St Peter the Apostle High

Mathematics Dept.

Higher Prelim Revision 5

Paper 2 - Calculator

Time allowed - 1 hour 30 minutes

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre (-g, -f) and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Trigonometric formulae: $sin(A \pm B) = sin A cos B \pm cos A sin B$ $cos(A \pm B) = cos A cos B \mp sin A sin B$ sin2A = 2sin A cos A $cos2A = cos^{2} A - sin^{2} A$ $= 2 cos^{2} A - 1$ $= 1 - 2 sin^{2} A$

ALL questions should be attempted



5. A company making commercial "glow sticks" have devised a method to test the brightness and consistency of the glow given off.

All glow sticks depend on a chemical process known as chemiluminesence to produce their light. Once a glow stick has been illuminated (by mixing two chemicals together) the brightness of the glow diminishes over a period of time.

When one of their glow sticks is ignited the initial brightness is rated at 200 gu (glow units).



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(a)	During any 1 hour period the glow light is known to lose 8% of its brightness at the beginning of the period.	
	Calculate the brightness remaining, in gu's, after a period of 4 hours.	2
(b)	At the end of each 4 hour period, the glow light is automatically passed through a tube which has an internal temperature of $-40^{\circ}C$. This lowering of the temperature of the glow light has the effect of allowing it to regain some of its lost brightness. A single pass through this refrigerated tube allows the glow stick to regain 32 glow units.	
	The 4 hour cycle described above is now left to run uniterrupted for a total of 16 hours.	
	By considering an appropriate recurrence relation, calculate the brightness remaining, in <i>gu</i> 's, after this 16 hour period has been completed. Your answer must be accompanied with the appropriate working.	3
(c)	If this cycle was left to run over a very long period of time would the brightness of the glow stick ever drop to below half of its initial brightness? Explain your answer.	
	Your answer and explanation must be accompanied with the appropriate working.	3
(a)	If $k = \frac{(x-1)^2}{x^2+4}$, where <i>k</i> is a real number, show clearly that	
	$(k-1)x^2 + 2x + (4k-1) = 0.$	3

(b) Hence find the value of k given that the equation $(k-1)x^2 + 2x + (4k-1) = 0$ has equal roots and k > 0.

6.

The floor plan of a rectangular greenhouse is shown below. All dimensions are in metres.The gardener places a rectangular wooden storage shed, of width *x* metres, in one corner.



(a) Given that the **area of the shed** is 3 square metres, show clearly that the area of greenhouse floor remaining, *A* square metres, is given in terms of *x* as

$$A(x) = 12 + 4x + \frac{9}{x} .$$
 3

(b) Hence find the value of *x* which **minimises** the area of the greenhouse floor remaining, **justifying your answer**.

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[END OF QUESTION PAPER]