■	□	□	□
6	■	□	□	□
7	□	□	□	■
8	■	□	□	□
9	□	■	□	□
10	□	■	□	□
11	□	□	□	■
12	□	□	■	□
13	□	□	■	□
14	■	□	□	□
15	□	■	□	□
16	□	□	□	■
17	□	■	□	□
18	□	□	□	■
19	■	□	□	□
20	□	■	□	□

RED denote Unit 3 Topics

21.

(a) $x^2+y^2-4x-8y+7=0$

$x=5y-5$

- For circle problems substitution is used for where two graphs meet.
- Straight line rearranged to read $x=$

$$(5y-5)^2+y^2-4(5y-5)-8y+7=0$$
$$25y^2-50y+25+y^2-20y+20-8y+7=0$$
$$26y^2-78y+52=0$$
$$y^2-3y+2=0$$
$$(y-1)(y-2)=0$$
$$\therefore\ y=1 \text{ or } 2$$

For $y=1 \Rightarrow x=5y-5=5\times1-5=0$ $(0,1)$

For $y=2 \Rightarrow x=5y-5=5\times2-5=5$ $(5,2)$

$\therefore$ Points of intersection are $(0,1)$ and $(5,2)$

(b) For equation of a circle you need centre and radius and use $(x-a)^2+(y-b)^2=r^2$

Centre is midpoint of diameter

$$\text{Centre}=\left(\frac{x_2+x_1}{2},\frac{y_2+y_1}{2}\right)\quad (0,1)(5,2)$$
$$=\left(\frac{5}{2},\frac{3}{2}\right)$$

Radius is distance from centre to either $(0,1)$ or $(5,2)$

$$\text{Radius}=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}\quad \text{using } \left(\frac{5}{2},\frac{3}{2}\right)(0,1)$$

$$= \sqrt{\left(\frac{5}{2}-0\right)^2+\left(\frac{3}{2}-1\right)^2}$$

$$= \sqrt{\frac{25}{4}+\frac{1}{4}}$$

$$= \sqrt{\frac{26}{4}}$$

$\therefore$ using radius $\sqrt{\frac{26}{4}}$ and centre $\left(\frac{5}{2},\frac{3}{2}\right)$

$$(x-a)^2+(y-b)^2=r^2$$

$$\left(x-\frac{5}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=\cancel{\frac{26}{4}}\frac{13}{2}$$

(c) Sub in $(1,0)$ to find inside $\left(<\frac{13}{2}\right)$ outside $\left(>\frac{13}{2}\right)$ or on $\left(=\frac{13}{2}\right)$ the circle.

$$\left(x-\frac{5}{2}\right)^2+\left(y-\frac{3}{2}\right)^2=\frac{13}{2}$$

$$\left(1-\frac{5}{2}\right)^2+\left(0-\frac{3}{2}\right)^2=\frac{13}{2}$$

$$\left(-\frac{3}{2}\right)^2+\left(-\frac{3}{2}\right)^2$$

$$= \frac{9}{4}+\frac{9}{4}=\frac{18}{4}=\frac{9}{2}$$

$\therefore$ since $\frac{9}{2}<\frac{13}{2}$

$\therefore (1,0)$ lies inside the circle

22.

Unit 3 – Logs!

(a) $G(t) = 6000000 - k\ln(t-1)$

[Sub in $t=21$]

$= 6000000 - k\ln 20$

$\Rightarrow 6000000 - 3k = 0$ ($G(t) = 0$ is loch emptied)

$3k = 6000000$

$\underline{\underline{k = 2000000}}$

(b) $G(t) = 6000000 - 2000000\ln 15$ (sub in $t=16$)

$= 6000000 - 5420000$

$= \underline{\underline{580000 \text{ gallons}}}$

$$\begin{array}{r} 2.71 \\ \times 2 \\ \hline 5.42 \end{array} \times 1000000$$

$= 5420000$

$$\begin{array}{r} 6000000 \\ -\ 5420000 \\ \hline 580000 \end{array}$$

(c) $G(t) \Rightarrow 6000000 - 2000000\ln(t-1) = 4000000$

$2000000 = 2000000\ln(t-1)$

$\ln(t-1) = 1$

$\log_e(t-1) = \log_e e$

$t - 1 = e$

$t = 2.72 + 1$

$= \underline{\underline{3.72 \text{ minutes.}}}$

[① Note
Solve log/ln equations one log/ln on each side]

[② Note
$e = 2.72$
You need to know this value]

23.

