St Peter the Apostle High

Mathematics Dept.

Higher Prelim Revision 6

Paper 2 - Calculator

Time allowed - 1 hour 30 minutes

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre (-g, -f) and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

$\sin A \pm B = \sin A \cos B \pm \cos A \sin B$
$\cos \mathbf{A} \pm B = \cos A \cos B \mp \sin A \sin B$
$\sin 2A = 2\sin A \cos A$
$\cos 2A = \cos^2 A - \sin^2 A$
$= 2\cos^2 A - 1$
$= 1 - 2\sin^2 A$

ALL questions should be attempted

1. (a) A function *f*, defined on a suitable domain, is given as $f(x) = (x-1)^2$. A second function *h* is such that $h(x) = [f(x-3)]x^2$. Show clearly that *h* can be written in the form $h(x) = x^4 - 8x^3 + 16x^2$.

(b) Part of the graph of y = h(x) is shown below.



Find the coordinates of point A.

2. Two unique sequences are defined by the following recurrence relations

 $U_{n+1} = pU_n + 6$ and $U_{n+1} = p^2 U_n + 9$, where p is a constant.

(b) For both sequences $U_0 = 100$, find the difference between their first terms.

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4. Part of the graph of $y = x^3 + 6x^2 + 12x + 8$ is shown in the diagram.



- (a) Find the coordinates of P.
- (b) Hence calculate the shaded area.

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6. From a square sheet of metal of side 30 centimetres, equal squares of side *x* centimetres are removed from each corner.

The sides are then folded up and sealed to form an open cuboid.



(a) Show that the volume of this resulting cuboid is given by

$$V(x) = 4x^3 - 120x^2 + 900x.$$
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- (b) If the cuboid is to have **maximum** possible volume, what size of square should be removed from each corner?
- (c) How many litres of water would this particular cuboid hold?

8. A designer is testing two model racing cars along a straight track.

Each car completes a single run and the following information is recorded.

	Speed	Distance
Car A	k - x	3
Car B	k	4x



(a) Given that both cars completed the run in **exactly the same time**, show clearly that the following equation can be constructed.

$$4x^2 - 4kx + 3k = 0$$
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- (b) Find the value of the constant k if the equation $4x^2 4kx + 3k = 0$ has equal roots and k > 0.
- (c) Hence find *x* when *k* takes this value.

9. The tangent to the curve $y = x + \frac{p}{\sqrt{x}}$, at the point where x = 4, is parallel to the line with equation x + y = 10.

Find the value of *p*.

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[END OF QUESTION PAPER]