

- 1 B  
2 D  
3 B  
4 C  
5 C  
6 C  
7 D  
8 B  
9 C  
10 A  
11 D  
12 D  
13 B  
14 C  
15 B  
16 D  
17 B  
18 C  
19 A  
20 C

	A	B	C	D
1	□	■	□	□
2	□	□	□	■
3	□	■	□	□
4	□	□	■	□
5	□	□	■	□
6	□	□	■	□
7	□	□	□	■
8	□	■	□	□
9	□	□	■	□
10	■	□	□	□
11	□	□	□	■
12	□	□	□	■
13	□	■	□	□
14	□	□	■	□
15	□	■	□	□
16	□	□	□	■
17	□	■	□	□
18	□	□	■	□
19	■	□	□	□
20	□	□	■	□

	Give 1 mark for each •	Illustration(s) for awarding each mark
21(a)	ans: $Q(9, 7); (\sqrt{45}) \text{ or } 3\sqrt{5}$ (3 marks) <ul style="list-style-type: none"><li>•<sup>1</sup> states centre of <math>C_2</math></li><li>•<sup>2</sup> knows how to find radius</li><li>•<sup>3</sup> evaluates</li></ul>	<ul style="list-style-type: none"><li>•<sup>1</sup> <math>Q(9, 7)</math></li><li>•<sup>2</sup> <math>r^2 = 9^2 + 7^2 - 85</math></li><li>•<sup>3</sup> <math>r = \sqrt{45} \text{ or } 3\sqrt{5}</math></li></ul>
(b)	ans: proof (3 marks) <ul style="list-style-type: none"><li>•<sup>1</sup> finds distance between centres</li><li>•<sup>2</sup> finds total of 2 radii</li><li>•<sup>3</sup> conclusion</li></ul>	<ul style="list-style-type: none"><li>•<sup>1</sup> <math>PQ^2 = 8^2 + 4^2; PQ = \sqrt{80} = 4\sqrt{5}</math></li><li>•<sup>2</sup> <math>\sqrt{5} + 3\sqrt{5} = 4\sqrt{5}</math></li><li>•<sup>3</sup> distance between centres = sum of radii so circles touch at one point</li></ul>
22	ans: $a = 2$ (5 marks) <ul style="list-style-type: none"><li>•<sup>1</sup> prepares to integrate</li><li>•<sup>2</sup> integrates</li><li>•<sup>3</sup> subs and equates to 8</li><li>•<sup>4</sup> factorises (uses synthetic division)</li><li>•<sup>5</sup> realises only solution is 2</li></ul>	<ul style="list-style-type: none"><li>•<sup>1</sup> <math>\int_0^a 16 - 24x + 9x^2 \, dx</math></li><li>•<sup>2</sup> <math>[16x - 12x^2 + 3x^3]_0^a</math></li><li>•<sup>3</sup> <math>16a - 12a^2 + 3a^3 = 8</math></li><li>•<sup>4</sup> <math>(a-2)(3a^2 - 6a + 4) = 0</math></li><li>•<sup>5</sup> <math>a = 2</math></li></ul>
23(a)	ans: $y = 4x - 9$ (4 marks) <ul style="list-style-type: none"><li>•<sup>1</sup> find coordinates of S</li><li>•<sup>2</sup> finds gradient of AB</li><li>•<sup>3</sup> knows to use parallel gradient</li><li>•<sup>4</sup> subs info into equation of straight line</li></ul>	<ul style="list-style-type: none"><li>•<sup>1</sup> <math>S(4, 7)</math></li><li>•<sup>2</sup> <math>m_{AB} = \frac{5+3}{-2+4} = 4</math></li><li>•<sup>3</sup> <math>m = 4</math></li><li>•<sup>4</sup> <math>y - 7 = 4(x - 4)</math></li></ul>
(b)	ans: $D(2, -1)$ (2 marks) <ul style="list-style-type: none"><li>•<sup>1</sup> evidence of 'stepping out' or other suitable method</li><li>•<sup>2</sup> answer</li></ul>	<ul style="list-style-type: none"><li>•<sup>1</sup> evidence of suitable strategy</li><li>•<sup>2</sup> <math>D(2, -1)</math></li></ul>