(a) $4\cos 2x = 2\cos x - 1$

$4\cos 2x - 2\cos x + 1 = 0$

HARD PROBLEM → FORMULAE SHEET

$4(2\cos^2 x - 1) - 2\cos x + 1 = 0$

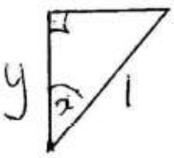
$8\cos^2 x - 2\cos x - 3 = 0$

$(4\cos x - 3)(2\cos x + 1) = 0$

$\cos x = \frac{3}{4}$ or $-\frac{1}{2}$

[Note: Question asked for $\cos x$ values]

(b)



FROM SOHCAHTOA

$\cos x = \frac{y}{1}$

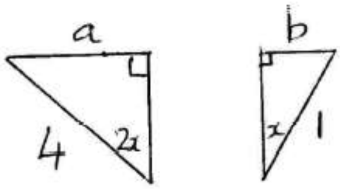
$\therefore y = \cos x$

$\therefore \cos x = \frac{3}{4} = y$

$\therefore y = \frac{3}{4}$

[Note: x must be less than 180° as its in a triangle $\therefore -\frac{1}{2}$ cannot be used]

(c)



$a + b$ = base of triangle

Using SOHCAHTOA

$\sin 2x = \frac{a}{4}$ and $\sin x = \frac{b}{1}$

$b = \sin x$

$\therefore a = 4\sin 2x$

$= 4(2\sin x\cos x)$

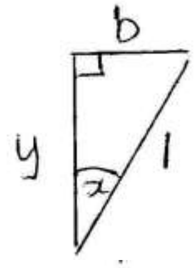
$= 8\sin x\cos x \Rightarrow \cos x = \frac{3}{4}$

$\therefore a = 8\sin x \times \frac{3}{4}$

$a = 6\sin x$

$\therefore a + b = 6\sin x + \sin x = 7\sin x$

Exact
Value



[Note
Need to find
exact value for $\sin x$]

[Previous answer] $y = \frac{3}{4}$

Pythagoras $\rightarrow$ $\therefore \quad b = \sqrt{1^2 - \left(\frac{3}{4}\right)^2}$

$$= \sqrt{1 - \frac{9}{16}}$$

$$= \sqrt{\frac{7}{16}}$$

$$= \frac{\sqrt{7}}{\sqrt{16}}$$

$$= \frac{\sqrt{7}}{4}$$

[From triangle
at top of
page] $\therefore \quad \sin x = \frac{\frac{\sqrt{7}}{4}}{1} \left(\frac{\text{opp}}{\text{hyp}}\right)$

$$= \frac{\sqrt{7}}{4}$$

$\therefore$ Exact value of base $= 7\sin x$

$$= 7 \times \frac{\sqrt{7}}{4}$$

$$= \frac{7\sqrt{7}}{4}$$

1.

(a) Perpindicular Bisector of AB

Note Lines parallel to x or y axis do not use $y - b = m(x - a)$ Answer by inspection

Mid pt of AB $= \left(\frac{x_2 + x_1}{2}, \frac{y_2 - y_1}{2}\right) = (7, 2)$

$\therefore$ Equation of $AB_{PB} \Rightarrow x = 7$

Perpendicular Bisector of AC

Mid point of AC $= \left(\frac{2+8}{2}, \frac{6+2}{2}\right)$

$= (5, 4)$

$(2,2)(8,6)$

$m_{AC} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-2}{8-2} = \frac{2}{3}$

$\therefore m_{PB} = -\frac{3}{2}$ (since $m_1 m_2 = -1$)

$\therefore$ Using $m = -\frac{3}{2}$ and $(5, 4)$

$y - b = m(x - a)$

$y - 4 = -\frac{3}{2}(x - 5)$

$2y - 8 = -3(x - 5)$

$2y + 3x = 23$

(b)

Note:

- AC and AB are chords of circle if circle passes through A, B and C.
- Also perpendicular bisector of chords pass through the centre.
- Two things required for the equation of a circle are <u>CENTRE</u> and <u>RADIUS</u>.

