Paper 1

1	D
2	В
3	С
4	С
5	B
6	D
7	A
8	С
9	D
10	В
11	В
12	С
13	D
14	Α
15	D
16	С
17	С
18	D
19	В

20

Α

	Α	В	С	D
1				-
2				
3			-	
4				
5				
6				
7				
8				
9				-
10				
11				
12				
13				-
14				
15				
16				
17				
18				
19				
20				

Higher Prelim Revision 4

Paper 1

	Give 1 mark for each •	Illustration(s) for awarding each mark
21(a)	<ul> <li>ans: proof (3 marks)</li> <li>•<sup>1</sup> finds expressions for 2 areas</li> <li>•<sup>2</sup> adds 4 to area of triangle and equates</li> <li>•<sup>3</sup> reorganises to given form</li> </ul>	• <sup>1</sup> $A_{rect} = x(2k-2); A_{tri} = x(x+k)$ • <sup>2</sup> $x(x+k)+4 = x(2k-2)$ • <sup>3</sup> $x^2 + (2-k)x + 4 = 0$
(b)	ans: $k = 6$ (3 marks)	
	<ul> <li><sup>1</sup> knows condition for equal roots</li> <li><sup>2</sup> substitutes values</li> <li><sup>3</sup> solves and discards</li> </ul>	• <sup>1</sup> $b^2 - 4ac = 0$ [stated or implied] • <sup>2</sup> $(2-k)^2 - 4 \times 1 \times 4 = 0$ • <sup>3</sup> $(k+2)(k-6) = 0; k = -2$ or 6; $k = 6$
(c)	ans: $x = 2$ ; 20cm <sup>2</sup> ; 16cm <sup>2</sup> (3 marks)	
	<ul> <li><sup>1</sup> substitutes value of k to form quadratic</li> <li><sup>2</sup> solves to x</li> <li><sup>3</sup> finds areas</li> </ul>	• $x^{2} - 4x + 4 = 0$ • $(x - 2)^{2} = 0; x = 2$ • $A_{rect} = 20cm^{2}; A_{tri} = 16cm^{2}$
22(a)	ans: $3y + x = -30$ (2 marks)	
	<ul> <li>identifies required gradient</li> <li>substitutes into general equation</li> </ul>	• $m_{CB} = -\frac{1}{3}$ • $y + 11 = -\frac{1}{3}(x-3)$ [or equivalent]
(b)	ans: D(-3,-9) (3 marks)	
	<ul> <li><sup>1</sup> knows to use systems of equations</li> <li><sup>2</sup> finds value for x</li> <li><sup>3</sup> finds value for y and states coordinates</li> </ul>	• vidence • $x = -3$ • $y = -9; (-3,-9)$
(c)	ans: C(-9,-7) (1 mark)	
	• <sup>1</sup> states coordinates of C	• $^{1}$ C(-9,-7)
( <b>d</b> )	ans: $(x+3)^2 + (y-1)^2 = 100$ (4 marks)	
	<ul> <li>identifies diameter</li> <li>finds centre</li> <li>finds radius or r<sup>2</sup></li> <li>subs into general equation</li> </ul>	• <sup>1</sup> AC is diameter [ $\angle$ ADC is right-angled] • <sup>2</sup> midpoint of AC is (-3,1) • <sup>3</sup> $r = 10$ or $r^2 = 100$ • <sup>4</sup> $(x+3)^2 + (y-1)^2 = 100$

	Give 1 mark for each •	Illustration(s) for awarding each mark
23(a)	ans: $(x-4)^2 - 15$ ; $p = -4$ , $q = -15$ (4 marks) • <sup>1</sup> finds derivative • <sup>2</sup> starts to complete square • <sup>3</sup> completes • <sup>4</sup> states values of $p$ and $q$	• $f(x) = x^2 - 8x + 1$ • $(x - 4)^2 \dots -15$ • $p = -4, q = -15$
(b)	ans: -15 when $x = 4$ (2 marks) • <sup>1</sup> states minimum rate of change • <sup>2</sup> states value of $x$	• <sup>1</sup> rate of change is -15 • <sup>2</sup> $x = 4$
24	ans: $\frac{2\pi}{3}$ , 0 (5 marks) • <sup>1</sup> collects terms to LHS and equates to 0 • <sup>2</sup> factorises quadratic • <sup>3</sup> finds values for cos <i>a</i>	• $2\cos^{2} a - \cos a - 1 = 0$ • $(2\cos a + 1)(\cos a - 1) = 0$ • $\cos a = -\frac{1}{2}$ or $\cos a = 1$
	<ul> <li><sup>4</sup> finds one value for a</li> <li><sup>5</sup> finds second value for a</li> </ul>	
		Total: 30 marks