	<b>Give 1 mark for each •</b>	<b>Illustration(s) for awarding each mark</b>
24(a)	ans: proof • <sup>1</sup> finds $g(f(x))$ • <sup>2</sup> finds $h(g(f(x)))$ • <sup>3</sup> completes proof  (3 marks)	• <sup>1</sup> $g(f(x)) = (\sin x)^2 = \sin^2 x$ • <sup>2</sup> $h(g(f(x))) = 1 - 2 \sin^2 x$ • <sup>3</sup> $1 - 2 \sin^2 x = \cos 2x$
(b)	ans: $-\frac{\sqrt{3}}{2}$  (3 marks)  • <sup>1</sup> subs value into formula • <sup>2</sup> finds equivalent angle • <sup>3</sup> evaluates	• <sup>1</sup> $\cos 2(\frac{5\pi}{12}) = \cos \frac{5\pi}{6}$ • <sup>2</sup> $-\cos \frac{\pi}{3}$ • <sup>3</sup> $-\frac{\sqrt{3}}{2}$
25	ans: $k = \frac{1}{2}$  (7 marks)  • <sup>1</sup> knows to sub line into circle • <sup>2</sup> multiplies • <sup>3</sup> simplifies • <sup>4</sup> solves for $y$ • <sup>5</sup> subs to find $x$ • <sup>6</sup> subs point into line • <sup>7</sup> solves for $k$	• <sup>1</sup> $(3y+10)^2 + y^2 - 4(3y+10) - 8y - 20 = 0$ • <sup>2</sup> $9y^2 + 60y + 100 + y^2 - 12y - 40 - 8y - 20 = 0$ • <sup>3</sup> $10y^2 + 40y + 40 = 0$ • <sup>4</sup> $10(y^2 + 4y + 4) = 0; (y+2)^2 = 0; y = -2$ • <sup>5</sup> $x = 3(-2) + 10 = 4$ • <sup>6</sup> $-2 = 4k - 4$ • <sup>7</sup> $k = \frac{1}{2}$

Total: 70 marks

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	ans: proof (3 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> subs one function into the other</li> <li>•<sup>2</sup> multiplies inner bracket</li> <li>•<sup>3</sup> multiplies to answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>f(x-3) = (x-3-1)^2 = (x-4)^2</math></li> <li>•<sup>2</sup> <math>h(x) = [x^2 - 8x + 16]x^2</math></li> <li>•<sup>3</sup> <math>x^4 - 8x^3 + 16</math></li> </ul>
(b)	ans: A(2, 16) (5 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> knows to make <math>\frac{dy}{dx} = 0</math></li> <li>•<sup>2</sup> differentiates</li> <li>•<sup>3</sup> solves for <math>x</math></li> <li>•<sup>4</sup> chooses correct values &amp; subs to find <math>y</math></li> <li>•<sup>5</sup> states coordinates of A</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{dy}{dx} = 0</math></li> <li>•<sup>2</sup> <math>\frac{dy}{dx} = 4x^3 - 24x^2 + 32x = 0</math> at SP</li> <li>•<sup>3</sup> <math>4x(x-4)(x-2) = 0; x = 2, 4</math></li> <li>•<sup>4</sup> <math>y = (2)^4 - 8(2)^3 + 16(2)^2 = 16</math></li> <li>•<sup>5</sup> A(2, 16)</li> </ul>
2(a)	ans: $p = 0.5$ (4 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> gives expression for both limits</li> <li>•<sup>2</sup> equates limits</li> <li>•<sup>3</sup> starts to solve</li> <li>•<sup>4</sup> solves and discards</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>L = \frac{6}{1-p}; L = \frac{9}{1-p^2}</math></li> <li>•<sup>2</sup> <math>\frac{6}{1-p} = \frac{9}{1-p^2}</math></li> <li>•<sup>3</sup> <math>6 - 6p^2 = 9 - 9p; 6p^2 - 9p + 3 = 0</math></li> <li>•<sup>4</sup> <math>3(2p-1)(p-1) = 0; p = 0.5 \text{ or } p = 1</math></li> </ul>
(b)	ans: 22 (3 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> finds 1<sup>st</sup> term for one RR</li> <li>•<sup>2</sup> finds 1<sup>st</sup> term for other RR</li> <li>•<sup>3</sup> calculates difference in terms</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>U_1 = \frac{1}{2}(100) + 6 = 56</math></li> <li>•<sup>2</sup> <math>U_1 = (\frac{1}{2})^2(100) + 6 = 34</math></li> <li>•<sup>3</sup> <math>56 - 34 = 22</math></li> </ul>
3	ans: $0^\circ, 113.6^\circ, 246.4^\circ, 180^\circ$ (5 marks) <ul style="list-style-type: none"> <li>•<sup>1</sup> subs for <math>\sin 2x^\circ</math> and simplifies</li> <li>•<sup>2</sup> factorises</li> <li>•<sup>3</sup> solves for <math>\sin x^\circ</math> and <math>\cos x^\circ</math></li> <li>•<sup>4</sup> solutions from <math>\sin x^\circ</math></li> <li>•<sup>5</sup> solutions from <math>\cos x^\circ</math></li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>5(2\sin x^\circ \cos x^\circ) + 4\sin x^\circ = 0</math></li> <li>•<sup>2</sup> <math>10\sin x^\circ \cos x^\circ + 4\sin x^\circ = 0</math></li> <li>•<sup>3</sup> <math>2\sin x^\circ(5\cos x^\circ + 2) = 0</math></li> <li>•<sup>4</sup> <math>\sin x^\circ = 0 \text{ or } \cos x^\circ = -\frac{2}{5}</math></li> <li>•<sup>5</sup> <math>x = 0^\circ, 180^\circ</math></li> <li>•<sup>5</sup> <math>x = 113.6^\circ, 246.4^\circ</math></li> </ul>