Point of intersection (centre)

$$x = 7 \quad \text{and} \quad 2y + 3x = 23$$

$$2y + 3\times 7 = 23$$

$$2y = 2$$

$$y = 1$$

$\therefore$ Centre is $(7, 1)$.

<u>Radius</u> (distance between 7, 1 and either A, B or C)

Distance between Centre and A

$$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (2,2)(7,1)$$

$$= \sqrt{(7-2)^2 + (1-2)^2}$$

$$= \sqrt{25 + 1}$$

$$= \sqrt{26}$$

$\therefore$ using $(x-a)^2 + (y-b)^2 = r^2$

and centre $(7,1)$ and $r = \sqrt{26}$

$$(x-7)^2 + (y-1)^2 = 26$$

Note: Check answer by sub in of either A, B or C and you would get 26

2.

(a) $x^2 - 10x + 27$

$= \boxed{x^2 - 10x + (-5)^2} - (-5)^2 + 27$

$= (x-5)^2 - 25 + 27$

$= \underline{\underline{(x-5)^2 + 2}}$

(b) For function increasing $g'(x) > 0$

$g(x) = \frac{1}{3}x^3 - 5x^2 + 27x - 2$

$g'(x) = x^2 - 10x + 27$

$\therefore$ For $g(x)$ increasing

$x^2 - 10x + 27 > 0$

using previous answer for (a) (Hence in the question)

$(x-5)^2 + 2 > 0$

Communication: For any value of x the squaring of the bracket will ensure an answer alway 0 or greater. The $+2$ will ensure answer alway greater than zero.

(c) Gradient equation is $x^2 - 10x + 27$

For any question asking for max/min, greatest/least, biggest/smallest find $f'(x) = 0$

$x^2 - 10x + 27$

$f'(x) = 0 \Rightarrow 2x - 10 = 0 \quad \therefore x = 5$

$\therefore$ minumum gradient is 5

(Always confirm answer with NATURE TABLE)

	5^-	5	5^+
$2x-10$	$-$	0	$+$
$f'(x)$	\	—	/

min

3. (a) $y = 3f(x)$ means y coordinate changing and y multiplied by 3.

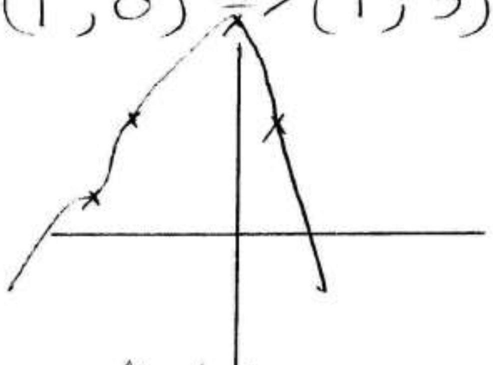
(i) $(-4,2) \Rightarrow (-4,6)$
$(-3,0) \Rightarrow (-3,0)$
$(0,-3) \Rightarrow (0,-9)$
$(1,0) \Rightarrow (1,0)$

(ii) $y = f(3x)$ means x coordinate changing x coordinate divided by 3.

$(-4,2) \Rightarrow (-\frac{4}{3},2)$
$(-3,0) \Rightarrow (-1,0)$
$(0,-3) \Rightarrow (0,-3)$
$(1,0) \Rightarrow (\frac{1}{3},0)$

(b) $y = 3 - f(x)$ means coordinate changing coordinate multiplied by -1 and add 3 to answer.