Higher Prelim Revision 4

Paper 2

Marking Scheme

	Give 1 mark for each •	Illustration(s) for awarding each mark
1(a)	ans: $2y - x = -2$ (3 marks)•1 finds midpoint of BC•2 establishes gradient of AM•3 substitutes in general equation	• midpoint BC: (10,4) • $m_{AM} = \frac{4+2}{10+2} = \frac{1}{2}$ • $y - 4 = \frac{1}{2}(x - 10)$
(b)	ans: $D(4,1)$ (3 marks)•1realising $y = 1$ •2substitutes into equation•3states coordinates of D	• <sup>1</sup> $y = 1$ • <sup>2</sup> $2(1) - x = -2; x = 4$ • <sup>3</sup> D(4,1)
(c)	<ul> <li>ans: proof (3 marks)</li> <li>•<sup>1</sup> finds gradient of BD</li> <li>•<sup>2</sup> knows condition for perp. lines</li> <li>•<sup>3</sup> makes statement re perpendicular</li> </ul>	• $m_{BD} = -2$ • $m_1 \times m_2 = -1$ [stated or implied] • $\frac{1}{2} \times -2 = -1$ so AM and BD are perp.
2(a) (b)	ans: $P(1, -\frac{25}{2})$ (4 marks)•1knows to take derivative and equate to 0•2takes derivative•3solves to find x - coordinate•4substitutes to find y - coordinateans: $Q(6,0)$ (3 marks)•1knows to make $y = 0$ •2uses synthetic division to find x•3states coordinates of Q	• $\frac{dy}{dx} = 0$ • $\frac{dy}{dx} = 0$ • $\frac{dy}{dx} = 15x + 12 = 0$ • $\frac{dy}{dx} = 1 \text{ [or 4]}$ • $\frac{dy}{dx} = 1^3 - \frac{15}{2}(1) + 12(1) - 18 = -\frac{25}{2}$ • $\frac{dy}{dx} = 1^3 - \frac{15}{2}(1) + 12(1) - 18 = -\frac{25}{2}$ • $\frac{dy}{dx} = -\frac{15}{2}(1) + 12(1) - 18 = -\frac{25}{2}$ • $\frac{dy}{dx} = -\frac{25}{2}$ • $\frac{dy}{dx} = -\frac{15}{2}(1) + 12(1) - 18 = -\frac{25}{2}$

	Give 1 mark for each •	Illustration(s) for awarding each mark	
3(a)	ans: $32.6$ gigatonnes(3 marks) $\bullet^1$ correct multiplier $\bullet^2$ completes calculation $\bullet^3$ calculation and correct rounding	• $^{1}$ 0.96 • $^{2}$ 0.96 <sup>5</sup> × 40 • $^{3}$ 32.6 gigatonnes	
(b)	ans: 31 gigatonnes (3 marks)		
	<ul> <li><sup>1</sup> sets up recurrence relation</li> <li><sup>2</sup> knows to calculate 3 figures</li> <li><sup>3</sup> final answer</li> </ul>	• $U_{n+1} = 0.96^5 U_n + 3.8$ • $1^{\text{st}}$ year: 36.4; $2^{\text{nd}}$ year: 33.4795 • $3^{\text{rd}}$ year: 31 gigatonnes	
(c)	ans: upper 20.6; lower 16.8 (3 marks)		
	<ul> <li><sup>1</sup> knows limit exists</li> <li><sup>2</sup> finds upper limit</li> <li><sup>3</sup> finds lower limit</li> </ul>	• limit exists since $-1 < 0.96^5 < 1$ • $L = \frac{3 \cdot 8}{1 - (0.96)^5} = 20.6$ • $20.6 - 3.8 = 16.8$	
<b>4</b> (a)	ans: $a = -2$ (2 marks)		
	<ul> <li>finds expression for f(g(-2))</li> <li>equates to -1 and solves for a</li> </ul>	• <sup>1</sup> $f(g(-2)) = f(-1) = 1 + a$ • <sup>2</sup> $a = -2$	
(b)	ans: $x = -2, 0, 2$ (5 marks)		
	<ul> <li><sup>1</sup> substitutes</li> <li><sup>2</sup> simplifies</li> <li><sup>3</sup> equates to 2</li> <li><sup>4</sup> factorises</li> <li><sup>5</sup> solves for x</li> </ul>	• <sup>1</sup> $f(f(x)) = (x^2 - 2)^2 - 2$ • <sup>2</sup> $x^4 - 4x^2 + 2$ • <sup>3</sup> $x^4 - 4x^2 + 2 = 2; x^4 - 4x^2 = 0$ • <sup>4</sup> $x^2(x^2 - 4) = 0$ • <sup>5</sup> $x = -2, 0, 2$	
5(a)	ans: $x = 1$ (2 marks)		
	• <sup>1</sup> realises $y = 0$ ; equates to 0 • <sup>2</sup> solves for x	• <sup>1</sup> $3x^2 - 6x + 3 = 0$ • <sup>2</sup> $3(x - 1)^2 = 0; x = 1$	
(b)	ans: $b = 2$ (5 marks)		
	<ul> <li>integrates expression</li> <li>substitutes values</li> <li>simplifies, equates to 1, rearranges</li> <li>uses synthetic division to solve</li> <li>realises one solution: discards b<sup>2</sup> - b + 1</li> </ul>	• <sup>1</sup> $[x^3 - 3x^2 + 3x]_1^b$ • <sup>2</sup> $(b^3 - 3b^2 + 3b) - (1 - 3 + 3)$ • <sup>3</sup> $b^3 - 3b^2 + 3b - 2 = 0$ • <sup>4</sup> $2 \begin{bmatrix} 1 & -3 & 3 & -2 \\ 2 & -2 & 2 \\ 1 & -1 & 1 & 0 \end{bmatrix}$ • <sup>5</sup> $b = 2$	
	- realises one solution, discutus $b = b + 1$	- 0 - 2	