	<b>Give 1 mark for each •</b>	<b>Illustration(s) for awarding each mark</b>
<b>4(a)</b>	<b>ans:</b> $P(-2, 0)$ <b>(3 marks)</b>	<ul style="list-style-type: none"> <li>•<sup>1</sup> equates function to 0</li> <li>•<sup>2</sup> solves using suitable strategy</li> <li>•<sup>3</sup> states coordinates of P</li> </ul>
<b>(b)</b>	<b>ans:</b> <b>4 square units</b> <b>(4 marks)</b>	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows how to find area</li> <li>•<sup>2</sup> integrates</li> <li>•<sup>3</sup> subs values</li> <li>•<sup>4</sup> evaluates</li> </ul> <p> <math>\bullet^1 \int_{-2}^0 x^3 + 6x^2 + 12x + 8 \, dx</math>  <math>\bullet^2 \left[ \frac{x^4}{4} + 2x^3 + 6x^2 + 8x \right]_{-2}^0</math>  <math>\bullet^3 0 - \left( \frac{(-2)^4}{4} + 2(-2)^3 + 6(-2)^2 + 8(-2) \right)</math>  <math>\bullet^4 4 \text{ square units}</math> </p>
<b>5(a)</b>	<b>ans:</b> $k = 1$ <b>(3 marks)</b>	<ul style="list-style-type: none"> <li>•<sup>1</sup> finds gradient of CP</li> <li>•<sup>2</sup> equates <math>m_{CP}</math> to expression for <math>m_{CP}</math></li> <li>•<sup>3</sup> solves</li> </ul>
<b>(b)</b>	<b>ans:</b> $(x - 2)^2 + (y - 1)^2 = 18$ <b>(3 marks)</b>	<ul style="list-style-type: none"> <li>•<sup>1</sup> finds midpoint of CP</li> <li>•<sup>2</sup> finds radius (length of CQ)</li> <li>•<sup>3</sup> subs into general equation of circle</li> </ul> <p> <math>\bullet^1 Q(-1, 4)</math>  <math>\bullet^2 r^2 = 3^2 + 3^2 = 18</math>  <math>\bullet^3 (x - 2)^2 + (y - 1)^2 = 18</math> </p>