$(-4,2) \Rightarrow (-4,1)$
$(-3,0) \Rightarrow (-3,3)$
$(0,-3) \Rightarrow (0,6)$
$(1,0) \Rightarrow (1,3)$



[Graph reflected in x axis and vertical movement +3]

4.

(a) $f(-3) = x^3 + 5x^2 - 10$

$= (-3)^3 + 5(-3)^2 - 10$

Graphical calculator could be used using
$-3 \to x$

$= -27 + 45 - 10$

$= \underline{\underline{8}}$

$g(-3) = 2x^2 + 4x + 2$

$= 2(-3)^2 + 4(-3) + 2$

$= 18 - 12 + 2$

$= \underline{\underline{8}}$ $\quad \therefore \underline{f(-3) = g(-3)}$

(b) To find where two graphs meet make the equations equal to each other

$f(x) = g(x)$

$x^3 + 5x^2 - 10 = 2x^2 + 4x + 2$

$x^3 + 3x^2 - 4x - 12 = 0$

Higher than quadratic use synthetic division

$$\begin{array}{r|rrrr} -2 & 1 & 3 & -4 & -12 \\ & & -2 & -2 & 12 \\ \hline & 1 & 1 & -6 & \boxed{0} \end{array}$$

$\therefore x^3 + 3x^2 - 4x - 12 = (x+2)(x^2 + x - 6)$

$= (x+2)(x+3)(x-2)$

$\therefore x = -2, -3, 2$

Sub in except for $x = -3$ (worked out earlier).

For $x = -2$, $y = 2x^2 + 4x + 2 = 2$

For $x = -3$, $y = 8$

For $x = 2$, $y = 2x^2 + 4x + 2 = 18$

Three points are $(-2,2)$, $(-3,8)$ and $(2,18)$

(c) $$\int_{\substack{x \text{ coordinate} \\ \text{where shading} \\ \text{starts}}}^{\substack{x \text{ coordinate} \\ \text{where shading finishes}}} (\text{graph above}) - (\text{graph below})\,dx$$

$$= \int_{-2}^{2} \left(2x^2 + 4x + 2 - (x^3 + 5x^2 - 10)\right) dx$$

$$= \int_{-2}^{2} \left(2x^2 + 4x + 2 - x^3 - 5x^2 + 10\right) dx$$

$$= \int_{-2}^{2} \left(-x^3 - 3x^2 + 4x + 12\right) dx$$

$$= \left[-\frac{x^4}{4} - \frac{3x^3}{3} + \frac{4x^2}{2} + 12x\right]_{-2}^{2}$$

$$= \left(-\frac{16}{4} - \frac{24}{3} + \frac{16}{2} + 24\right) - \left(-\frac{16}{4} + \frac{24}{3} + \frac{16}{2} - 24\right)$$

$$= \left(\cancel{-4} - 8 \cancel{+8} + 24 \cancel{+4} - 8 - 8 + 24\right)$$

$$= 48 - 16$$

$$= 32 \text{ units}^2$$

5.

(a) $3x^2 - 8x + 11$

Note Coefficient of x^2 must be 1 before completing the square

$= 3\left[\left(x^2 - \frac{8}{3}x + \frac{11}{3}\right)\right]$

$= 3\left[\boxed{x^2 - \frac{8}{3}x + \left(\frac{4}{3}\right)^2} - \left(-\frac{4}{3}\right)^2 + \frac{11}{3}\right]$

$\frac{1}{2} \times \frac{8}{3} = \frac{8}{6} = \frac{4}{3}$

$= 3\left[\left(x - \frac{4}{3}\right)^2 - \frac{16}{9} + \frac{11}{3}\right]$

$-\frac{4}{3} \times -\frac{4}{3} = \frac{16}{9}$

$= 3\left[\left(x - \frac{4}{3}\right)^2 - \frac{16}{9} + \frac{33}{9}\right]$

$= 3\left[\left(x - \frac{4}{3}\right)^2 + \frac{17}{9}\right]$

$= 3\left(x - \frac{4}{3}\right)^2 + \frac{17}{3}$

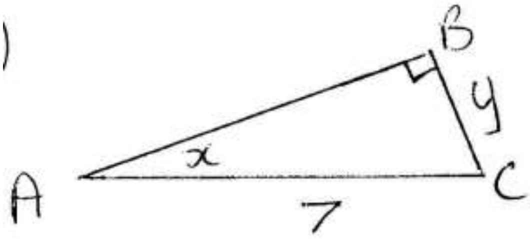
$\cancel{3} \times \frac{17}{\cancel{9}_3} = \frac{17}{3}$