	Give 1 mark for each •	Illustration(s) for awarding each mark
5(c)	ans: $y = 6x - 9$ (4 marks)	
	<ul> <li><sup>1</sup> finds 'c'</li> <li><sup>2</sup> knows to differentiate</li> <li><sup>3</sup> substitutes to find gradient</li> <li><sup>4</sup> substitutes into general equation</li> </ul>	• when $x = 2$ , $y = 3$ ; Q(2,3) • $\frac{dy}{dx} = 6x - 6$ • when $x = 2$ ; $\frac{dy}{dx} = 6$ • $y - 3 = 6(x - 2)$
6(a)	ans: (-2,1) (3 marks)	
	<ul> <li><sup>1</sup> substitutes eq.of line in eq. of circle</li> <li><sup>2</sup> simplifies and solves for x</li> <li><sup>3</sup> substitutes to find y</li> </ul>	• $x^{2} + (2x+5)^{2} - 4x + 2(2x+5) - 15 = 0$ • $5(x+2)^{2} = 0; x = -2$ • $y = 2(-2) + 5; y = 1$
(b)	ans: $(x+6)^2 + (y-3)^2 = 20$ (3 marks)	
	<ul> <li>•<sup>1</sup> establishes coordinates of B</li> <li>•<sup>2</sup> finds r<sup>2</sup></li> <li>•<sup>3</sup> substitutes into general circle equation</li> </ul>	• <sup>1</sup> B(-6,3) • <sup>2</sup> $r^2 = 20$ • <sup>3</sup> $(x+6)^2 + (y-3)^2 = 20$
7(a)	ans: $AE = \sqrt{5}$ ; $BC = 4$ (2 marks)	
	<ul> <li><sup>1</sup> knows to use Pythagoras'</li> <li><sup>2</sup> finds two lengths</li> </ul>	• evidence • AE = $\sqrt{5}$ ; BC = $\sqrt{(25-9)} = 4$
(b)	ans: proof (4 marks)	
	<ul> <li><sup>1</sup> knows to expand</li> <li><sup>2</sup> finds expression for ratios</li> <li><sup>3</sup> subs and starts to simplify</li> <li><sup>4</sup> completes simplification to answer</li> </ul>	• $\cos(x + y) = \cos x \cos y - \sin x \sin y$ • $\cos x = \frac{4}{5}; \cos y = \frac{2}{\sqrt{5}}; \sin x = \frac{3}{5}; \sin y = \frac{1}{\sqrt{5}}$ • $(\frac{4}{5} \times \frac{2}{\sqrt{5}}) - (\frac{3}{5} \times \frac{1}{\sqrt{5}}); \frac{5}{5\sqrt{5}}; \frac{1}{\sqrt{5}}$ • $\frac{1}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{5}$
8(a)	ans: $m = 2400 \text{ml}$ (4 marks)	
	<ul> <li><sup>1</sup> knows to differentiate and equate to 0</li> <li><sup>2</sup> differentiates</li> <li><sup>3</sup> solves for <i>x</i></li> <li><sup>4</sup> justifies maximum</li> </ul>	• $H'(m) = 0$ • $4 - \frac{1}{600}m = 0$ • $m = 2400$ • $4$ table of values; second derivative
(b)	<ul> <li>ans: 4800 teet (1 mark)</li> <li>•<sup>1</sup> knows to sub into function and evaluate</li> </ul>	• <sup>1</sup> 4(2400) $-\frac{(2400)^2}{1200} = 4800$ feet

Total: 60 marks