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6(a)	<b>ans: proof</b> (3 marks)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> gives expression for length and breadth</li> <li>•<sup>2</sup> subs into formula and starts to simplify</li> <li>•<sup>3</sup> completes simplification to answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>(30 - 2x)</math></li> <li>•<sup>2</sup> <math>x(30 - 2x)^2</math></li> <li>•<sup>3</sup> <math>x(900 - 120x + 4x^2)</math></li> </ul>
(b)	<b>ans: <math>x = 5</math></b> (5 marks)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows to make derivative = 0</li> <li>•<sup>2</sup> takes derivative</li> <li>•<sup>3</sup> factorises and solves</li> <li>•<sup>4</sup> discards</li> <li>•<sup>5</sup> justifies answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>V'(x) = 0</math></li> <li>•<sup>2</sup> <math>12x^2 - 240x + 900 = 0</math></li> <li>•<sup>3</sup> <math>12(x - 5)(x - 15) = 0</math></li> <li>•<sup>4</sup> <math>x = 5</math></li> <li>•<sup>5</sup> nature table or 2<sup>nd</sup> derivative</li> </ul>
(c)	<b>ans: 2 litres</b> (1 mark)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> calculates volume</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>20 \times 20 \times 5 = 2000\text{cm}^3 = 2 \text{ litres}</math></li> </ul>
7	<b>ans: proof</b> (5 marks)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> finds length of AB and <math>\cos p</math></li> <li>•<sup>2</sup> realises angle DBC = <math>2p</math></li> <li>•<sup>3</sup> replaces double angle</li> <li>•<sup>4</sup> subs value for <math>\cos p</math></li> <li>•<sup>5</sup> evaluates to answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>AB = \sqrt{10}; \cos p = \frac{3}{\sqrt{10}}</math></li> <li>•<sup>2</sup> <math>\cos DBC = \cos 2p</math></li> <li>•<sup>3</sup> <math>\cos 2p = 2\cos^2 p - 1</math> [or alternative]</li> <li>•<sup>4</sup> <math>2\left(\frac{3}{\sqrt{10}}\right)^2 - 1</math></li> <li>•<sup>5</sup> <math>2\left(\frac{9}{10}\right) - 1; \frac{18}{10} - 1 = \frac{8}{10} = \frac{4}{5}</math></li> </ul>

	<b>Give 1 mark for each •</b>	<b>Illustration(s) for awarding each mark</b>
8(a)	<b>ans:</b> proof (3 marks)	
	<ul style="list-style-type: none"> <li>•<sup>1</sup> states expression for both distances</li> <li>•<sup>2</sup> equates</li> <li>•<sup>3</sup> rearranges to answer</li> </ul>	<ul style="list-style-type: none"> <li>•<sup>1</sup> <math>\frac{3}{k-x}</math> and <math>\frac{4x}{k}</math></li> <li>•<sup>2</sup> <math>\frac{3}{k-x} = \frac{4x}{k}</math></li> <li>•<sup>3</sup> <math>3k = 4x(k-x); 3k = 4xk - 4x^2 \dots\dots</math></li> </ul>
(b)	<b>ans:</b> $k = 3$ (3 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> knows discriminant = 0 for equal roots</li> <li>•<sup>2</sup> finds discriminant</li> <li>•<sup>3</sup> solves and discards</li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>b^2 - 4ac = 0</math> for equal roots</li> <li>•<sup>2</sup> <math>b^2 - 4ac = (-4k)^2 - 4.4.3k = 0; 16k^2 - 48k = 0</math></li> <li>•<sup>3</sup> <math>16k(k-3) = 0; k = 3</math></li> </ul>
(c)	<b>ans:</b> $x = \frac{3}{2}$ (2 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> subs value for <math>k</math> and rewrites expression</li> <li>•<sup>2</sup> factorises and solves</li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>4x^2 - 12x + 9 = 0</math></li> <li>•<sup>2</sup> <math>(2x-3)^2 = 0; x = \frac{3}{2}</math></li> </ul>
9	<b>ans:</b> $p = 32$ (5 marks)	<ul style="list-style-type: none"> <li>•<sup>1</sup> differentiates</li> <li>•<sup>2</sup> subs value to find gradient</li> <li>•<sup>3</sup> finds gradient of given line</li> <li>•<sup>4</sup> equates gradients</li> <li>•<sup>5</sup> solves</li> </ul> <ul style="list-style-type: none"> <li>•<sup>1</sup> <math>y = x + px^{-\frac{1}{2}}; \frac{dy}{dx} = 1 - \frac{1}{2}px^{-\frac{3}{2}} = 1 - \frac{p}{2x^{\frac{3}{2}}}</math></li> <li>•<sup>2</sup> <math>1 - \frac{p}{2(4)^{\frac{3}{2}}} = 1 - \frac{p}{16}</math></li> <li>•<sup>3</sup> <math>x + y = 10; m = -1</math></li> <li>•<sup>4</sup> <math>1 - \frac{p}{16} = -1</math></li> <li>•<sup>5</sup> <math>-\frac{p}{16} = -2; p = 32</math></li> </ul>
		<div style="border: 1px solid black; padding: 5px; display: inline-block;">Total: 60 marks</div>