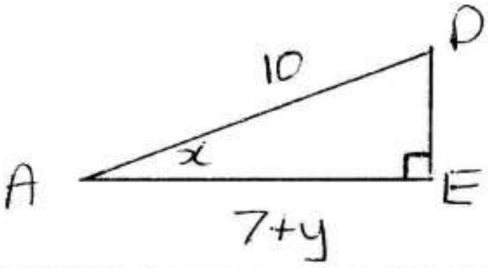
(b)

Coordinates of turning point is $\left(\frac{4}{3}, \frac{17}{3}\right)$ minimum

6.

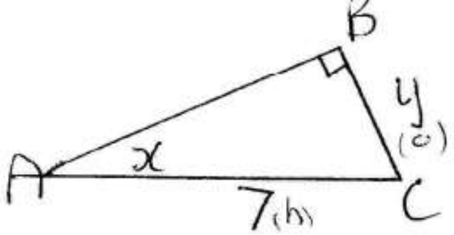
)





Since BC = CE have called them both y

$\therefore AE = 7 + y$



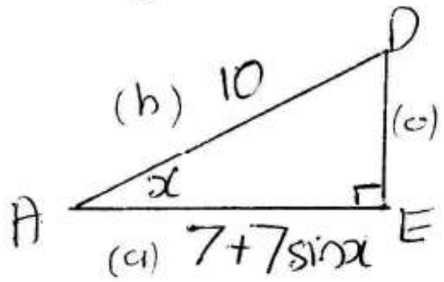
=7 using SOHCAHTOA to obtain expression for y

$$\sin x = \frac{opp}{hyp}$$

$$\sin x = \frac{y}{7}$$

$$y = 7\sin x \qquad \therefore AE = 7 + y$$

$$= 7 + 7\sin x.$$



using SOHCAHTOA to obtain expression for final answer.

$$\cos x = \frac{adj}{hyp}$$

$$\cos x = \frac{7+7\sin x}{10}$$

$$10\cos x = 7 + 7\sin x$$

Rearranging to match question answer

$$10\cos x - 7\sin x = 7$$

7.

Unit 3 – Wave Function!

(a) $10\cos x - 7\sin x = k\sin(x - a)$

$$\boxed{\begin{aligned} k\sin(x-a) &= k\sin x\cos a - k\sin a\cos x \\ k\cos a &= -7 \\ k\sin a &= -10 \end{aligned}}$$

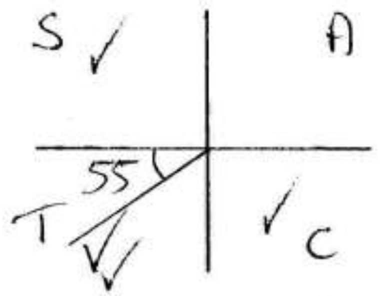
$k = \sqrt{a^2 + b^2}$

$= \sqrt{10^2 + (-7)^2}$

$= \sqrt{149}$

$\tan \alpha = \frac{a}{b}$

$= \frac{-10}{-7} = \frac{10}{7}$ acute angle $= 55°$



$a = 235°$

$\therefore \quad 10\cos x - 7\sin x = k\sin(x - a)$

$= \sqrt{149}(x - 235)$

$10\cos x - 7\sin x = \sqrt{149}\left(x - \frac{235 \times \pi}{180}\right)$

$= \sqrt{149}\left(x - \frac{47\pi}{36}\right)$

<u>Note</u> a fraction that is "awkward" should be written as a decimal ie replace $\pi = 3.14$

$10\cos x - 7\sin x = \sqrt{149}(x - 4.1)$

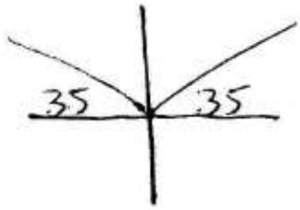
(b) Solve $10\cos x - 7\sin x = 7$

$$\sqrt{149}\,\sin(x-235) = 7 \qquad \left(\begin{array}{c}\text{SIMPLE}\\ \downarrow \\ \text{CAST}\end{array}\right)$$

$$\sin(x-235) = \frac{7}{\sqrt{149}}$$

$$\sin(x-235) = \frac{7}{\sqrt{149}}$$

acute angle $= 35°$



$x - 235 = 35$ $\qquad\qquad$ $x - 235 = 145$

$x = 270°$ $\qquad\qquad$ $x = 380$

(Remember you can add or subtract the period for more answers)

$$x = 380 - 360$$
$$= 20°.$$

$$\therefore \; x = 20° \text{ or } 270°$$

$$= \frac{20\times\pi}{180} \text{ or } \frac{270\times\pi}{180}$$

$$= \frac{\pi}{9} \text{ or } \frac{3\pi}{2}$$

8.

(a) $C_0 = 200$ and $B_0 = 0$

After 1 day

$C_1 = 0.1B_0 + 100 = 0.1 \times 0 + 100 = 100$

$B_1 = 2C_0 - 100 = 2 \times 200 - 100 = 300$

After 2 days

$C_2 = 0.1B_1 + 100 = 0.1 \times 300 + 100 = 130$

$B_2 = 2C_1 - 100 = 2 \times 100 - 100 = 100$

After 3 days

$C_3 = 0.1B_2 + 100 = 0.1 \times 100 + 100 = 110$

$B_3 = 2C_2 - 100 = 2 \times 130 - 100 = 160$

After 4 days

$C_4 = 0.1B_3 + 100 = 0.1 \times 160 + 100 = 116$

$B_4 = 2C_3 - 100 = 2 \times 110 - 100 = 120$

After 5 days

$C_5 = 0.1B_4 + 100 = 0.1 \times 120 + 100 = 112$

$B_5 = 2C_4 - 100 = 2 \times 116 - 100 = 132$

(b) C_n to B_{n-1}

Note m_{n+1} to m_n V_n to v_n C_{n+1} to C_n B_n to B_{n-1}	All mean next term on LHS Doesn't always have to be written as m_{n+1} to m_n

$\therefore$ If $C_{n+1} = 0.1B_n + 100$

then $\underline{\underline{C_n = 0.1B_{n-1} + 100}}$ is the same.

(c) B_{n+1} to B_{n-1} means
B_3 to B_1
or
B_4 to B_2
etc.

$$C_2 = 0.1B_1 + 100$$
$$B_3 = 2C_2 - 100$$
$$\therefore \ B_3 = 2(0.1B_1 + 100) - 100$$

(Sub in $C_2 = 0.1B_1 + 100$)

$$= 0.2B_1 + 200 - 100$$
$$B_3 = 0.2B_1 + 100$$
$$\therefore \ B_{n+1} = 0.2B_{n-1} + 100$$

(d) LIMIT since $-1 < 0.2 < 1$

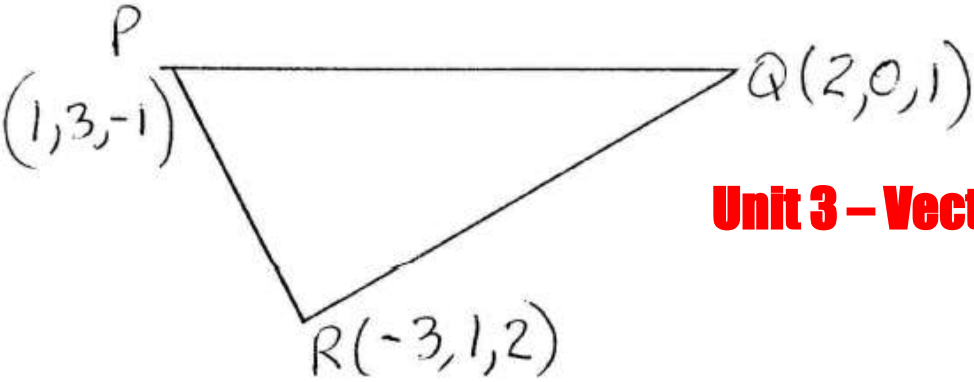
$$L_B = 0.2L + 100$$
$$0.8L_B = 100$$
$$L_B = \frac{100}{0.8} = \frac{1000}{8} = \underline{\underline{125}}$$

$$\therefore \ L_c = 0.1L_B + 100 = 0.1 \times 125 + 100$$
$$= \underline{\underline{112.5}}$$

$\therefore$ <u>125 Birds to 112caterpillars on any particular day.</u>

9.

(a)



Unit 3 – Vectors!

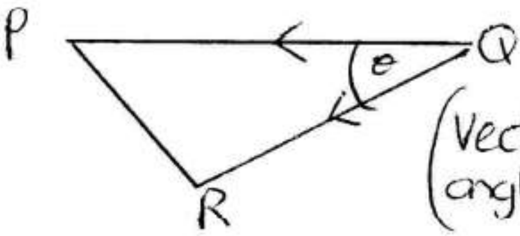
$$\overrightarrow{QP} = p - q = \begin{pmatrix}1\\3\\-1\end{pmatrix} - \begin{pmatrix}2\\0\\1\end{pmatrix} = \begin{pmatrix}-1\\3\\-2\end{pmatrix}$$

$$|\overrightarrow{QP}| = \sqrt{(-1)^2+(3)^2+(2)^2} = \underline{\underline{\sqrt{14}}}$$

$$\overrightarrow{QR} = r - q = \begin{pmatrix}-3\\1\\2\end{pmatrix} - \begin{pmatrix}2\\0\\1\end{pmatrix} = \begin{pmatrix}-5\\1\\1\end{pmatrix}$$

$$|\overrightarrow{QR}| = \sqrt{(-5)^2+(1)^2+(1)^2} = \underline{\underline{\sqrt{27}}}$$

(b)



(Vectors always away from angle being calculated)

$$\cos\theta = \frac{a \cdot b}{|a||b|}$$

$$= \frac{\overrightarrow{QP} \cdot \overrightarrow{QR}}{|\overrightarrow{QP}||\overrightarrow{QR}|}$$

$$\boxed{\overrightarrow{QP} \cdot \overrightarrow{QR} = \begin{pmatrix}-1\\3\\-2\end{pmatrix} \cdot \begin{pmatrix}-5\\1\\1\end{pmatrix} = 5 + 3 - 2 = \underline{\underline{6}}}$$

$$\therefore \cos\theta = \frac{6}{(\sqrt{14}\sqrt{27})} = \frac{6}{\sqrt{378}}$$

$$\therefore \theta = 72^\circ$$

10.

Unit 3 – Logs!

10 (a) $f(t) = 1 - e^{-0.0072t}$

For $t = 60$

$f(60) = 1 - e^{-0.0072 \times 60}$

$= 1 - e^{-0.432}$

$= 1 - 0.649209$

$= 0.3508$

$\therefore$ Since $f(t)$ is the fraction of the remaining then

Amount remaining $= 0.3508 \times 750$

$= 263$ micrograms. (to the nearest microgram)

(b) $0.8 = 1 - e^{-0.0072t}$

$e^{-0.0072t} = 0.2$

$\log_e e^{-0.0072t} = \log_e 0.2$

$\boxed{\log_e e = 1}$

$-0.0072t \log_e e = \log_e 0.2$

$-0.0072t = \log_e 0.2$

$t = \dfrac{\log_e 0.2}{-0.0072}$

$= \underline{223.5 \text{ years}}$